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CORONAL LOOPS

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Abstract

The statistical treatment of the magnetohydrodynamically turbulent plasma is used to study the steady state configuration of the coronal loops. The stability of the plasma system is defined in terms of certain invariants like total energy, magnetic helicity and magnetic fluxes. The steady state is represented by a superposition of two Chandrasekhar-Kendall functions. This representation defines the three dimensional temperature structure of the coronal loops. We present only two dimensional (r,z) variation of temperature in a cylindrical geometry of the loop. The radial as well as the axial variation of the temperature in a constant density loop is calculated. These variations are found to conform to the observed features of cool core and hot sheath of the loops as well as to the location of the temperature maximum at the apex of the loop. We find that these features are not present uniformly all along either the length of the loop or across the radius. We further study the nature of and relationship between the velocity field and magnetic field fluctuations using the nonlinear MHD equations and the statistical description of MHD turbulence. It is found that these fluctuations are of Alfvenic type. A possible existence of Alfven waves with group velocity along and opposite to the average magnetic field is shown. This study has important bearing on the heating mechanism of coronal loops.

1. Introduction

The study of solar coronal loops has the potential of revealing the interior as well as the exterior structure of the solar atmosphere as the foot points of these structures lie deep in the convection zone with their tops extending out in the corona sometimes up to 2 solar radii. Gas contained in such loops can attain temperature $\geq 10^6$ °K and emit electromagnetic radiation over a wide range of frequencies in the form of lines as well as in the continuum. Loops break through regions of strong magnetic field as in the case of sunspot loops. The foot points of these loops are continuously jostled by the eddies in the convection zone and through this mechanism, a part of the mechanical energy in the convection zone gets transferred up which then heats the corona. It is found that a very small fraction of the sun's total output is used in heating the corona and thus solar atmosphere is considered an example of an inefficient atmosphere in contrast to the atmospheres of some other stars where the loops are better tuned to respond to the motions in the convection zone as discussed by Mullan (1985).

Solar coronal loops have been seen in the emission at UV, EUV and X-ray wavelengths (Fourkal, 1975, 1976, 1978; Foukal et al. 1974; Levine and Withbroe, 1977; Vaiana and Rosner, 1978). The current carrying plasma in the loop supports a helical form of the magnetic field (Levine and Altschuler, 1974; Poleto et al. 1975; Hood and Priest, 1979). In this paper one dimensional and two dimensional steady state pressure profiles of a coronal loop are derived based on the Taylor's conjecture (1974, 1975, 1976) that the decay of energy to a minimum value compatible with a conserved value of the magnetic helicity leads to a force free state. Based on this selective decay hypothesis, a proposal for the probable initial configuration of a flaring loop has also been made (Krishan 1985b). We have also studied the statistical mechanics of the velocity and the magnetic fields in a coronal loops (Krishan 1985; Montgomery et al. 1978).

The statistical mechanical formulation of the MHD turbulence enables us to study the nature of fluctuations in the fields and their correlations if any. Here we also study the time evolution of the velocity and magnetic field fluctuations using fully nonlinear MHD equations. The loop plasma is assumed to be in the minimum energy state in which the pressure gradient is balanced by the convective derivative of the velocity field. A relationship between the velocity and magnetic fluctuations is sought which brings out the Alfvenic nature of these fluctuations. This is quite interesting since there are several ways in which Alfven waves have been held responsible for the heating of the loop plasma, Kuperus, Ionson and Spicer (1981), Chiuderi (1981), Wentzel (1981). Some of the theoretical concepts used in the present and earlier work on coronal loops have been scrutinized more directly in the case of solar wind where insitu observations of fluctuations are available (Matthaeus and Goldstein 1982; Dobrowolny et al. 1980; Grappin et al. 1982 and Matthaeus et al. 1983). Further, the statistical description of MHD turbulence (Krishan, 1985; Montgomery et al. 1978) has been used to investigate the relative amounts of energy in the velocity and magnetic fluctuations and thus discuss equipartition of energy.

