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Letter to the Editor

Predictions of strengths of long-term variations in sunspot activity

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Abstract. Recently, using Greenwich data (1879–1976) and SOON/NOAA data (1977–2002) on sunspot groups we found a big or a moderate drop in the solar equatorial rotation rate, A, occurred after every four solar cycles suggesting the existence of "double Hale cycle (DHC)" and "Gleissberg cycle (GC)" in A. We also found the existence of "Hale cycle (HC)" and GC in the latitude gradient of the rotation, B (Javaraiah 2003). Using these results here we made forecasts for the following: (i) epochs of the forthcoming big and moderate drops in A; (ii) the epoch of maximum $|B|$ during the current GC of B; (iii) the strengths of DHCs and HCs of sunspot activity which follow the big and the moderate drops in A; (iv) violation of the Gnevyshev & Ohl rule during the current HC 11 which consists of cycles 22 and 23; and (v) deduced the near complete absence of sunspot activity during the deep Maunder minimum.

Key words. Sun: rotation – Sun: activity – Sun: sunspots

1. Introduction

Study of variations in solar activity is not only important for understanding the physical process inside the Sun, but also provides information on variations of the solar-terrestrial environment. A vast amount of research is carried out in the worldwide to understand the underlying mechanism of the 11 yr sunspot cycle and to predict its amplitude well in advance. A wide variety of statistical methods have been proposed to predict the amplitudes of 11 yr sunspot cycles (e.g., see Li et al. 2001; Kane 2001).

It is well known that a 11 yr sunspot cycle represents half of a Hale's 22-yr magnetic cycle. A number of statistical studies of solar activity also suggested a physical relationship between neighboring 11 yr activity cycles. The well known Gnevyshev-Ohl rule or G–O rule (Gnevyshev & Ohl 1948) states that the sum of sunspot numbers over an odd-numbered sunspot cycle exceeds that of its preceding even-numbered cycle. However, some pairs of the even and the odd numbered cycles violate this rule. Recently, Komitov & Boney (2001) found that violation of the G–O rule could be not random phenomena but occurring under special conditions, the main factor being the very high maximum of the even-numbered cycle. Figure 1 shows the variation of the monthly Wolf number during 1749–2002. In Table 1 we give the $R_{\rm sum}$, $H_{\rm sum}$ and $D_{\rm sum}$, viz., the sums of the monthly averaged sunspots over the durations of sunspot cycle, "double sunspot cycle" or HC and DHC, respectively. In this table we also give the durations or lengths (PERs) of the sunspot cycles. We have taken the values of $R_{\rm sum}$, $H_{\rm sum}$ and PER for cycles 1–21 from the paper by Wilson (1988). For cycles 22 and 23 we determined them from the average monthly values which were taken from the website: `http://science.nasa.gov/ssl/pad/solar/greenwich.htm`. In Table 1 and Fig. 1, it can be seen that the G–O rule was violated by the 11-yr cycles pair 4, 5 (HC 2) and likely to be violated by the cycles pair 22, 23 (HC 11).

By using the G–O rule, it is possible to predict the $R_{\rm sum}$ of an odd number cycle from that of its preceding even numbered cycle with a reasonable accuracy (e.g., see Wilson 1988). To the best of our knowledge, so far no reliable methods are available to predict the strengths of even numbered activity cycles and the variations of activity on time scales longer than a 11-yr cycle.

Interactions of the Sun's differential rotation and magnetic field play a basic role in generation of all solar activity (Babcock 1961). However, role of the differential rotation in cyclic variation of activity is not yet clear. The differential rotation can be determined accurately by fitting a large set of the data on sunspot or sunspot groups to the standard form: $\omega(\phi) = A + B\sin^2\phi$, where $\omega(\phi)$ is the solar sidereal angular velocity at latitude ϕ, the coefficients A and B represent the equatorial rotation rate and the latitudinal gradient of the rotation, respectively. Recently, we studied cycle-to-cycle modulations in A and B using Greenwich data (1879–1976) and SOON/NOAA data (1977–2002) on sunspot groups (Javaraiah 2003). We found, besides the known big drop in A from cycle 13 to cycle 14, the existence of a moderate drop

