Comment on "Observation of matter wave beat phenomena in the macrodomain for electrons moving along a magnetic field"

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I show that the claim of the observation of matter wave beat phenomena in the classical macrodomain by Varma *et al.* [Phys. Rev. E **65**, 026503 (2002)] is based on a mistaken interpretation of effects arising from multiple focusing of an electron beam in an axial magnetic field. I present the basic physical facts that mimic wavelike phenomena and suggest a classical explanation of modulations reported by Varma *et al.* Realization that the macroscopic "de Broglie wavelength" used by Varma *et al.* is the same as the focusing distance of a monoenergetic electron beam in the uniform magnetic field leads to a full classical explanation of all the effects reported by Varma *et al.* The reported observations are not evidence for any quantumlike phenomenon in the macrodomain, and their results do not indicate any violation of the Lorentz equation of motion.

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I. INTRODUCTION

In a recent paper [1] and in several earlier papers [2,3], Varma *et al.* have advocated that there are quantumlike effects (interference, tunneling, resonances, Aharonov-Bohm effect, etc.) in the classical macrodomain corresponding to the motion of electrons in a uniform magnetic field, with a source to detector distance of the order of 30–50 cm. This distance is orders of magnitude larger than the coherence length of electrons in such a beam. Varma *et al.* claim that quantumlike effects have been observed with the adiabatic invariant $\mu = \frac{1}{2}mv^2/(eB/mc)$ playing the role of Planck's constant (μ is typically 10⁸ \hbar in these experiments) and with an effective wavelength of the relevant macroscopic matter waves of the order of 2–5 cm. Their interpretation also implied that there are violations of the Lorentz force law, Maxwell's equations, and classical electrodynamics.

If this is true, then it signals serious gaps in our present understanding of the physical world even in those domains that are considered well understood and well tested. Therefore it becomes important to closely check the results and interpretation. The purpose of this paper is to present an analysis of their experiments and interpretation and to show that the observed effects are entirely within the classical domain.

We examined both the experiments and interpretations of results by Varma *et al.* once earlier when one-dimensional interference and resonance effects were reported [4,5]. We first reproduced their results in a set of independent experiments and then *discovered that the entire observations could be explained as arising from the multiple focusing of a beam of secondary electrons generated at various electrodes.* This paper follows the same thread of reasoning to arrive at results that explain the beats observed by Varma *et al.* entirely using classical physics.

II. FOCUSING, SECONDARY EMISSION, AND A PSEUDOWAVE

For the "one-dimensional interference effects" reported earlier, Varma et al. had used a monoenergetic electron beam in a uniform axial magnetic field and the detector was a Faraday cup with several grids in front [2]. They observed oscillatory patterns in the current detected at various electrodes as the various parameters in the experiment were varied (electron energy, magnetic field, grid retardation voltage, etc.). The peak-to-peak distance in the oscillatory pattern varied as the square root of the energy of the beam, and Varma et al. interpreted this as due to quantumlike effects in the macrodomain, with an effective wavelength of λ $=2\pi v/(eB/mc)$. It is this expression for the "wavelength" that gives the crucial clue as to what is the basic physical mechanism underlying the results obtained by Varma et al. For a monoenergetic electron beam with energy $E = \frac{1}{2}mv^2$ and small angular spread, multiple focusing occurs in a uniform magnetic field B with "focal length" $l_f = 2\pi v/(eB/mc)$ [6,4]. This is just the distance traveled by the electron with velocity v over a time scale equal to the Larmour time. For a magnetic field of about 100 G and electron energy of 1000 eV, l_f is approximately 7 cm, and therefore multiple focusing will occur over macroscopic lengths between the source and detector. This creates a pseudo-standing-wavelike pattern in space, as sketched in Fig. 1 [4]. Passage of the beam through small finite apertures or wire grids then crucially depends on whether the aperture or the grid wire is close to a focal point or not. Also secondary electron emission from each electrode depends on the intensity of the beam, which in turn depends on the proximity of the focal point to the electrode. Naturally and entirely classically, one obtains a oscillatory pattern as the electron energy or the

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FIG. 1. The pseudo-standing-wave arising from multiple focusing of an electron beam in a magnetic field. The lower panel shows the shift in focal points when the energy of the beam is increased.

magnetic field is varied since the focal points move in space axially as these parameters are varied. An oscillatory pattern is obtained also when the detector or grids are moved through this pseudo-standing-wave. Thus it was already shown that the one-dimensional wavelike aspects seen by Varma *et al.* are due to a simple and well-known phenomenon due to the Lorentz force in classical charged particle dynamics [4,5]. The role of secondary electrons has been emphasized by Ito and Yoshida as well [7] in a later experiment.

