THE ASTROPHYSICAL JOURNAL, 311:680–683, 1986 December 15

EFFECTS OF SPACETIME CURVATURE AND ROTATION ON ARRIVAL TIMES OF PULSES FROM FAST PULSARS

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Received 1985 December 26; accepted 1986 May 22

ABSTRACT

General relativistic effects due to spacetime curvature and rotation on the arrival times of pulsar signals are investigated using a rotationally perturbed spherical metric. We find that for the milliscecond pulsar PSR 1937+214, the advancement in arrival time (assuming the usual polar cap model for emission mechanism and a radius-to-frequency mapping) due to these effects is nontrivial. The results are compared with recent multi-frequency timing measurements on PSR 1937+214, and the implications regarding emission region thickness and emission mechanism for fast pulsars are pointed out.

Subject headings: pulsars — relativity

I. INTRODUCTION

The discovery of fast pulsars (Backer *et al.* 1982; Boriakoff, Buccheri, and Fauci 1983) has generated a renewed interest in the effects of rapid rotation on pulsar properties. The requirement of stability against large rotation implies that such pulsars be compact (Datta and Ray 1983). In this paper we point out that large spacetime curvature and rotation-induced dragging of inertial frames will play an important role in deciding the arrival times of pulsed signals from a fast pulsar. We do this using a rotationally perturbed spherical metric and in the context of the usual polar cap model of pulsed emission with a radius-to-frequency mapping. We demonstrate that the net arrival time advancement of a lower frequency pulse over a higher frequency one due to the above effects is significantly higher than what will result from classical considerations. For the fast pulsar PSR 1937+214 we find that, for a reasonable choice of the emission region thickness, the advancement in arrival time is several tens of microseconds, comparable in magnitude to the pulse duration. Possible implications of our results in relation to emission region thickness and emission mechanism for fast pulsars are discussed.

II. PULSE ARRIVAL TIME ADVANCEMENT

We make the assumption that magnetic polar cap models (Radhakrishnan and Cooke 1969; Sturrock 1971; Ruderman and Sutherland 1975) describe the pulse emission mechanism; that is, the observed pulsar radiation is emitted in a cone pattern centered along the magnetic axis. In such models, the frequency of the emitted coherent radiation depends on the radial location ($r = r_e$) of the source, a higher frequency pulse having a smaller r_e, and pulses at different frequencies correspond to a set of nested cones with a common axis. Spacetime curvature and rotation will imply that there will be a finite difference in the arrival times of pulses at two frequencies ν_1 and ν_2 ($<\nu_1$) emitted from $r = r_e$ and $r = r_e + \Delta r$. The arrival time of the latter will be advanced over that of the former by an amount given by

$$\Delta\tau = \Delta\tau_{\rm ret} + \Delta\tau_{\rm drag} . \qquad (1)$$

The retardation time $\Delta\tau_{\rm ret}$ is the time for the radiation to travel the distance Δr, assuming that the pulses from r_e and $r_e + \Delta r$ start out simultaneously. The term $\Delta\tau_{\rm drag}$ will arise from the effect of rotation. In the absence of general relativistic corrections, $\Delta\tau_{\rm drag}$ corresponds to the classical aberration formula.

A proper determination of $\Delta\tau$, relevant for fast pulsars, will require a solution of equations of motion for photons in a curved spacetime and incorporating the effects of rotation. For this purpose, it is reasonable to use a rotationally perturbed spherical metric of the following form (Hartle and Thorne 1968; Thorne 1971) (signature: $+ - - -$), with its interior form matching to its exterior form at the surface of the pulsar:

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$
$$= e^{2\nu}dt^2 - e^{2\psi}(d\phi - \omega dt)^2 - e^{2\mu}d\theta^2 - e^{2\lambda}dr^2 , \qquad (2)$$

where ω is the the r-dependent angular velocity of the cumulative dragging of inertial frames induced by rotation and given by

$$\omega = 2J/r^3 ,$$

where J is the angular momentum of the pulsar, and ν, ψ, μ, and λ are terms that depend on the structure of the pulsar. The metric given by equation (2) is valid for strong gravitational fields and for uniform rotation rates that have as the upper bound the critical speed for centrifugal breakup. Neutron star models with rotation at the secular instability limit (assuming the star to be homogeneous), relevant in the context of fast pulsars, are within this bound (Ray and Datta 1984).