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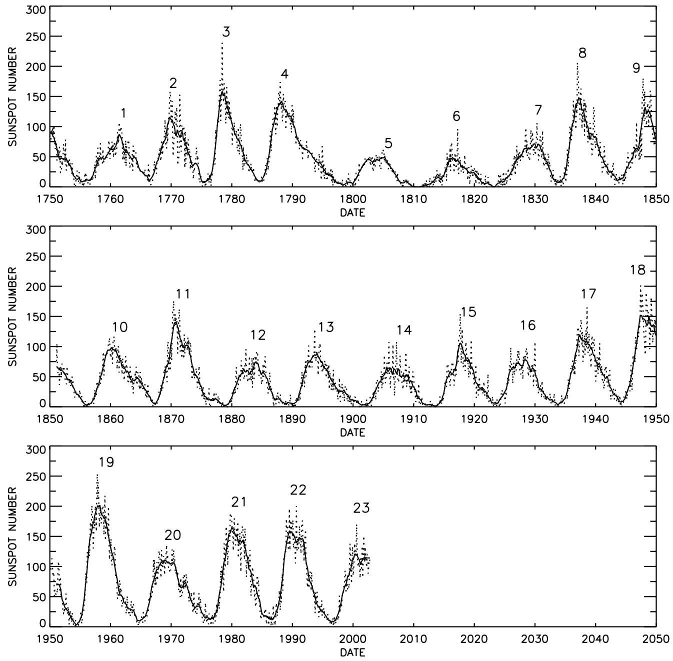

Fig. 1. Monthly-averaged (dotted curve) and smoothed (continuous curve) international sunspot numbers during 1749–2002 (`ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS`). The Waldmeier cycle number is marked near the top of each peak.

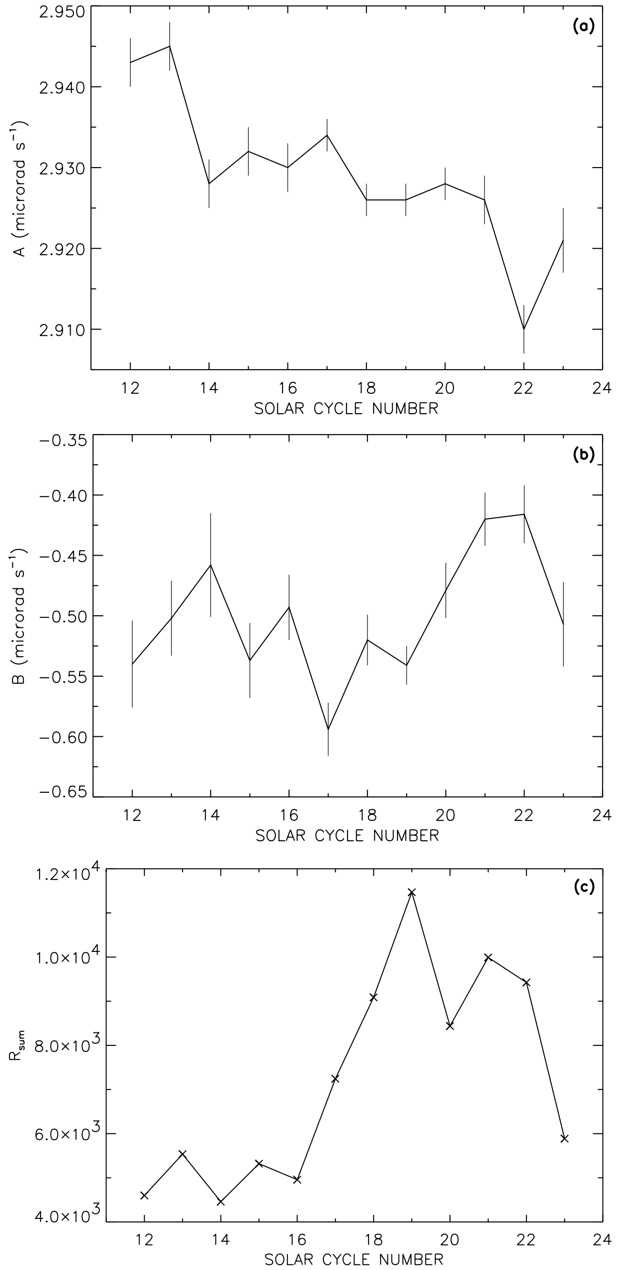

Fig. 2. Cycle-to-cycle variations of A, B and $R_{\rm sum}$. (Note: cycle 23 is not yet complete.)

from cycle 17 to cycle 18 and a big drop from cycle 21 to cycle 22 as that of the one from cycle 13 to cycle 14 (see Fig. 2a). Also, in the paper by Pulkkinen & Tuominen (1998) we noticed that the value of A (Carrington/Spörer data) in cycle 10 is considerably lower than those in cycles 11, 12 and 13. This low value suggests that a big or a moderate drop in A might have occurred from cycle 9 to cycle 10. Hence, we concluded that a big or a moderate drop in A is occurring after every four cycles suggesting the existence of "44-yr" cycles or DHCs in A. The gap between the aforesaid two big drops suggests the existence of a "90-yr" cycle or GC in A. We also found the existence of a "90-yr" cycle in B from cycle 14 to cycle 22, with maximum $|B|$ during cycle 17 (see Fig. 2b). Using these results in the present letter we made forecasts for the strengths of the long-term variations in sunspot activity.