In the recent paper Varma *et al.* reported further results showing a beatlike phenomenon in similar experiments, with a fixed source and detector plate and a movable grid in between [1]. With the source-plate distance and source-grid distance fixed, beatlike modulations were seen riding on the faster oscillatory pattern when the energy of the beam was scanned. Varma *et al.* claim that beatlike phenomena can only be due to the manifestation of wave aspects and that there could not be any classical explanation for such an observation. Therefore, *according to them, their results are a clear indication of a departure from classical electrodynamics and are evidence for quantumlike effects in the macroscopic domain.*

We will show that it is possible to get beatlike modulations entirely within a classical scenario, and that we can explain all the features of the results seen by Varma *et al.* as due to multiple focusing of the electron beam in the magnetic field. Their observations do not indicate any new physical phenomenon, let alone quantumlike effects in the classical macrodomain.

It is a well-known and easily observed fact that any two periodic structures when overlapped can give beatlike patterns. Moiré patterns are formed like this, and therefore observation of beatlike phenomena indicates only that there are two patterns with some periodicity, but does not necessarily imply that wave phenomena are involved. The issue is whether it is possible to get quantitatively the beat patterns that were observed in the electron beam experiment. We have already seen that multiple focusing of the electron beam creates a periodic pattern in space that is purely classical. This can be treated as a pseudo-standing-wave with a node-to-node distance or more accurately focus-to-focus distance of $l_f = 2\pi v/(eB/mc)$. This primary beam with a spatial modulation is a classical source that can generate secondary electrons at various electrodes. Using these two features—

that of periodicity and secondary electron generation—we will be able to explain all the features seen by Varma *et al.* within the classical paradigm.

III. CLASSICAL SCENARIO FOR BEATS

A. Feature to be explained

Before we present a full discussion, we note the various features that need an explanation in the results obtained by Varma *et al.* [1].

(i) There is an oscillatory pattern in the current detected by the grid and plate and the peak-to-peak distance changes as $E^{1/2}$ as the electron energy *E* is varied.

(ii) The spatial frequency of the oscillatory pattern is proportional to the separation between the source and detection electrode (this is explicit in some of the earlier results by the same authors [2]).

(iii) The currents at the grid and plate are anticorrelated.

(iv) There are slow amplitude modulations ("beats") of the oscillatory current at the grid and plate and these modulations are in phase. The spatial frequency of the beats is proportional the separation between the plate and grid.

(v) The period of the slow modulation decreases as the distance between the grid and source is decreased, or equivalently the distance between the grid and the plate is increased.

(vi) When the separation between the plate and grid is much larger than half the separation between the plate and source, the beats disappear and the higher-frequency current oscillations ride over a low-frequency modulation.

B. Current modulations and dependence on energy

First we try to model these main features and then go on to look at more detailed characteristics. Our aim is to show that there is at least one well-understood classical mechanism that explains all the features seen in the experiments by Varma *et al.* It is possible that there are additional classical effects that may have a bearing on the fine details of the experimental results, but even our simple model shows that their results are certainly not evidence for any macroscopic quantum phenomena. We want to derive the main results of Varma *et al.* by making the simplest and physically reasonable assumption relevant to their experiment. In fact, we will try to explain all main features as resulting entirely from the multiple focusing in the magnetic field and from the way secondary electron production at each electrode is dependent on the intensity of the electron beam.

Since the detecting electrodes are not biased (both the grid and plate are at ground potential in their experiment), there is no field to attract back low-energy secondary electrons of typically 10-30 eV. Secondary electrons generated at the electrodes escape and then are detected at various other electrodes and metal parts at ground or at more positive potentials. In addition secondary electron production itself depends on the properties of the electrodes as well as on the local intensity of the primary beam. This means that the actual current detected by the plate and the grid will depend on whether the beam is focused or not at their respective planes.