2. Modeling of the Coronal Loops

In the existing models of the coronal loops, the radial and the longitudinal pressure or temperature (for constant density) structure are studied separately. Radial temperature structure is derived using the emission measure analysis of the lines observed in the loop. The intensity I of the emission line is given by:

$$I = C \, Ne \, Ni \, d$$

where C contains all the atomic parameters, Ne and Ni are the electron and the ion densities and d is the spatial width of the line. By measuring I and assuming a reasonable value of the electron density, one can find the value of d, the extent over which a line is excited and hence the extent over which the temperature is the formation temperature of the line. Thus, from the observations of lines of different formation temperatures one can reconstruct the radial temperature variation in the loops. It is concluded from these studies that the temperature increases with the radius such that the lines with higher formation temperature have larger spatial widths.

The longitudinal temperature structure of the coronal loop is derived by considering energy balance between thermal conduction, Radiation losses and the mechanical heating in the loop as:

$$\frac{d}{ds}\left[K_{11}\frac{dT}{ds}\right] = n^2 \chi T^{\alpha} - hn$$

Conduction Radiation Heat.

Using the boundary conditions: T = Constant at the base of the loop and $dT/ds = 0$ at the top, scaling laws relating temperature, pressure and the length of the loop are derived. An important outcome of this exercise was the realization that heating is a function of the geometry of the loop and therefore the heating mechanism is necessarily related to the magnetic field. Although this procedure of modeling is quite revealing one desires to describe the loopl through magnetohydrodynamic processes where the coronal loop is taken as volume of plasma isolated by defining magnetic field with radial and longitudinal variations. One should solve MHD equations subject to boundary conditions at the foot points and at the surface with additional constraints upon internal transport processes. Taking this line of thought, we propose to model the loops through statistical description of MHD turbulence.

In the steady state, the coronal loop should be in a state of minimum energy corresponding to a conserved value of the magnetic helicity. The steady state pressure structure of the coronal loop can be studied following Montgomery et al. (1978). The magnetic and velocity fields are expanded in terms of Chandrasekhar-Kendall functions. We represent the loop plasma by a minimum energy state which is the sum of the lowest mode (m = 0) axisymmetric state and the m = 1, non-axisymmetric state. This representation describes the three dimensional spatial pressure variation in the loop. Since the observations inform us only of the (r,z) pressure dependence, we shall not consider azimuthal dependence. The pressure (p) profile of an incompressible MHD plasma is given by

$$\frac{\nabla^2 p}{\rho} = \nabla \cdot [(\nabla \times \underset{\sim}{B}) \times \underset{\sim}{B}] - \nabla \cdot [(\underset{\sim}{V} \cdot \underset{\sim}{\nabla})\underset{\sim}{V}] \tag{A}$$

where $\underset{\sim}{V}$, $\underset{\sim}{B}$, are velocity and magnetic fields respectively and B is defined in Alfven speed units. The $\underset{\sim}{V}$ and $\underset{\sim}{B}$ for m = 0 and m = 1 state are given by Montgomery et al (1978) as:

$$\underset{\sim}{B} = \underset{\sim}{B}_0 + \underset{\sim}{B}_1$$

$$\underset{\sim}{B}_0 = \xi_0\lambda_0 C_0[\hat{e}_\theta \lambda_0 J_1(\gamma_0 r) + \hat{e}_z \lambda_0(\gamma_0 r)$$

$$\underset{\sim}{B}_1 = \xi_1\lambda_1 C_1 \left[-\hat{e}_r \left(\frac{iK_1\gamma_1}{\lambda_1}\right) J_1(\gamma_1 r) + \hat{e}_\theta \gamma_1 J_1(\gamma_1 r) + \hat{e}_z \frac{\lambda_1^2 - K_1^2}{\lambda_1} J_0(\gamma_1 r)\right] e^{iK_1 z}$$

$$\underset{\sim}{V} = \underset{\sim}{V}_0 + \underset{\sim}{V}_1 , \quad \underset{\sim}{V}_0 = \frac{\eta_0}{\xi_0}\underset{\sim}{B}_0$$

$$\underset{\sim}{V}_1 = \frac{\eta_1}{\xi_1}\underset{\sim}{B}_1 \tag{B}$$

Substituting in equation (A) one finds:

$$p = p_0 + (C_0\lambda_0\eta_0\gamma_0)^2 \tfrac{1}{2}[1 - J_0^2(\gamma_0 r) - J_1^2(\gamma_0 r)] - 2C_0C_1\lambda_0\lambda_1\eta_0\eta_1[\gamma_0\gamma_1 J_1(\gamma_0 r)J_1(\gamma_1 r)$$
$$\cos K_1 z + \frac{\lambda_0}{\lambda_1}\gamma_1^2 J_0(\gamma_0 r) J_0(\gamma_1 r) \cos K_1 z - \frac{\lambda_0}{\lambda_1}\gamma_1^2]$$

