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CONVECTION AND THE PHENOMENON OF KILOGAUSS MAGNETIC FIELDS ON THE SUN

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Abstract

The role played by convection in the formation of slender magnetic flux tubes and in the dynamics of the gas within the tube is discussed in the case of simplified models. Convection instability cannot drive systematic downflows whereas convective buffetting of the tube can. The inclusion of heat transport reduces the efficiency of convective collapse for the formation of strong fields. The implications of these two results for the solar magnetic flux tubes is pointed out.

1. Introduction

The interaction of velocity and magnetic fields has been well studied in the context of magnetoconvection. The work of Parker (1963), Weiss (1966), Proctor and Galloway (1979), Galloway and Weiss (1981), all indicate that in a Boussinesq fluid the final result of the interaction of a circulatory velocity field and a uniform magnetic field is the formation of a structured magnetic field devoid of flow and a region of flow bereft of magnetic field. It is natural to ask the question whether such structures can be formed even in the case of compressible magnetoconvection. This question is particularly important in the light of the highly structured nature of the magnetic fields observed on the solar surface.

One way of answering this question would be to assume a family of equilibrium structures and examine their stability as a function of some parameter characterising the family, e.g. the magnetic field. Stability analysis of two dimensional structures (e.g. an axisymmetric tube with area of cross-section varying with height) is exceedingly difficult. The slender flux tube approximation, introduced conceptually by Defouw (1976) and established rigorously by Roberts and Webb (1978), helped in the linear stability analysis of such structures. Webb and Roberts (1978) showed that tubes with field strengths smaller than a critical value are prone to a convective instability. The critical value depends on the depth of the tube as well as on the superadiabatic temperature gradient. Spruit and Zweibel (1979) calculated the growth rates of the instability for tubes embedded in a realistic convection zone environment. Their critical field strength was ≈ 1500 G. These values compare well with the observed field strengths.

However, one does not know a priori whether the convective collapse of weaker tubes would result in a hydrostatic tube or hydrodynamic tube within the time scales of interest in the solar context. Hasan (1983) calculated the nonlinear development of convective instability of a slender flux tube and obtained final hydrodynamic states independent of the initial value of the magnetic field. In this paper we review some of our results of similar calculations for polytropic tubes, both for adiabatic (Venkatakrishnan, 1983) as well as non-adiabaic (Venkatakrishnan, 1984a) conditions.

Furthermore, the magnetic structures observed in the photosphere are constantly buffetted by granulation and waves. We shall also consider the nonlinear response of magnetic tubes to such perturbations.

2. Nonlinear Development of Convective Instability with Slender Flux Tubes

The various parameters that determine the evolution of a polytropically stratified slender flux tube are β_0, the ratio of initial gas pressure of magnetic pressure, δ_0 the initial superdiabatic gradient and the boundary conditions. The basic equations for a slender flux tube were first rigorously formulated by Roberts and Webb (1978). These equations are:

$$\frac{\partial}{\partial t}(\rho/B) + \frac{\partial}{\partial z}(\rho v/B) = 0, \qquad (2.1)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial}{\partial z}v + \frac{1}{\rho}\frac{\partial}{\partial z}p + g = 0 \qquad (2.2)$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p - \frac{\gamma p}{\rho}\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho = -(\gamma-1)\,(\nabla\cdot\underset{\sim}{F}) \qquad (2.3)$$

$$B^2 = 8\pi\,(p_e - p), \qquad (2.4)$$

where p, ρ, v, B are the pressure, density velocity and magnetic field inside the tube, p_e is the pressure outside the tube and $\underset{\sim}{F}$ is the energy flux. We shall first consider the case $\underset{\sim}{F} = 0$ corresponding to adiabatic motions.

2.1. Adiabatic Flow

The equations (2.1) through (2.4) with $\underset{\sim}{F} = 0$ form a system of hyperbolic partial differential equations which can be cast in the characteristic form. We integrated these equations using a backward marching algorithm based on the method of characteristics (For details see Venkatakrishnan, 1984b). In this way we obtained the values of all the dynamical variables as a function of z for various instants of time. In this paper we shall mainly discuss the evolution of velocity and magnetic field of the tube at a few spatial locations.