This point has been already noted in Ref. [4], from results of experiments done specifically to check this point (see the last paragraph in Sec. 3.2 of [4]). So we make the simple assumption, which is experimentally supported, that the classical current detected at the electrode is a function of the intensity of the primary electrons on the electrode. As the energy of these electrons is varied, their velocity varies as $v \propto E^{1/2}$, and the focus-to-focus distance changes as $l_f \propto E^{1/2}$. Therefore, the number of focusings that occur within a particular length (like the source-to-plate distance) varies as $E^{-1/2}$. It is also proportional to the magnetic field and distance between the source and electrode, since the focal length is inversely proportional to the magnetic field. Periodicity in the focal points then can translate to periodicity in the detected currents, as the energy, magnetic field, or separation of the electrodes is varied. This immediately explains two important features seen in these experiments: (a) there is a frequency associated with the distance between the source and detector electrode (L), which is simply the number of focusings within the distance, and (b) the frequency varies as $LBE^{-1/2}$. Since this frequency is the inverse of the peak-topeak distance in the current, we have now explained features (i) and (ii) in the list of features to be explained (referred to as the "list" in the rest of the paper).

Since we have seen that there are multiple focusings in the magnetic field, we can estimate the focal length $l_f = 2\pi v/(eB/mc)$ and see whether it matches well with the observations. Written in terms of the energy,

$$l_f = 2\pi (2E/m)^{1/2} / (eB/mc).$$
(1)

For a magnetic field of 69 G and electron energy of about 200-250 eV, this is on the average 4.6 cm (4.35-4.86 cm). Therefore there are approximately 10-11 focusings that the electron beam does from the source to the grid and detector plate. Let us start with a situation where the last focus-say, n=11—is close to the plate. This means that we are close to a minimum in the plate current since the surface density of electrons is high and there is lot of loss of electrons from the metal surface due secondary electron emission. As the energy is increased, the focus moves to a virtual position past the plane of the plate and the electron density decreases, decreasing the secondary electron loss and increasing the net current detected by the plate. This repeats itself as a new focus point (10th in this case) comes close to the plate as the energy is increased further. Thus the current detected is a periodic function of $E^{-1/2}$. To check the quantitative agreement, we can calculate the change in the energy required to go from one peak to the next in the oscillatory current pattern, around some energy value-say, 200 eV. We have

$$\frac{\delta l_f}{l_f} = -\frac{\delta E}{2E},\tag{2}$$

$$n\,\delta l_f = l_f.\tag{3}$$

The second relation comes from the fact that with *n* focusings, the net change in the focal distance is $n \delta l_f$, and when this is equal to the focal length, the pattern repeats. From data around 200 eV and from n=10, we get $\delta l_f \approx l_f/10$



FIG. 2. The variation of the current at the detector as a function of energy of the beam derived by assuming that the secondary electron emission (loss) depends on the intensity of the beam at the detector.

=0.46 cm. Therefore, $\delta E/E = 2 \, \delta l/l = 2/n \approx 0.2$. This gives $\delta E \approx 0.2E \approx 40$ eV. We examine Fig. 2 in which Varma *et al.* report the primary results and see that the energy change required to go from a minimum to the next around 200 eV energy is approximately 40 eV, in good agreement with the prediction from our classical model.

The explicit functional form of the dependence of the detected current on the energy can be derived knowing the secondary electron emission characteristic, but the details are not needed to conclude that the current at each electrode is a periodic function of $E^{1/2}$. Since a segment of the multiply focused pattern can be approximated as $\sin(2\pi x/l_f)$, the cross-sectional area of the beam at a point x on this segment is simply $\pi[\sin(2\pi x/l_f)]^2$ and the intensity is given by

$$\mathcal{I}_b = I_s / \pi [\sin(2\pi x/l_f)]^2, \qquad (4)$$

where I_s is the primary current from the source reaching the detector. Since the focus is blurred, due to the finite size of the source as well as due to the finite spread in the energy of the beam, the physically relevant intensity is given by a function that accounts for this blur, without a singularity. This will look like