2. Big drops in *A and strengths of "double Hale cycles" in sunspot activity*

Using Table 1 and Fig. 2a one can see that the big drop in A from cycle 13 to cycle 14 was followed by DHC 4 whose $D_{\rm sum}$ is considerably lower than those of both DHC 3 and DHC 5, the moderate drop in A from cycle 17 to cycle 18 was followed by DHC 5 whose $D_{\rm sum}$ is considerably larger than that of DHC 4. The $D_{\rm sum}$ of DHC 2 is relatively lower than those of both DHC 1 and DHC 3. So, a big drop and a modern drop in A might have occurred from cycle 5 to cycle 6 and from cycle 1 to cycle 2, respectively. Thus, using the epochs of the big and the moderate drops in the cycle-to-cycle modulation of A shown in Fig. 2a and the pattern of modulation in the strengths of the DHCs given in Table 1, one can predict the epochs of big and moderate drops in A several decades ahead, and hence can predict the strengths of DHCs of sunspot activity which follow big and moderate drops in A. So, the $D_{\rm sum}$ of the current DHC 6 which consists of the cycles 22, 23, 24 and 25, and follows the big drop in A from cycle 21 to 22, is expected to be less than that of DHC 5. We can predict a moderate drop in A from cycle 25 to cycle 26. This will be followed by DHC 7 which consists of the cycles 26, 27, 28 and 29 and whose $D_{\rm sum}$ is expected to be relatively larger than that of DHC 6. Obviously, the aforementioned patterns in activity variation constitute the well known GCs of sunspot activity.

In the empirical rule, suggested above, size of a drop in A is the predictor of the strength of the DHC which follows the drop. However, if we assume that a change in activity leads to

Table 1. Values of cycle length (PER, in months), $R_{\rm sum}$ (sum of monthly means of Wolf numbers), $H_{\rm sum}$ and $D_{\rm sum}$.

Cycle	PER	$R_{\rm sum}$	HC	$H_{\rm sum}$	DHC	$D_{\rm sum}$
1	135	5647.2				
2	109	6438.7				
			1	13845.8		
3	111	7407.1				
					1	27359.1
4	163	10100.6				
			2	13513.3		
5	147	3412.7				
6	153	2820.5				
			3	7588.6		
7	127	4768.1				
					2	23712.1
8	116	7813.2				
			4	16123.5		
9	149	8310.3				
10	135	6549.7				
			5	14056.1		
11	141	7506.4				
					3	24190.0
12	134	4598.2				
			6	10133.9		
13	143	5535.7				
14	138	4459.1				
			7	9778.9		
15	120	5319.8				
					4	21979.2
16	122	4956.9				
			8	12200.3		
17	125	7243.4				
18	122	9087.4				
			9	20556.5		
19	126	11469.1				
					5	38986.2
20	140	8438.2				
			10	18429.7		
21	123	9991.5				
22	124	9424.7				
			11			
23	69[a]	5886.3[a]				

[a] indicates the incompleteness of the present cycle 23.

a variation in rotation, then it implies a weak DHC is followed by a moderate drop in A and a strong DHC is followed by a big drop in A. In this scenario, the strength of a DHC is a predictor of the size of a drop in A. However, it seems a drop in A occurs in the beginning cycle of a DHC and then seems to be persisting during a few more cycles. Hence, the size of a drop in A may be a plausible predictor of the strength of the DHC during which the drop occurs rather than the strength of the preceding DHC is a predictor of the size of a drop in A.

The big drop ($\approx$0.017 microrad s^{-1}) in A from cycle 13 to cycle 14 is about 0.58%. The $D_{\rm sum}$ of DHC 4 which followed the big drop in A from cycle 13 to cycle 14 is about 9.1% less than that of DHC 3. The big drop ($\approx$0.016 microrad s^{-1}) in A from cycle 21 to cycle 22 is about 0.55% and almost equal to the drop from cycle 13 to cycle 14. Hence, the $D_{\rm sum}$ of DHC 6 is expected to be also about 9.1% less than that of DHC 5, i.e., $\approx$35 477.

