Strong gravity, black holes, and hadrons

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Analogies between the properties of black holes (in the framework of strong gravity) and those of elementary particles are discussed especially in connection with recent works on black holes with gauge charges and black-hole thermodynamics.

Recent renewal[1-6] of interest in general relativity playing some role in elementary-particle physics has drawn attention to certain striking resemblances between the properties of black holes and those of elementary particles. For instance, black holes are characterized by only a few observable parameters such as mass, angular momentum, and charge. These are the measurable parameters for an elementary particle where these quantities occur in discrete or quantized units. One could picture particles as quantum black holes of the strong gravity field (i.e., mediated by massive spin-2) particles as was done in Refs. 2, 3, 6, 7, and 8. Moreover, it is known that a charged rotating black hole, a Kerr-Newman black hole, has a gyromagnetic ratio of 2, the same value as that for an elementary particle. Although they have a magnetic moment, charged black holes with angular momentum do not have an electric dipole moment, which is also true for elementary particles. In a recent paper, Tennakone[8] has pictured the proton to be a black-hole singularity of the Reissner-Nordström metric in the strong gravitational field, assuming that the usual results of general relativity are applicable in the case of strong gravity.

If the structure of space-time in the immediate vicinity of hadrons is presumed to be determined by strong gravity, it is natural to replace the Newtonian constant G_N by the strong gravitational coupling constant G_f, the dimensionless constant then being of the same magnitude as the strong-interaction dimensionless constant. Now the Einstein field equations $G_{\mu\nu}=\kappa T_{\mu\nu}$ relate a geometrical invariant quantity (i.e., the Einstein tensor $G_{\mu\nu}$) on the left-hand side to an invariant physical quantity (i.e., the conserved energy-momentum tensor $T_{\mu\nu}$) on the right-hand side through a proportionality (coupling) constant κ. It must be emphasized that the derivation of the equation places no restriction whatsoever on the numerical value of the constant κ. For instance, in the standard derivation of the field equation from an action principle with the Lagrangian density $\mathcal{L}=\kappa^{-1}R\sqrt{-g}+\mathcal{L}_m$ [where R is the curvature scalar and $g=\det(g_{\mu\nu})$, $\mathcal{L}_m=$ matter Lagrangian density], κ is a factor of dimensions $\mathrm{g^{-1}\,cm^{-1}\,sec^2}$ whose numerical value is entirely undetermined at this stage. It is only when one uses the field equations as the basis for a relativistic theory of gravitation that one relates to the Newtonian constant G_N. Since Einstein used his field equations to describe a theory of gravitation, he chose $\kappa=8\pi G_N/c^4$, so as to be consistent with Newtonian gravitation theory. Following this it has become customary to always relate κ to the Newtonian constant as a matter of habit since all applications of general relativity have hitherto been to macrophysics.

Apart from this there is no other compelling reason why the coupling parameter κ should be related only to the Newtonian constant. The existence of massive spin-2 meson states (like the f meson) in nature does suggest the possibility of a short-range strong gravitational interaction which would determine the metrical properties of space-time in the region near an elementary particle. This field would then also be described by an Einstein-type field equation (since by now it is well known that starting from a linear spin-2 relativistic theory and successively adding all self-interactions in a consistent way one does recover field equation of the Einstein type).

With a new metric $f_{\mu\nu}$ and a strong coupling constant $\kappa_f=8\pi G_f/c^4$ to make it consistent with strong-interaction physics, one would now have a Lagrangian density

$$\mathcal{L}=\frac{1}{\kappa_f}\sqrt{-f}\,R(f)+\frac{1}{\kappa_g}\sqrt{-g}\,R(g)+\mathcal{L}(fg)\,,$$