The retardation correction for the spacetime metric equation (2) is given by

$$\Delta\tau_{\rm ret} = \int_{r_e}^{r_e+\Delta r} \frac{dt/d\Gamma}{dr/d\Gamma}\, dr , \qquad (3)$$

where Γ is an affine parameter. The equations of motion for photons corresponding to the spacetime metric provide $dt/d\Gamma$ and the null condition for photons provides $dr/d\Gamma$ (Kapoor and Datta 1985), so that corresponding to the metric (2), equation (3) becomes

$$\Delta\tau_{\rm ret} = \int_{r_e}^{r_e+\Delta r} \frac{(1+\omega q_e)dr}{e^{2\nu}[(1+\omega q_e)^2 - q_e{}^2 e^{2\nu-2\psi}]^{1/2}} , \qquad (4)$$

where the quantity q_e is the photon impact parameter (Kapoor

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and Datta 1984):

$$q_e = \left. \frac{e^{\psi-\nu}(v_s + \sin\delta)}{1 + e^{\psi-\nu}(\omega v_s + \Omega \sin\delta)} \right|_{r=r_e}, \tag{5}$$

$$v_s = e^{\psi-\nu}(\Omega - \omega) . \tag{6}$$

Here Ω is the angular velocity of the pulsar as seen by a remote observer, δ is the azimuthal angle of the emitted photon with respect to the radius vector of the source through the origin of coordinates seen in the rest frame of the emitter such that it increases in a direction opposite to that of rotation, and θ_s is the polar angle of emission made by the line of sight through the center of the pulsar with respect to the axis of rotation (Datta and Kapoor 1985). Since polar deflection will be negligible for the rotationally perturbed spherical metric (2), θ_s can be taken to be identical to the angle of inclination as seen by a distant observer.

We wish to emphasize that equation (3) is the relativistic generalization of the simple retardation-correction formula $\Delta r/c$ that is usually adopted in the literature (see, e.g., Cordes and Stinebring 1984; Davis *et al.* 1985), and this leads to corrections amounting to several microseconds.

When the rotation rate of the pulsar is large, the use of the classical aberration formula (Cordes 1978) to calculate the rotation-induced arrival time advancement is necessarily an underestimation. To deal with the problem correctly, it is imperative that a general relativistic generalization of the classical aberration formula be employed. Such a generalization should take into consideration the rotation-induced general relativistic phenomenon of dragging of inertial frames in a curved spacetime.

The inertial frame drag effect refers to rotation of local inertial frames near a rotating gravitating body with respect to inertial frames at infinity (Misner, Thorne, and Wheeler 1973). For a photon trajectory near a fast pulsar, this will imply a change in the usual gravitational light deflection formula. With reference to the polar cap model, the inertial frame drag effect will imply a tilt of the pulse cone axis toward the direction of rotation of the pulsar (Datta and Kapoor 1985; Kapoor and Datta 1985). The tilt angle is equivalent to the net deflection of a photon that is emitted radially outwards (i.e., $\delta = 0$) in the local rest frame of emission.

To evaluate $\Delta\tau_{\text{drag}}$, we first note that the net deflection of a photon trajectory will be given by (since polar bending is negligible):

$$-\Phi(r_e, \theta_s, \delta) = \int_{r_e}^{D} \frac{d\phi/d\Gamma}{dr/d\Gamma}\, dr , \tag{7}$$

where D is the radial location of the distant observer. The quantity $d\phi/d\Gamma$ is obtained once the equations of motion for photon are written down. Equation (7) is general, applicable both to the polar cap models (for which $\delta \approx 0°$) as well as to the relativistic beaming model (for which $\delta \approx -\pi/2$) of the pulsed emission mechanism. The effect, however, is more appreciable in the former case. For the metric (2), equation (7) gives

$$-\Phi(r_e, \theta_s, \delta) = \int_{r_e}^{D} \frac{[\omega(1 + \omega q_e) - q_e e^{2\nu-2\psi}]dr}{e^{\nu-\lambda}[(1 + \omega q_e)^2 - q_e^2 e^{2\nu-2\psi}]^{1/2}} . \tag{8}$$

The tilt of the pulse cone axis caused by the inertial frame drag will be given by $|\Phi(r_e, \theta_s, \delta = 0)|$. The corresponding arrival time advancement is then given by

$$\Delta\tau_{\text{drag}} = \frac{P}{2\pi} [\,|\Phi(r_e + \Delta r_e, \theta_s, 0)| - |\Phi(r_e, \theta_s, 0)|\,] , \tag{9}$$

where P is the period of the pulsar.