We applied two different sets of boundary conditions. In one set the Eulerian pressure was kept constant at both ends of the tube. These conditions could be more appropriate to closed magnetic regions of the sun (Venkatakrishnan, 1983). The tubes on which such conditions were applied will be called 'closed' tubes in what follows. In the other set of boundary conditions the Eulerian pressure was kept constant at the base of the tube while the Lagrangian pressure was kept constant at the top of the tube. This type of boundary conditions would be more appropriate for open vertical tubes in the sun. These tubes will be denoted as 'open' tubes in what follows.

The general behaviour of velocity for the adiabatic case consisted of an initial slow increase till a time τ and then a rapid increase beyond that time. The onset of the nonlinear phase was delayed for stronger initial magnetic fields (small values of β_0). It was apparent that $\tau\to\infty$ as β_0 tended to some finite value ≈2.0 (Figure 4 of Venkatakrishnan, 1983). Furthermore, the intensification of the field was not seen to be sustained during the nonlinear phase for β_0 = 4.0, 5.0 and 6.0. An interesting example of overstability was seen for β_0 = 2.0 in the case of an open tube. When β_0 = 4.0, however, it was seen that steady magnetohydrodynamic states are attained. For β_0 = 6.0, the nonlinear regime showed unsteady behaviour even until 30 free fall times.

The following picture emerged out of these calculations. The magnetic field inhibits the convective collapse of flux tubes. For some value of the field, overstable oscillations set in while for weaker fields steady dynamic states are attained. Even this is not possible for the weakest fields studied here, and in these cases unsteady behaviour persists for large times. The actual values of field strengths demarcating these regimes depends on the boundary conditions, with open tubes exhibiting greater stability compared to closed tubes.

2.2. Effect of Radiative Heat Transport

The energy equation for an optically thick slender flux tube is:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)p-\left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)\rho=(\gamma-1)K$$

$$\left[\left\{\frac{4(Te-T)}{r_0^2}+\frac{\partial^2 T}{\partial Z^2}\right\}+\left(\frac{\partial T}{\partial z}\right)^2\left(\frac{\partial}{\partial T}+\frac{\partial p}{\partial T}\frac{\partial}{\partial p}\right)\ln K\right].\quad(2.5)$$

(For a derivation of this equation see Venkatakrishnan 1984a). The first term on the R.H.S. of the equation (2.5) represents the lateral heat exchange. The second term represents longitudinal heat diffusion for constant heat conductivity K, while the third term represents the influence of variable conductivity. Here we shall discuss only the case of constant K.

When lateral heat exchange alone was considered in an open tube, it was seen that oscillations were produced. The amplitude of these oscillations increased with decreasing radius r_0 of the tube. For r_0 < 0.5 the oscillations became so large that local pressure enhancements rendered the magnetic field imaginary and precluded further computations. The amplitude of oscillation was seen to be drastically reduced for β_0 = 4.0, as compared to the case for β_0 = 6.0. When longitudinal heat transport was included, the frequency of the oscillations was seen to be doubled and the amplitude reduced. This perhaps could be due to the excitation of a new mode or harmonic.

Thus heat transport was seen to shift the state of marginal stability to lower values of the magnetic field as compared to the adiabatic case. Wherever the instability was evidenced, it was only in the form of oscillations and not monotonic as seen in the case of adiabatic flow.

3. Response of Tubes to External Turbulence

Let us for a moment forget the considerations of stability and assume that a stable magnetostatic flux tube exists in the solar atmosphere. This tube cannot remain static since it is embedded in a highly chaotic environment. The environment can influence the tube broadly in 3 ways. It can twist, bend or squeeze the tube. A twist cannot influence the dynamics of the gas inside the tube while bending and squeezing motions can. In what follows we report some results concerning the response of this gas to bending motions of the tube (Hasan and Venkatakrishnan 1980) as well the response to squeezing motions (Venkatakrishnan, 1984b).

3.1. Response of Squeezing Motions

The equations (2.1) through (2.4) described the state of a tube with a time independent environment. If external pressure perturbations are applied then they would induce changes in the internal gas pressure and, therefore, create a flow of gas along the field. We applied two forms of external perturbations. In one form the fluctuations were oscillatory in time but decreasing exponentially with height. The response of the gas within the tube is maximum when the scale length of the perturbation is approximately equal to the distance travelled by a tube wave in one period of the oscillation (Figure 1).