$$\mathcal{I}_b = I_s / \{\delta + \pi [\sin(2\pi x/l_f)]^2\},\tag{5}$$

where δ depends on the source characteristics, beam energy, and energy spread. According to our classical hypothesis of secondary electron loss dependent on the primary beam intensity, we see that the current at the electrode is a periodic function, a constant current modulated by the function above with some strength α depending on the characteristics of the beam and the electrode material. The actual current detected at the detector coordinate x is the primary flux minus the secondary electron loss, and this is given by

$$I = I_s - \alpha I_s / \{\delta + \pi [\sin(2\pi x/l_f)]^2\}$$

= $I_s - \alpha I_s / (\delta + \pi \{\sin[x(eB/mc)/(2E/m)^{1/2}]\}^2).$

This analysis applies to the grid as well. A typical expected pattern, as a function of the position of the detector elec-



FIG. 3. Current detected in an axial magnetic field when the secondary electron loss depends on the intensity at the detector and thus on the sharpness of the focus. The amplitude is smaller at lower energies due to the blur of the focus after the larger number of focusings before the electrons reach the detector.

trode, due to such a modulation of the secondary electron emission is shown in Fig. 2.

Note that the pattern is not a symmetrical sinusoidal modulation and a comparison with the results of Varma et al. shows that this pattern fits their observations better than a simple sinusoidal dependence in $E^{1/2}$ (note the region of large energies). In any case, we will simplify the rest of the analysis by using a sinusoidal pattern for the current in each detector, determined completely by classical considerations like secondary electron emission (the pattern can be approximated well by a sinusoid when the value of δ is large and when the dependence of secondary electron emission on the intensity is mild). This will help in doing the analysis accurately enough to explain the main features and without the complication of asymmetric oscillations. The important point we wish to show is that it is indeed possible to have beatlike patterns in the classical domain with beat frequency ω_b $=\omega_1-\omega_2$, where ω_1 and ω_2 indicate frequencies of modulations of the classical current.

At this stage it is also important to point out that a more realistic model should include the physical fact that the sharpness of the focus reduces as more and more focusings take place because of the finite spread in the energy of the beam from the source. Since the number of focusings within a fixed length increases as $1/E^{1/2}$, the modulation depth for very low energy is expected to be small. For higher and higher energies the modulation depth increases, but only as $E^{1/2}$. Once this is incorporated into the model, the classical current modulation due to secondary electron emission looks like that shown in Fig. 3. This is close to what is observed in all the relevant experiments [1,2,4,7]—the modulations are weaker in amplitude at lower energies.

C. Currents at various electrodes

Since we are interested in explaining the main observations regarding beats, from now on we will use a simpler sinusoidal function of L/L_f to represent the modulations in current due to secondary electron loss. We can write the current detected at an electrode approximately as

$$I_i = I_{0i} [1 - \eta_i \cos(l_i / L_f)] + \sum I_j,$$
(6)

where η_i represents an efficiency factor for secondary electron emission and the cosine term is the intensity-dependent modulation. L_i is the distance of the electrode from the source. The second term represents the current at electrode *i* due to secondary emission from other electrodes, which also will be oscillatory due to the dependence on focusing at those electrodes. Since $l_f = 2\pi (2E/m)^{1/2}/(eB/mc)$, the current is

$$I_i = I_{0i} [1 - \eta_i \cos(\alpha B L_i E^{-1/2})] + \sum I_j,$$
(7)

where we have absorbed the constant factors into α . For example, the current at the plate in this experiment will be

$$I_p = I_{0p} [1 - \eta_p \cos(\alpha B L_p E^{-1/2})] + I_{gs}, \qquad (8)$$

where I_{gs} is the oscillatory secondary electron emission from the grid. While some fraction of the emitted secondary electrons can get back to the electrodes due to the confining nature of the axial magnetic field, most of these electrons will end up in other electrodes in the experiments. In this particular experiment, the electrons that are emitted by the plate electrode will mostly end up on the grid (two absorptions in the forward and backward passages). Therefore, if Eq. (8) represents the current in the plate, then the current in the grid, which is the sum of the primary electron current at the grid, secondary electron loss from the grid (oscillatory), and the secondary electrons received from the plate, will be approximately

$$Ig \simeq I_{0g} - I_{0g} \eta_g \cos(\alpha B L_g E^{-1/2}) + I_{0p} \eta_p \cos(\alpha B L_p E^{-1/2}).$$
(9)

Clearly, the two currents add up to a constant current, I_{0p} + I_{0g} , and *the currents in the two electrodes are anticorrelated always*. Thus we have a simple and physically transparent explanation for item (iii) in the list—namely, anticorrelation of the oscillatory currents at the grid and plate.