where, $p = p_0$ at (r=0, z=0) ,

$$|C_0|^2 = (2\pi L)^{-1}\left[\frac{(\gamma_0 R)^2}{2}\left\{2 J_1^2(\gamma_0 R) - J_0(\gamma_0 R)J_2(\gamma_0 R) + J_0^2(\gamma_0 R)\right\}\right]^{-1}$$

$$|C_1|^2 = (2\pi L)^{-1}\left[\left(\frac{\gamma_1 R}{2}\right)^2 (1 + K_1^2/\lambda_1^2)\{J_1^2(\gamma_1 R) - J_0(\gamma_1 R) J_2(\gamma_1 R)\}\right.$$
$$\left. + \frac{(\gamma_1 R)^2}{2}\{J_0^2(\gamma_1 R) + J_1^2(\gamma_1 R)\}\right]^{-1}$$

λ_1 is given by the Zeroes of $J_1(\gamma_1 R)$. λ_0 is determined from the relationship

$$\psi_t/\psi_p = -\frac{R}{L}\,\frac{\gamma_0 J_0'(\gamma_0 R)}{\lambda_0 J_0(\gamma_0 R)}$$

where ψ_t and ψ_p are the toroidal and poloidal magnetic fluxes. For particular numerical values of the plasma parameters one can calculate axial and radial variation of the pressure. The details of these calculations are given in Krishan (1983, 1985a). The conclusion is that the pressure increases towards the surface of the loop at the bottom (Z=0). It is approximately constant across the radius at the mid point of the half loop (Z=L/4) and at the apex of the loop (Z=L/2) the pressure is maximum at the axis and decreases outwards towards the surface. On the axis at r=0, the pressure increases from Z=0 to Z=L/2. This behavior clearly indicates the twisted configuration of the loop. The calculated pressure profile agrees quite well with the observed pressure structure.

3. Alfvenic Fluctuations in the Loop Plasma

We represent a coronal loop by a cylindrical column of plasma with periodic boundary conditions at the ends of the cylinder. The magnetohydrodynamic equations for an incompressible medium are:

$$\frac{\partial}{\partial t} \underset{\sim}{z}^{\pm} + (\underset{\sim}{z}^{\mp}\cdot\underset{\sim}{\nabla})\underset{\sim}{z}^{\pm} = -\underset{\sim}{\nabla}[p/\rho + b^2/2] \quad (1)$$

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{z}^{\pm} = 0 \quad (2)$$

where $\underset{\sim}{z}^{\pm} = \underset{\sim}{V} \pm \underset{\sim}{B}/\sqrt{4\pi\rho} = \underset{\sim}{V} \pm \underset{\sim}{b}$ are Elsasser varibles, p is the mechanical pressure, ρ is the density and $\underset{\sim}{B}$ the magnetic field. Now, taking the divergence of Equation (1) and using Equation (2), one gets

$$\nabla^2[p/\rho + \frac{b^2}{2}] = -\nabla\cdot[(\underset{\sim}{z}^{\mp}\cdot\nabla)\underset{\sim}{z}^{\pm}] \quad (3)$$

Separating quantities into average and fluctuating parts:

$$\underset{\sim}{V} = \langle\underset{\sim}{V}\rangle + \delta\underset{\sim}{V} \quad \text{and} \quad \underset{\sim}{B} = \langle\underset{\sim}{B}\rangle + \delta\underset{\sim}{B} \quad (4)$$

One finds:

$$\underset{\sim}{z}^{\pm} = \langle\underset{\sim}{V}\rangle \pm \langle\underset{\sim}{b}\rangle + \delta\underset{\sim}{z}^{\pm} \quad (5)$$