where $R(f)$ is the curvature scalar constructed from $f_{\mu\nu}$ and its derivatives and $\mathcal{L}(fg)$ describes interaction between f-mesons and gravitons. If one drops this term, and considers that $\kappa_g/\kappa_f \ll 1$, the only dominant term is $\kappa_f^{-1}\sqrt{-f}\,R(f)$ which leads to an Einstein-type equation for the $f_{\mu\nu}$ field with a constant κ_f. This merely suggests that strong interactions curve the space much more strongly than in the usual Newtonian case, an idea also expressed and justified (independently of strong grav-gravity) in Refs. 9 and 10. Again, as we are interested in the strong gravity field in the immediate neighborhood of hadronic matter, any exponential factors of the form e^{-mr} occurring in the metric components due to the finite range of the strong gravity can be neglected. This is also done in Ref. 8. Thus we see that the solutions of the new field equations after these considerations will be the same as those of the usual Einstein equations, or in any case are very likely to be a good approximation to them. The reasonable results obtained by the author of Ref. 8 as well as the results of this paper seem to support this.

However, the metric used by Tennakone does not enable us to incorporate the angular momentum or spin of the particle. For this we have to invoke the Kerr metric in the context of strong gravity; that is, space-time in the vicinity of hadronic matter is assumed to be dominated by a strong gravitational field with a coupling constant[2,6,7] $G_f \approx 6.7\times 10^{30}$ cgs units. In earlier papers,[3,6] we had indicated that, by using quantized values for the angular momentum J occurring in the Kerr metric, a mass formula for the hadronic resonance states is obtained as

$$(G_f/\hbar)m_h^2 = Jc\,. \tag{1}$$

It is worth noting that Eq. (1) implies a quadratic dependence of angular momentum on the resonance mass m_h; i.e., a plot of J versus m_h^2 would appear as straight lines with a universal slope given by

$$S=(G_f/\hbar c)\approx (1\text{ GeV})^{-2}\,. \tag{2}$$

Now, it is a generally accepted experimental feature of hadron spectroscopy that all hadron resonances lie on rising Regge trajectories, i.e., appear in a Chew-Frautschi plot as straight lines, the angular momentum being linear in the mass squared, with a universal slope $\alpha\approx(1\text{ GeV})^{-2}$. Equations (1) and (2) automatically imply a linear relationship between J and m_h^2 (the usual Regge theory makes no clear-cut prediction on this dependence) with a slope of the right order of magnitude. We can also consider mass formulas resulting from the charged Kerr metric, i.e., the Kerr-Newman metric which is an exact solution of the Einstein-Maxwell equations. It seems plausible as pointed out by Salam[11] that if internal symmetries are incorporated into the structure of Einstein's field equations then in the corresponding charged Kerr-type solution, the charge Q^2 would be replaced by $I(I+1)$, where I is the isotropic spin, and J^2 would be replaced by $J(J+1)$. In effect this would give a mass formula of the type

$$m_h^2 b = \frac{J(J+1)}{m_h^2 b} + I(I+1)\,, \tag{3}$$

where a is a constant involving G_f, $\hbar$, and c. This is a typical SU(4)-type formula for a Regge hadronic trajectory. A Kerr-Newman-type metric is thus seen to provide hadronic mass formulas involving SU(6)-type combinations such as $I(I+1)$ and $J(J+1)$. That an equation like (3) gave reasonable numerical agreement with actual particle masses when using the strong-gravity coupling constant G_f was shown in Refs. 3, 6. It is of interest to note that Eq. (3) also arises by imposing the condition that a Kerr-Newman black hole be extremely charged and rotating.

Further, it is interesting to note that the Christodoulou-Ruffini mass formula[12] for charged rotating black holes (the formula holding for any general black hole, not merely extreme ones), i.e.,

$$m^2 = \left(m_{\text{ir}} + \frac{Q^2}{4m_{\text{ir}}}\right)^2 + \frac{J^2}{4m_{\text{ir}}^2} \tag{4}$$