III. RESULTS AND DISCUSSION

To illustrate the effect discussed above, we consider the fastest rotating pulsar discovered so far, namely PSR 1937+214 with $P = 1.5577$ ms, taking its mass to be 1.4 $M_\odot$, which is a representative choice consistent with the analysis of binary pulsar data. For the present calculation, the relevant structural parameters of this pulsar corresponding to the interior form of metric (2) have been taken from our previous work (Ray and Datta 1984) for five representative models of the equation of state for neutron star matter—(*a*) Reid-Pandharipande (RP), (*b*) Friedman-Pandharipande (FP), (*c*) Canuto-Datta-Kalman (CDK), (*d*) Bethe-Johnson model I (BJ), and (*e*) tensor interaction (TI)—and summarized in Table 1.

Most pulsar emission mechanism models place the region of emission well within the light cylinder. So, for numerical calculations we take $r_e = R'$, $2R'$, and $3R'$, where R' is the radius of the rotating neutron star. Recent observations (Cordes and Stinebring 1984) suggest that the emission region for the millisecond pulsar PSR 1937+214 is thin. being $\sim 0.25R'$ for the frequency range 0.3–1.4 GHz. To illustrate the magnitude of the effect discussed in this paper, we, therefore, choose $\Delta r = 0.25R'$ and $0.5R'$ for the sake of seeing the trend of the results. A choice of $D = 700R'$ was found to be adequate to numerically evaluate equation (8), which gives the value of the tilt angle. As already mentioned, tilt is the bending of the radially emitted photon in the emitter's frame of reference and can be regarded as referring to the peak of the pulse or the midpoint of an average pulse that possesses a more detailed structure.

The results for the tilt angle and the arrival time advancement are shown in Tables 2 and 3. As can be seen from these tables, the net arrival time advancement is substantial: $\sim$(20–50) μs, a range comparable to the observed pulse duration, since the pulse width for PSR 1937+214 is $\sim$(15°–30°). Values of the retardation correction $\Delta\tau_{\text{ret}}$, calculated according to equation (4), are found to differ significantly from a simple $\Delta r/c$ value and possess a dependence on the polar emission angle. Since the emitter describes a corotating orbit around the pulsar, the tilt increases as r_e increases. However, $\Delta\tau$ first decreases with increasing r_e, has a minimum around $r_e \approx 2R'$, and thereafter increases with r_e. This behavior can be seen to follow from an examination of equations (4) and (8). The physi-

TABLE 1

RADIUS AND ANGULAR MOMENTUM OF ROTATING NEUTRON STAR MODELS FOR PSR 1937+214 ($M = 1.4\ M_\odot$) CORRESPONDING TO DIFFERENT EQUATIONS OF STATE

Equation of State	Radius (10^6 cm)	Angular Momentum (10^{48} g cm^2 s^{-1})
RP	1.029	6.084
FP	1.119	6.492
CDK	1.176	6.675
BJ	1.323	6.105
TI	1.691	6.636

TABLE 2

TILT $|\Phi(r, \theta_s, 0)|$, RETARDATION CORRECTION ($\Delta\tau_{ret}$), AND NET ARRIVAL TIME ADVANCEMENT ($\Delta\tau$) CORRESPONDING TO EMISSION REGION THICKNESS $\Delta r = 0.25R'$

		$r_e = R'$				$r_e = 2R'$				$r_e = 3R'$			
EOS	θ_s	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)
RP	54°	9°17	12°14	13.06	25.91	18°73	20°73	10.58	19.23	26°66	28°65	10.22	18.83
	90	9.16	12.15	13.10	26.04	18.82	20.87	10.75	19.62	27.02	29.11	10.60	19.64
FP	54	10.26	13.23	12.72	25.57	20.20	22.37	10.51	19.90	28.82	30.99	10.26	19.65
	90	10.25	13.24	12.77	25.71	20.32	22.55	10.70	20.35	29.29	31.60	10.74	20.74
CDK	54	10.89	13.87	12.48	25.37	21.09	23.36	10.45	20.27	30.16	32.46	10.29	20.24
	90	10.88	13.89	12.53	25.55	21.24	23.58	10.67	20.80	30.71	33.18	10.83	21.52
BJ	54	12.56	15.52	11.85	24.66	23.33	25.87	10.30	21.29	33.57	36.20	10.38	21.76
	90	12.57	15.55	11.92	24.81	23.55	26.18	10.58	21.96	34.39	37.28	11.12	23.62
TI	54	15.69	19.12	11.02	25.86	28.95	32.23	10.24	24.43	42.53	46.18	10.99	26.78
	90	15.73	19.22	11.12	26.22	29.41	32.90	10.72	25.82	44.56	49.05	12.65	32.08