Substituting for $\underset{\sim}{z}^{\pm}$ in Equation (1) and Equation (3) one gets:

$$\frac{\partial}{\partial t}\delta z^{\pm} \mp (\underset{\sim}{C_A'}\cdot\underset{\sim}{\nabla})\,\delta\underset{\sim}{z}^{\pm}\ (\delta\underset{\sim}{z}^{\mp}\cdot\nabla)\,\delta\underset{\sim}{z}^{\pm} = -\nabla[p/\rho + b^2/2] \quad (6)$$

$$\text{and}\quad \nabla^2[p/\rho + b^2/2] = -\underset{\sim}{\nabla}\cdot[(\delta\underset{\sim}{z}^{-}\cdot\underset{\sim}{\nabla})\,\delta\underset{\sim}{z}^{+}] \quad (7)$$

$$\text{where}\quad \underset{\sim}{C_A'} = \langle\underset{\sim}{V}\rangle \pm \langle\underset{\sim}{b}\rangle \quad (8)$$

$$\text{and}\quad \delta\underset{\sim}{z}^{\pm} = \delta\underset{\sim}{V} \pm \delta\underset{\sim}{b} \quad (9)$$

Equation (9) represents the two possible Alfvenic modes. In order to study the propagation characteristics of the modes defined by Equation (9), one has to solve for pressure in Equation (7) and substitute in Equation (6). We choose to represent velocity and magnetic fields by a single Chandrasekhar-Kendall function which provide a good description of a coronal loop in its minimum energy state as discussed in earlier papers, Krishan (1983, 1985, 1985a). Therefore one writes

$$\underset{\sim}{B} = \xi_0 \lambda_0 C_0 \underset{\sim}{a}_0 \tag{10}$$

$$\underset{\sim}{V} = \eta_0 \lambda_0 C_0 \underset{\sim}{a}_0 \tag{11}$$

$$\underset{\sim}{\hat{a}} = \hat{e}_\theta \lambda_0 J_1(\lambda_0 r) + \hat{e}_z \lambda_0 J_0(\lambda_0 r) \tag{12}$$

$\hat{e}_\theta$ and $\hat{e}_z$ are the unit vectors, J_0 and J_1 are Bessel functions. Using Equations (10) and (11) we find:

$$\delta \underset{\sim}{z}^{\pm} = [(\eta_0 - <\eta_0>) \pm (\xi_0 - <\xi_0>)]\lambda_0 C_0 \underset{\sim}{a}_0 \tag{13}$$

From equation (12) it follows that $\delta z_r^\pm = 0$. From Equation (7) one finds for the pressure profile:

$$\frac{\partial}{\partial r}(p/\rho + b^2/2) = \frac{\delta z_\theta^- \, \delta z_\theta^+}{r} \tag{14}$$

and $$<\partial p/\partial r> = [<\eta_0^2> - <\eta_0>^2 + <\xi_0>^2] \times \lambda_0^4 C_0^2 \frac{J_1^2(\lambda_0 r)}{r} \tag{15}$$

where the angular brackets represent the ensemble average. The spatial variation of the pressure given by Equation (15) has been discussed in earlier papers, Krishan (1983, 1985a) where $<\eta_0> = 0$ was assumed. Substituting equation (14) in Equation (6), we find

$$\begin{aligned} &\frac{\partial}{\partial t}\delta z_r^\pm \mp \left(-\frac{C'_{A\theta}}{r}\delta z_\theta^\pm\right) = 0 \\ &\frac{\partial}{\partial t}\delta z_\theta^\pm \mp \left(-\frac{C'_{A\theta}}{r}\delta z_r^\pm\right) = 0 \\ &\frac{\partial}{\partial t}\delta z_z^\pm = 0 \end{aligned} \tag{16}$$

Using Equation (13), $\delta z_r^\pm = 0$, one finds:

$$C'_{A\theta}\delta z_\theta^\pm = 0 \text{ and } \frac{\partial}{\partial t}\delta z_\theta^\pm = 0 \tag{17}$$

(i) If $\delta z_\theta^\pm = 0$ then $\delta V_\theta = \pm \delta b_\theta$ (17)

or $\eta_0 - <\eta_0> = \pm(\xi_0 - <\xi_0>)$

and from Equation (14)

$$\frac{\partial}{\partial r}(p/\rho + b^2/2) = 0$$

$$\text{and } \left\langle \frac{1}{\rho}\frac{\partial p}{\partial r} \right\rangle = <\xi_0^2> \lambda_0^2 C_0^2 \frac{a_{0\theta}^2}{r} \tag{18}$$

One observes from Equatiojns (16) and (17), $\delta V_\theta = \pm \delta b_\theta$ and $\delta V_z = \pm \delta b_z$ represents Alfvenic fluctuations propagating along $\underset{\sim}{b}$ with group velocity $\mp <b>$ and $\delta \underset{\sim}{V} = \pm \delta \underset{\sim}{b}$ respectively, with frozen spectrum as discussed in Dobrowolny et al (1980) and Matthaeus et al (1983). The non-linear interactions between δz^- and δz^+ vanish. The spatial variation of pressure Equation (18) is the same as in Equation (15).