Interestingly, $\frac{D_{\rm sum}\ \text{of DHC 2}}{D_{\rm sum}\ \text{of DHC 1}} \approx \frac{D_{\rm sum}\ \text{of DHC 4}}{D_{\rm sum}\ \text{of DHC 3}}$. Hence, we may have $\frac{D_{\rm sum}\ \text{of DHC 4}}{D_{\rm sum}\ \text{of DHC 3}} \approx \frac{D_{\rm sum}\ \text{of DHC 6}}{D_{\rm sum}\ \text{of DHC 5}}$, which also leads to the aforesaid estimated value of $D_{\rm sum}$ of DHC 6.

In Fig. 2 it can be seen that over a time-scale of the order of 100 years or more, i.e., substantially longer than a GC, activity and A are strongly increased and decreased with time, respectively. The GCs or DHCs seem to be superposed on this relatively strong variation on a time scale of substantially longer than a GC. There exists about 60% anticorrelation between A and amount of activity. However, the correlation between A and activity seems not negative throughout a GC. In fact, we find about 92% possitive and 66% negative correlations between A and activity within DHC 4 and DHC 5, respectively. Within DHC 5 the variation in A is insignificant and ambiguous. The slope of the variation in A within DHC 4 is relatively very small compared to the size of a big drop or the slope of the relatively strong decrease in A over a time scale of longer than the length of a GC. So, it seems not possible to use the small variations in A within the DHCs 4 and 5 to derive a statistical measure of significance of modulations in strengths of DHCs in activity. For this purpose the available data (number of drops in A) are inadequate.

3. Strengths of "Hale cycles" in sunspot activity

In Table 1 one can also see that within DHC 4 of activity, which followed the big drop in A from cycle 13 to cycle 14, the $H_{\rm sum}$ of the "preceding" HC 7 is considerably less than that of the "following" HC 8. This is opposite in DHC 5 which followed the moderate drop in A from cycle 17 to cycle 18, i.e., the $H_{\rm sum}$ of HC 9 is larger than that of HC 10. This property exists in the earlier DHCs also, including even DHC 1 during which HC 2 violated the G–O rule. Thus, the $H_{\rm sum}$ of the current HC 11 (cycles pair 22, 23) is expected to be considerably less than that of HC 12 (cycles pair 24, 25) and the $H_{\rm sum}$ of HC 13 (cycles pair 26, 27) is expected to be considerably larger than that of HC 14 (cycles pair 28, 29).

The strength of a HC in a particular DHC seems to be related to the closest HC of the adjacent DHC in such way that the weak and the strong HCs of the DHC are close to the weak HC of the preceding DHC and the strong HC of the following DHC, respectively. Thus, the weak HC 10 in DHC 5 leads to a predicted weak HC 11 in DHC 6.

Since $D_{\rm sum}$ of DHC 6 $\approx$ 35 477 (estimated in see Sect. 2), $H_{\rm sum}$ of HC 11 is expected to be less than $\approx\frac{1}{2}\times 35\,477$, i.e., less than $\approx$17 738. Since $R_{\rm sum}$ of cycle 22 = 9425 (from Table 1), the $R_{\rm sum}$ of cycle 23 is expected to be less than $\approx$8313. Thus, we predict violation of the G–O rule during the current HC 11.

The G–O rule relates only strength of an even cycle to that of its following odd cycle. A relationship between the strength of an odd cycle to that of its following even cycle is not known so far. It is worthwhile to note here that a big or a moderate drop in A seems to be always taking place from an odd cycle to its following even cycle. Between the odd and even cycles

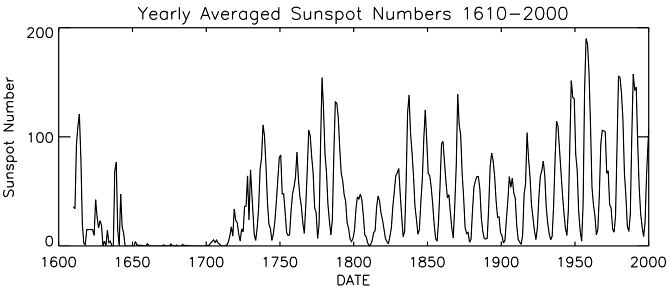

Fig. 3. Yearly-averaged Wolf sunspot numbers 1610–2000 (`http://science.msfc.nasa.gov/ssl/pad/solar/sunspots.htm`).

during which a big drop in *A* is occurring, always the former seems stronger than the latter. Hence, cycle 29 is expected to be stronger than cycle 30. Between the odd and even cycles during which a moderate drop in *A* is occurring, the former and the latter seem to be alternatively weaker or stronger during alternative moderate drops in *A*, indicating relatively stronger cycle 25 than cycle 26.