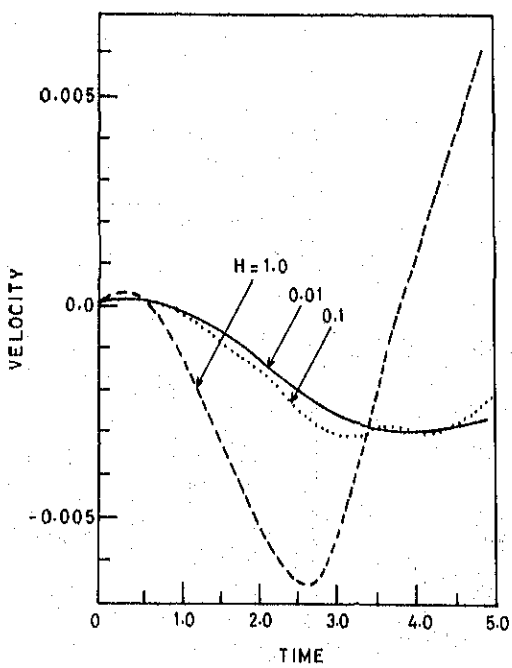

Fig.1. Response of gas to squeezing motions on the tube - effect of exponentially decaying fluctuations with different values of scale height H.

Moreover, the resulting flow along the tube is oscillatory. When the scale length is smaller than the tube travel length, the velocity amplitude is smaller and the flow tends to drift into a down flow.

The second form of the pressure perturbation was chosen to be oscillatory in space as well as in time. In this case, the maximum response is seen to occur when the wavelength of the perturbation matches with the tube travel length (Figure 2). This confirms the resonance predicted by Roberts (1979) on the basis of linear analysis. It must

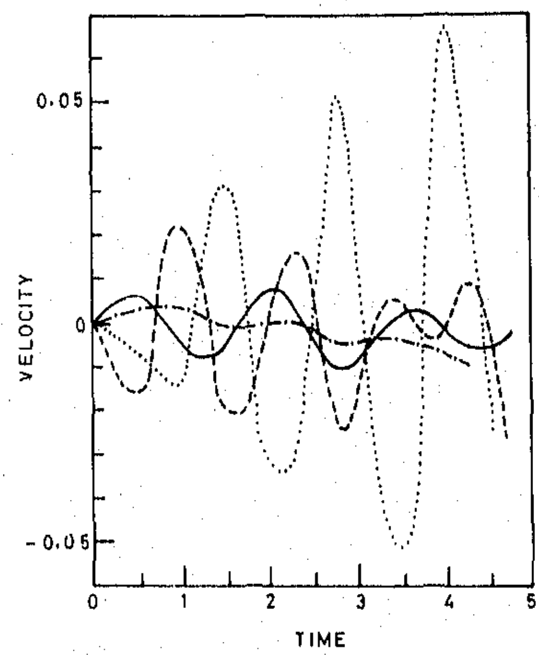

Fig.2. Same as in Figure 1 but for wavelike pressure fluctuations for various ratios λ of wave to tube speed.

be seen, however, whether the resonant build up of amplitudes leads to a shock wave or whether the resonance is destroyed at large times.

For the solar atmosphere if one takes the dimensions of the scale height of granular velocity decay into account then it would seem from these results that for strong tubes the most preferred response would be an oscillatory flow.

3.2. Response to Bending Motions

In order to study bending motions,

it is convenient to consider the motion of infinitesimal flux tubes with area of cross-section δA which is inversely proportional to the local field strength. For this case, we imposed a lateral velocity for the field line and studied the resulting flow along the field. The equations relevant to this study were written in a frame of reference moving with the field line, These euations are (Hasan and Venkatakrishnan, 1980):

$$\frac{dV_s}{dt} + V_s \frac{dV_s}{ds} = -\frac{1}{\rho}\frac{dp}{ds} - g\cos\theta + V_n\frac{d\theta}{dt} + V_n V_s \frac{d\theta}{ds}, \tag{3.1}$$

$$\frac{d\ln\rho}{dt} + V_s\frac{d\ln\rho}{ds} + \frac{dV_s}{ds} = -V_s\frac{\partial\theta}{\partial n} + V_n\frac{d\theta}{ds} - \frac{\partial V_n}{\partial n}, \tag{3.2}$$

$$p/\rho^{\Gamma} = \text{constant}, \tag{3.3}$$

where p, ρ, V_s and θ are the gas pressure, density, velocity along the field, and the angle made by the field to the vertical respectively. The coordinate 's' is the distance measured along the field and V_n is the velocity normal to the field direction. Once again these equations were integrated by the method of characteristics. A detailed description of the resulting motion for two assumed forms of V_n is given in Hasan and Venkatakrishnan (1980). The main result is that gas is seen to be accelerated in the direction of increasing V_n. Physically this effect is because of the centrifugal acceleration on the gas constrained to move in a curved path because of the non-uniform V_n.