D. Classical explanation of "beats"

What about the beatlike modulation which Varma et al. point out as crucial for their interpretation of results as macroscopic quantum like effects? These beats have been highlighted by Varma et al. as the most crucial nonclassical effect in their experiments. They claim that no classical scenario can explain such beats and therefore "they establish unambiguously the existence of macroscopic matter waves." In the rest of the discussion we show unambiguously that the entire set of beatlike modulations arises in a simple classical addition of oscillatory currents determined by secondary electron emission and multiple focusings of the primary electron beam. In fact, all the beatlike structures seen in their experiment result from a simple sum of two periodic classical currents with frequencies ω_1 and ω_2 . Contrary to the assertion by by Varma et al. that no classical scenario can get beats with frequency $\omega_1 - \omega_2$, the beat frequency in such a situation is indeed $\omega_1 - \omega_2$ as we will show now.

Consider two classical currents $I_1 = I_{01} + a \cos \omega_1 t$ and $I_2 = I_{02} + b \cos \omega_2 t$, with $I_{01} \approx I_{02} = I_0$, and $a \approx b$. The amplitude



FIG. 4. The top panel shows the beats resulting from the addition of two classcal oscillatory currents. The bottom panel shows how the total current behaves when the difference in the frequencies of the oscillatory currents is large.

of modulation is smaller than the average current, as required by any source of particles. The sum of the two currents is

$$I = I_{01} + a \cos \omega_1 t + I_{02} + b \cos \omega_2 t$$

$$\approx 2I_0 + 2a \left\{ \cos \frac{1}{2} (\omega_1 + \omega_2) t \cos \frac{1}{2} (\omega_1 - \omega_2) t \right\}.$$
(10)

This represents a oscillatory signal at frequency $\frac{1}{2}(\omega_1+\omega_2)$ modulated at frequency $\frac{1}{2}(\omega_1-\omega_2)$. The beats themselves are at frequency $(\omega_1-\omega_2)$, since the separation between the maxima of the modulated pattern is at time intervals $1/(\omega_1-\omega_2)$. Since this is an important point to be clarified in this context, we have plotted in Fig. 4 the classical sum for frequencies $\omega_1=100$ Hz and $\omega_2=90$ Hz in the upper panel and $\omega_1=100$ Hz and $\omega_2=10$ Hz in the lower panel. Note that the beat frequency is indeed $(\omega_1-\omega_2)$ and not $\frac{1}{2}(\omega_1-\omega_2)$ as Varma *et al.* claim.

As derived earlier, the currents in the plate and grid are

$$I_p = I_{p0} - I_{0p} \eta_p \cos(\alpha B L_p E^{1/2}) + I_{0g} \eta_g \cos(\alpha B L_g E^{1/2}),$$
(11)

$$I_{g} = I_{0g} - I_{0g} \eta_{g} \cos(\alpha B L_{g} E^{1/2}) + I_{0p} \eta_{p} \cos(\alpha B L_{p} E^{1/2}).$$
(12)

The important point to note is the different periodicities associated with the two cosine terms in the current at each electrode. If $E^{1/2}$ is taken as the variable directly, then the "frequencies" associated with the current in the plate and grid are, respectively,

$$\omega_p = \alpha B L_p,$$

$$\omega_g = \alpha B L_g,$$

$$\Delta \omega = \omega_p - \omega_g = \alpha B (L_p - L_g).$$
(13)

