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A Degenerate Parton Gas Model for the Proton

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The quark-parton language for explaining the deep inelastic phenomena is generally accepted by now.[1] Apart from the 'valence' quarks (u, u and d) needed for taking account of the observed quantum numbers of the proton, one needs a large or even infinite number of partons-sea-quarks.[2] Several experiments tend to show[3] that the masses of the valence quarks, u and d, can be taken to be rather small, of the order of a few MeV. In fact, some of the unified models for strong, electromagnetic and weak interactions suggest identification of quarks with leptons.[4] Unlike the leptons, however, the quarks-partons presumably interact by exchanging gluons, thus generating the strong interactions. The gluons are assumed to be massless vector bosons, their coupling strength depending on the energy-momentum of the partons. Deep inelastic phenomena are then explained by using 'asymptotic freedom', the coupling becoming weaker in the limit of large momenta (small distances). The non-observance of free quarks is expressed as 'infra-red slavery', the interaction becoming strong at low momenta (large distances), thus giving rise to quark confinement.[5]

The question then arises as to the large observed mass of the proton (compared to that of the electron), when the constituents themselves are small mass. Moreover, a cloud of partons undergoing attractive interactions via gluonic exchanges would collapse. In analogy with the classical Lorentz-Poincare-Abraham model for the electron, one would need a kind of pressure to balance this collapsing force and thus make the proton stable, as observed.

We present here a simple model for the proton wherein the proton is described as a miniature 'white dwarf', the Fermi pressure of the parton gas balancing the attractive gluonic interaction. Itoh[6] has considered a model for the quark stars where the hydrostatic equilibrium conditions are imposed on a degenerate quark gas. In his case, it was the gravitational interaction balancing the pressure of the degenerate gas. To us, it seems more reasonable to take account of the strong interaction due to gluonic exchange and neglect the gravitational interaction between the light partons.

The light partons (mass of the order of a few MeV at most), confined in a region of the size of the proton, can be described as constituting a degenerate ultrarelativistic Fermi gas. The expressions for the pressure and the energy of such a gas are well-known to be given by:

$$P=-\frac{1}{3}\,\varepsilon=-\frac{1}{3}\left(\frac{\partial E}{\partial V}\right), \tag{1}$$

and

$$E=\frac{8\pi V}{h^3}\int^{E_F/c} p^2(pc)\,dp\,, \tag{2}$$

where ε is the energy density and V is the volume. E_F is obtained, as usual by integrating over the phase space. Essentially similar equations of state for the quark gas have been used in the literature.[7],[8] One thus gets, for the pressure of such a gas, the relation

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$$P=\frac{3}{16\pi}\left(\frac{9}{32\pi^2}\right)^{1/3}\frac{N^{4/3}\hbar c}{R^4}\,, \tag{3}$$

where N is the number of the partons in the proton and R is the radius of the spherical proton. The gluonic interaction can be approximated by a $1/r$ Coulomb like attractive potential,[9] at least for small distances. Thus we can write for the energy density (pressure) of the gluonic interaction for a gas of N partons:

$$\frac{g^2}{16\pi R^4}N^2\,, \tag{4}$$

where g denotes the coupling constant.

Equating (3) and (4) for a stable proton, we get

$$N=3^{3/2}\left(\frac{9}{32\pi^2}\right)^{1/2}\frac{1}{(g^2/c\hbar)^{3/2}}\,. \tag{5}$$

This result is independent of R, the size of the proton. In the spirit of the unified strong-electromagnetic-weak interaction models, $g^2/c\hbar$ should asymptotically tend to the value α, where $\alpha=(e^2/c\hbar)$ is the electromagnetic fine structure constant. Thus, we get

$$N=3^{3/2}\left(\frac{9}{32\pi^2}\right)^{1/2}\frac{1}{\alpha^{3/2}}\simeq 1700\,.$$

Therefore, one would need about 1700 partons for a stable proton. This is very close to the observed proton to the electron mass ratio, thus suggesting the equality of the parton mass with the electron mass. This is consistent with the idea of quark-lepton identification.

It is interesting to note that the mass of the proton turns out to be approximately $\alpha^{-3/2}$ times the mass of the electron in our model. In the case of the usual white dwarf stars,[10] where the gravitational interaction balances the degenerate Fermi gas pressure, the equilibrium Chandrasekhar mass is of the order of $\alpha_G{}^{-3/2}$ times the proton mass, where α_G is the gravitational fine structure constant $(=Gm^2/c\hbar\simeq 10^{-39}$, m being the mass of the proton).

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