This effect can be applied to the case of granule flux tube interaction. It is known that the rms granular velocity decreases with height (Durrant et al. 1979). Thus if the tube is bent with a corresponding lateral velocity decreasing with height, then a downflow would result. A numerical simulation of this effect was performed by Venkatakrishnan and Hasan (1981). The resulting velocities (Figure 3) compared favourably with some observations (e.g. those of Giovanelli et al. 1978). A detailed account of the limitations of such an approach of modelling the interaction can be found in Venkatakrishnan (1984b).

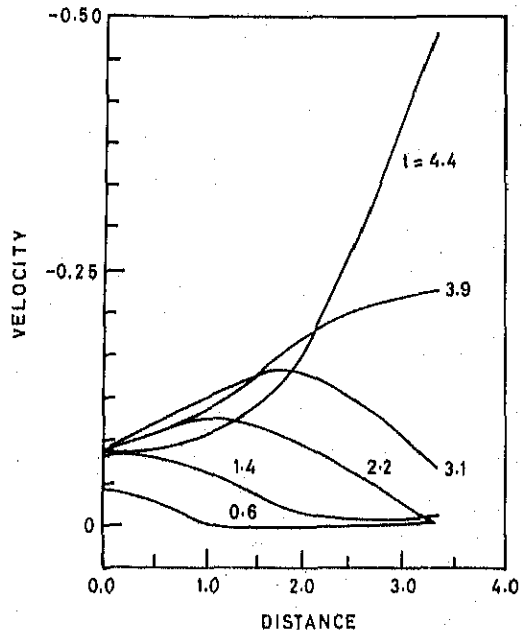

Fig.3. Interaction of granules with magnetic flux tubes - evolution of spatial velocity profiles.

4. Application to Solar Kilogauss Fields

There are two ways in which one can consider these results in the context of the tiny kilogauss flux tubes found in the solar photosphere. If one considers the dynamics of the gas within these flux tubes, then it would appear from all the above results that convective collapse with heat transport and external pressure fluctuations both produce only oscillatory motion. It is only the buffetting of granules that can be expected to drive the observed downflows in these tubes. The problem of gas supply exists, however, and there is a school of thought which even doubts the reality of the downflows. However, the fact remains that the stochastic jostling of flux tubes with a spatially stationary gradient in the rms amplitude would necessarily lead to a downflow. The question only remains as to the exact magnitude of the downflow. Recent observations using the 1-m FTS at Kitt Peak National Observatory (Stenflo et al. 1984) do demonstrate the necessity of including velocity gradients in flux tube models.

The second aspect concerns the problem of convective collapse. For polytropic tubes we have shown that radiative heat diffusion shifts the state of marginal stability to lower values of the field and introduces oscillations of the field strength as well. It remains to be seen whether calculations for non-polytropic realistic tubes would yield similar results. Such calculations would put better theoretical limits on the minimum field strength of the flux tubes and also provide the exact amplitudes and periods of the oscillations, if any.

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DISCUSSION:

BALASUBRAMANIAM: What sort of observations can one make, to look for these oscillations in sunspots? How do you think they will show up?

VENKATAKRISHNAN: Oscillations are already known to occur in sunspots. My work is aimed at tiny flux tubes. Only observations of high spatial resolution can detect the oscillations of field strength. Velocity oscillations can be detected using the shift in the zero point of the V-Stokes profile but one needs high spectral resolution in this case.

GOPALSWAMY: Can your computations be used for some inferences regarding the three tier nature of solar convection zone? For example, in the middle convection tier a single strong flux tube gives rise to many thin flux tubes in the upper tier, where the radiation becomes very important. Can the oscillation of the middle tier flux tubes be considered as the temporal shaking of the thin upper tier flux tubes?

VENKATAKRISHNAN: One can always think of a scenario where the hierarchy of magnetic structures is governed by the hierarchy of convective cells. However this is beyond the scope of my work. Moreover my work is concerned with the dynamics of individual tubes and can therefore say nothing regarding aggregates of tubes coupled to each other.