4. Gleissberg cycle in *B*

In Fig. 2 one can see that there exists a considerable anti-correlation (corr. coeff. = −0.54) between *A* and *B*. This anticorrelation implies that larger latitude gradient in the rotation is associated with faster equatorial rotation rate and vice versa. The GC in *B* from cycle 14 to cycle 22 seems to be in phase with the GC of sunspot activity which consists of DHC 4 and DHC 5. So, it seems the present GC in *B* is started from cycle 22, expected to have maximum $|B|$ during cycle 25 and ends during cycles 29–30.

5. Absence of activity during the deep Maunder minimum

If we extend our discussion in Sect. 2 to the earlier data, we can find that a big drop in *A* might have occurred from cycle 1611–1618 to cycle 1619–1633 (here DHCs were counted backward from cycle 1 using the file "maxmin.new" available in the website, cited in the caption of Fig. 1) and the systematic behavior in the amplitude modulation of DHC's in activity seems to hold good for the earlier DHCs also. However, it seems there were violations of the systematic behavior of amplitude modulation (cf. Sect. 3) of the HCs within some earlier DHCs. In view of there are large errors in the lengths and amplitudes of many earlier cycles, we expect that a big drop in *A* might have occurred near the beginning of the Maunder minimum (1645–1715, see Fig. 3) rather than the one cycle earlier. A big drop in *A* is about 0.5% during the modern time and is followed by a DHC whose $D_{\rm sum}$ is about 9% less than that of its preceding DHC (see Sect. 2). Recently, Vaquero et al. (2002) showed that during the deep Maunder minimum (1666–1700) the solar rotation rate near the equator was about 5% lower than during the modern time. Hence, the expected big drop in *A* near the beginning of the Maunder minimum might be about 10 times larger than a big drop during the modern time. This implies that the $D_{\rm sum}$ of the DHC which began at the beginning of the Maunder minimum might be about 90% lower than that of its preceding DHC, and also to that of a DHC during the modern time. This is in consistent with the observational evidence of the near complete absence of activity during the deep Maunder minimum.

6. Conclusions

Using the results in Javaraiah (2003), here we have made the following predictions: (i) The $D_{\rm sum}$ of the current DHC 6 in sunspot activity which follows the big drop in *A* from cycle 21 to cycle 22 is expected to be less than that of the DHC 5. The $D_{\rm sum}$ of the DHC 7 which will follow a moderate drop in *A* from cycle 25 to cycle 26 is expected to be larger than that of DHC 6; (ii) within DHC 6 the $H_{\rm sum}$ of the preceding HC 11 is expected to be less than that of the following HC 12, within DHC 7 the $H_{\rm sum}$ of the preceding HC 13 is expected to be larger than that of the following HC 14; (iii) HC 11 is most likely violate the G–O rule; (iv) cycles 25 and 29 are expected to be relatively stronger than cycles 26 and 30, respectively; (v) it seems the present GC of *B* is started during cycle 22, expected to have maximum $|B|$ during cycle 25 and ends during cycles 29–30; and (vi) the beginning of the Maunder minimum might have followed a big drop in *A* which might be about 10 times larger than a big drop during the modern time and related to the near complete absence of activity during the deep Maunder minimum.

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References

Babcock, H. W. 1961, ApJ, 133, 572
Gnevyshev, M. N., & Ohl, A. I. 1948, AZh, 25, 18
Javaraiah, J. 2003, Sol. Phys., 212, 23
Kane, R. P. 2001, Sol. Phys., 202, 395
Komitov, B. J., & Boney, B. 2001, ApJ, 554, L119
Li, K. J., Yun, H. S., & Gu, X. M. 2001, A&A, 368, 285
Pulkkinen, P., & Tuominen, I. 1998, A&A, 332, 748
Vaquero, S. D., Sánchez-Bajo, F., & Gallego. M. C. 2002, Sol. Phys., 207, 219
Wilson, R. M. 1988, Sol. Phys., 117, 269