(where m_{ir} is the irreducible mass which does not decrease in black-hole interactions), strongly resembles hadronic mass formulas involving SU(6) combinations like Eq. (3), if Q^2 and J^2 are replaced by $I(I+1)$ and $J(J+1)$. However, in order to have a complete SU(6) formula, we need an additional parameter to identify with the hypercharge Y, which does not occur in Eqs. (3) and (4). This could possibly be done through a recent work of Bekenstein,[13] who has shown through a new solution that an additional scalar charge could also be an observable parameter of a black hole in addition to mass, angular momentum, and electric charge. This could then be identified with the hypercharge. We would then have a complete SU(6)-type formula. In this connection, it is of interest to note that recently[14] solutions were constructed for the coupled Einstein-Yang-Mills field equations which describe the exterior of a rotating black hole having gauge charges such as isospin and hypercharge. These solutions for the Einstein-Yang-Mills equations are the analog of the Kerr-Newman solution for the Einstein-Maxwell equations and now correspond to spinning black holes with gauge charges instead of or in addition to electric charge, the equation of the event horizon being written as

$$\Delta = r^2 - 2Mr + a^2 + \gamma_{pq}Q^pQ^q\,, \tag{5}$$

where the constants M, a, Q^p are interpreted as the mass, angular momentum per unit mass, and gauge charges, respectively. The mass formula for these black holes will now resemble Eq. (4) with appropriate additional terms for the gauge charges Q^p. The singularity structure of the new solutions will be the same as that for the Kerr-Newman solution with the same inequalities between M, a, and $(Q^0)^2=\gamma_{pq}Q^pQ^q$ to prevent or permit naked singularities. The gauge fields considered can belong to any Lie group which has an invariant group metric γ_{pq}, i.e., gauge groups usually considered by particle physicists. These gauge groups include the vector mesons mediating strong and other interactions, the gauge charges then being the conserved quantities like isospin and hypercharge derived by applying Noether's theorem to the appropriate gauge symmetries. Moreover, the generalization of the Higgs mechanism to curved space; i.e., solutions of the Einstein-Higgs equations can generate black-hole solutions characterized by massive gauge fields. Such solutions are not yet known but one would reasonably expect them to exist. The Israel-Carter uniqueness conjecture[12] can be extended to these new black-hole solutions and it leads to the identification of various gauge charges as distinct observable parameters of the black holes. For the Kerr-Newman solution Wald[15] has shown that the inequality $m^2 \geqslant a^2 + Q^2$ is always maintained, if it is satisfied initially. This would also hold for the new black-hole solutions. We thus see that with quantized values for a, Q^p, etc., and in the context of strong gravity, the mass formulas for these black holes [Eqs. (3) and (4)] will really look like the SU(6) mass formulas for hadrons and will give reasonable numerical values with the strong gravitational constant. Of course, we note that we end up with squared-mass formulas for all hadrons, i.e., for both mesons and baryons, whereas the usual Gell-Mann–Okubo (GMO) mass formula is quadratic for mesons and linear in the mass for baryons. Whether we should use the linear mass or (mass)^2 has been an unsettled question since then. Although the linear mass was used for the original GMO mass formula for baryons the latest indications[16] are that the (mass)^2 formula is numerically more accurate even for the baryonic case. It must be remarked that the unitary symmetries [SU(3) or SU(6)] used for the classification of strongly interacting particles are approximate, their experimental success being attributed to the rather phenomenological introduction of symmetry-breaking terms (which give additional degrees of freedom such as I, Y, etc.) as a perturbation on the mass operator; i.e., we have a theory with a flat-space invariant $P_\mu P^\mu$ (Poincaré invariant or mass operator) together with an interaction which gives a nondegenerate mass spectrum, i.e., mass splitting.[9] An "elementary particle" *without* intrinsic degrees of freedom such as isospin, hypercharge, etc., can be described by a single value for its mass (i.e., a degenerate state) given by the invariant mass operator $P_\mu P^\mu = m^2$, i.e., characterized by an irreducible representation of the Poincaré group as its symmetry group. Symmetry-breaking interaction terms would now perturb this flat-space mass operator giving rise to a mass spectrum (nondegenerate state) describing elementary particles *with* intrinsic degrees of freedom. The theory in flat space with an interaction can be reduced to an equivalent description of a group of motions in curved space; i.e., it can be related to geometry completely in the spirit of general relativity where the curvature of space-time is caused by the presence of energy-momentum tensors generated by various interactions. Using a curved-space group (such as the de Sitter group) instead of the flat-space Poincaré group one automatically gets a "generalized" mass operator $P_\mu P^\mu$ which has additional terms[9,10] giving rise to a mass spectrum, i.e., $P_\mu P^\mu$ is no longer a Poincaré invariant, but terms like $P_\mu P^\mu + J(J+1) + \cdots$ are now invariants of this curved-space group. This seems more natural than introducing symmetry-breaking terms. An elementary particle with intrinsic degrees of freedom would now be described by representations of the curved-space group (say de Sitter group) as the symmetry group and would go by contraction[17] into an elementary particle with a single degenerate mass state given by the usual Poincaré invariant. The use of curved-space groups of motion would be necessitated[18] by the presence of the strong gravitational interaction in the immediate vicinity of hadronic matter. This is one way of understanding the above results. The irreducible mass in (4) could be regarded as the rest mass of the degenerate ground state (say proton for the baryon spectrum).