TABLE 3

TILT, RETARDATION CORRECTION, AND NET ARRIVAL TIME ADVANCEMENT FOR $\Delta r = 0.5R'$ [a]

		$r_e = R'$				$r_e = 2R'$				$r_e = 3R'$			
EOS	θ_s	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)	Φ_{r_e}	$\Phi_{r_e+\Delta r}$	$\Delta\tau_{ret}$ (μs)	$\Delta\tau$ (μs)
RP	54°	9°17	14°52	24.92	48.06	18°73	22°72	20.87	38.13	26°66	30°64	20.24	37.44
	90	9.16	14.55	24.99	48.29	18.82	22.91	21.16	38.87	27.02	31.23	20.95	39.16
FP	54	10.26	15.71	24.42	47.98	20.20	24.52	20.75	39.43	28.82	33.19	20.33	39.21
	90	10.25	15.75	24.49	48.27	20.32	24.78	21.10	40.36	29.29	33.97	21.21	41.46
CDK	54	10.89	16.41	24.04	47.95	21.09	25.62	20.65	40.26	30.16	34.78	20.39	40.37
	90	10.88	16.46	24.13	48.28	21.24	25.92	21.04	41.33	30.71	35.71	21.38	43.02
BJ	54	12.56	18.20	23.01	47.39	23.33	28.41	20.39	42.37	33.57	38.88	20.36	43.35
	90	12.57	18.28	23.12	47.83	23.55	28.85	20.88	43.85	34.39	40.31	21.92	47.54
TI	54	15.69	22.41	21.61	50.72	28.95	35.57	20.28	48.94	42.53	50.02	21.73	54.14
	90	15.73	22.60	21.81	51.54	29.41	36.59	21.17	52.26	44.56	54.09	24.77	65.99

[a] Formulas, etc., same as in Table 2.

cal interpretation of this is that, for $r_e > 2R'$, special relativistic effects start becoming more important and the general relativistic effects start losing their prominence.

Timing observations on the millisecond pulsar PSR 1937+214 between the frequencies 0.3 and 1.4 GHZ limit the time delay discrepancies to within 6 μs (Cordes and Stinebring 1984). Our results show that the net arrival time advancement, taking into account spacetime curvature and relativistic effects of rotation, is substantially larger than the reported discrepancy of 6 μs. Therefore, to be in conformity with the observations, one possible conclusion that can be inferred on the basis of our calculations is that the emission region is even *thinner* than that suggested by Cordes and Stinebring (1984). On the other hand, if one insists on retaining a value $\sim 0.25R'$ for the emission region thickness, alternate implications are that (*a*) a polar cap model *without* a radius-to-frequency mapping (Arons 1979) is perhaps a more viable model of pulsed emission in the case of fast pulsars, and (*b*) the radiation mechanism is *altogether different* from the standard polar cap model, as also suggested by the extremely narrow width of PSR 1937+214 (Backer 1984).

To summarize, our investigations establish that the arrival time advancement in the context of the polar cap model for a fast pulsar (like PSR 1937+214), induced by spacetime curvature and inertial frame drag effect due to rotation, is nontrivial. In relation to available observational information, this seems to suggest a need for a revision of current notions regarding the emission region thickness or perhaps the emission mechanism itself. We would like to add here that it is quite possible that large rotation tends to bunch together the pulsar magnetic field lines (Ruderman 1985), which will then push the emission region radially further outward. However, this would not wash away the arrival time advancement pointed out in this paper, as is evident from Tables 2 and 3. The calculations reported here will be refined if the vertical distribution of plasma is taken into account, for in that case the emission region is more exactly specified.

The authors thank Dr. S. Krishnamohan and the anonymous referee for useful suggestions. B. D. acknowledges the Indian National Science Academy for the award of Biren Roy Trust Fellowship, and R. C. K. thanks Prof. B. F. Schutz for hospitality at the Department of Applied Mathematics and Astronomy, University College, Cardiff.

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