In Fig. 5, we plot the results of this classical model for L_p $-L_g \ll L_p$. We have also included the fact that the sharpness of the focus reduces when the number of focusings is large at lower energies. This is a simple sum of two oscillatory currents at two frequencies ω_p and ω_g , both variations inversely dependent on $E^{1/2}$, and the amplitude of oscillations also in-



FIG. 5. Classical model for the observation of beats by Varma *et al.* The physical inputs are classical focusings of the electron beam in the axial magnetic field and the intensity dependence of secondary electron emission from unbiased electrodes.

creasing as $E^{1/2}$ due the dependence on the sharpness of the focus. Remarkably, all main features of the results seen by Varma *et al.* are reproduced. We stress that the periodicity of the beatlike modulations happens at the frequency $\omega_p - \omega_g$. This explains the features (iv) and (v) in the list excellently.

If we plot the behavior of the current at any of the electrodes when $L_p - L_g \ge L_p/2$, corresponding to the situation when the distance between the source and grid is much smaller than the distance between the grid and plate, we get a different pattern that again reproduces what Varma *et al.* have observed for such parameter values (Fig. 6).

IV. SOME CONCEPTUAL ISSUES

Now that we have established that the results obtained by Varma *et al.* are fully explained within the classical paradigm, it is worth pointing out some serious conceptual flaws in the analysis by Varma *et al.* of their data using their "quantum wave algorithm." Varma *et al.* sought to explain the oscillatory behavior in the currents at the detectors by assigning complex quantum amplitudes to the various ways in which the electrons can reach the electrode. For this they use three amplitudes $\gamma \exp[ik(x-L_g)]$, and



FIG. 6. Results of the classical model plotted for the situation when the distance between the grid and plate is larger than half the distance between the source and plate. The agreement with the features observed by Varma *et al.* is very good.

 $\beta \exp[ik(x-L_n)]$, which represent the contributions, of "direct from the source," "forward scattered from the grid," and "forward scattered from the plate," respectively. Already at this point there is a serious problem since these amplitudes are written down (obviously to match the results) assuming that the detection is done at a point x that is beyond the real detector plate. Since the plate is a solid plate of steel in the experiment, the detection is at the front surface of the plate itself, at the coordinate $x=L_p$, and consequently the forwardscattering amplitude is negligibly small beyond that point. Even if one wants to assume the existence of such an amplitude and to write it formally, an extra term will multiply the factor in the exponential due to the different velocity of the electrons in the metal (a term equivalent to a refractive index). Inside the metal, the wave vector is not k. Therefore, their whole analysis based on a wave algorithm is not valid. Another point to note is that their wave algorithm also predicts oscillatory current with periodicity characterized by the sum of the two lengths, $L_p + L_g$, and this means that oscillations at almost half the periodicity and amplitude comparable to that of the first harmonic should be visible in their data when $L_p \approx L_g$. This is not seen. There are even more severe problems when other consequences of a wave algorithim are explored. The forward-scattering amplitude from the wire grid is written as a single-exponential plane wave whereas the "wavelength" in their wave algorithm is much larger than the wire-to-wire separation of the grid used in these experiments. Severe diffraction effects and almost complete reflection is expected using the same wave algorithm, and the current detected at the plate should have been nearly zero when the grid is a few wavelengths away from the plate. This is not what is seen in their data, clearly indicating that the picture of the macroscopic quantum wave the authors are trying to advocate has neither experimental evidence nor theoretical validity.

V. SUMMARY

This completes a full classical explanation of the features listed earlier. We have proved unambiguously that the results obtained by Varma et al. are easily reproduced within the classical electrodynamics of charged particles in a magnetic field. We conclude that features like the one-dimensional interference and beats observed by Varma et al. are explained adequately within the standard paradigm without any need to invoke nonstandard physical phenomena, let alone new quantum effects in the macroscopic domain. Their observations neither indicate the existence of matter waves in the macroscopic domain nor any new effect that contradicts the classical Lorentz equation of motion. Varma et al. have severely misinterpreted simple classical effects arising from multiple focusing of an electron beam in an axial magnetic field and its effects on secondary electron emission at various detecting electrodes. We have also been able to ascertain that the macroscopic analog of the Aharonov-Bohm effect claimed to be observed by Varma et al. [3] is spurious, and this is discussed elsewhere [8].

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