We would now mention a few more amusing analogies between the properties of black holes and those of hadrons. While considering the collision of two black holes, we have a theorem[19] due to Hawking which states that in the interactions involving black holes the total surface area of the boundaries of the black holes can never *decrease*. In stationary processes they remain unchanged at best. This seems consistent with the observed increase in cross sections in hadron collisions, hadrons here being assumed to be quantum black holes of the strong gravity field. Further, there are elegant analogies between the laws of black-hole dynamics and the laws of ordinary thermodynamics. One remarkable conclusion arising from this analogy is that a black hole radiates like a black body whose temperature is inversely proportional to its mass.[19,20] We would, therefore, associate with a black hole of mass M a temperature given by

$$T_b = \frac{\hbar c^3}{8\pi G M k_B}, \tag{6}$$

where k_B is the Boltzmann constant. That a black hole should be assigned a temperature (proportional to its surface gravity) as given by Eq. (6) was first suggested by Bekenstein[20] on the basis of analogies between thermodynamic quantities and the parameters of black-hole dynamics. Direct calculations by Hawking[19] showed that a black hole does radiate like a body with temperature given by Eq. (6), in agreement with the temperature defined by Bekenstein. The temperature is seen to depend (inversely) only on the black-hole mass and on no other parameter. The smallest black-hole mass will therefore give rise to the highest temperature. In the case of strong gravity, G in Eq. (6) would be replaced by the strong gravitational constant G_f and M would correspond to the mass of a hadron, hadrons being considered as black or "grey" holes of the strong gravity field. Now the hadron with the smallest mass is the (spin-0) pion having $m_\pi \approx 140$ MeV. Equation (6) then gives (with $M = m_\pi$, $G = G_f$)

$$T_{b\pi} \approx \frac{\hbar c^3}{8\pi G_f m_\pi k_B} \simeq 2 \times 10^{12} \,{}^\circ\mathrm{K}. \qquad (7)$$

This would be the upper limit for the temperature of hadrons or hadronic matter. It is remarkable that this upper limit for the temperature is the same as the limiting temperature arising in thermodynamic bootstrap models of hadrons (we also recall in this connection Fermi's blackbody radiation model for hadronic matter) such as the Hagedorn model, where usually $k_B T_{\max} \sim m_\pi c^2 \sim 2 \times 10^{12}\,{}^\circ$K. There is a measure of experimental support for the existence of such a temperature as manifested by a cutoff in the transverse momenta of colliding hadrons in high-energy experiments. The existence of an upper limit for the temperature of hadrons can be conceived of as the fourth law of thermodynamics.[21] Equation (7) giving the upper limit for the temperature of black holes would then be a statement of the fourth law of black-hole dynamics, the other three laws being already known.[22] Many analogies, therefore, exist between the behavior of black holes and those of elementary particles.

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