(ii) If $\delta z_\theta^\pm \neq 0$ and $C'_{A\theta} = 0$ which gives

$$<\eta_0> \pm <\xi_0> = 0 \tag{19}$$

Substituting in Equation (15) one gets for the pressure variation:

$$\left\langle \frac{\partial p}{\partial r} \right\rangle = <\eta_0^2> \lambda_0^2 C_0^2 \frac{a_{0\theta}^2}{r} \tag{20}$$

Equation (20) is same as Equation (18) except for the factors $<\xi_0^2>$ and $<\eta_0^2>$. In this case the pressure balance is through nonlinear interaction of δz_θ^+ and δz_θ^-. The conclusion of the above exercise is that in a coronal loop the pressure balance occurs through the nonlinear interactions of the Alfvenic type fluctuations. This supports the existence of Alfvenic waves excited within the loop and, therefore, the heating mechanisms based on Alfven waves.

The state of the coronal loop described by Equations (10), (11) and (12) can be treated as an ensemble and one can calculate the correlations between velocity and magnetic fields. The statistical description of this state was given by Krishan (1985). Here we use the results in order to calculate the correlation coefficient of the velocity and the magnetic field. The probability distribution function for the expansion coefficient ξ_0 and η_0 are:

$$P_{\xi_0,\eta_0} = K \exp[-(\alpha\lambda_0^2 + \beta\lambda_0)|\xi_0|^2 - \delta\rho_0\xi_0 - (\alpha\lambda_0^2|\eta_0|^2) - \gamma\lambda_0^2\xi_0\eta_0] \tag{21}$$

where α, β, γ and γ are respectively the Lagrange multipliers corresponding to the four invariants.

the total energy $E = \lambda_0^2|\xi_0|^2 + \lambda_0^2|\eta_0|^2$

the magnetic helicity $H_m = \lambda_0|\xi_0|^2$

the toroidal flux $\psi_t = \rho_0\xi_0$ (22)

and the cross helicity $H_c = \lambda_0^2\xi_0\eta_0$

and $\rho_0 = 2\pi R C_0 \lambda_0 J_1(\lambda_0 R)$, R = radius of the cylindrical plasma.

The normalization constant determined from the condition

$$\int_{-\infty}^{\infty} P_{\xi_0,\eta_0}\, d\xi_0\, d\eta_0 = 1$$

is found to be:

$$K = \frac{(\alpha\lambda_0^2)^{1/2}}{\pi}\left[\alpha\lambda_0^2 + \beta\lambda_0 - \frac{\gamma^2\lambda_0^2}{4\alpha}\right]^{1/2} \exp\left[-\frac{\delta^2\rho_0^2}{4(\alpha\lambda_0^2 + \beta\lambda_0 - \frac{\gamma^2\lambda_0^2}{4\alpha})}\right]$$

The correlation coefficient of the velocity and magnetic field fluctuation is defined as

$$\rho_c = \frac{<\delta\underset{\sim}{V}.\delta\underset{\sim}{b}>}{(<\delta V^2><\delta b^2>)^{1/2}} \tag{23}$$

In order to determine ρ_c, we calculate the ensemble average of the expansion coefficients of the magnetic and velocity fields as:

$$<\xi_0> = -\delta\rho_0/2A$$

$$<\eta_0> = \delta\rho_0\gamma/4\alpha A$$

$$<\xi_0\eta_0> = -\frac{1}{2}\frac{\gamma}{\alpha}\left[\frac{1}{A} + \frac{\delta^2\rho_0^2}{2A^2}\right]$$

$$<\xi_0^2> = \left[\frac{1}{2A} + \frac{\delta^2\rho_0^2}{4A^2}\right]$$

$$<\eta_0^2> = \frac{1}{\alpha\lambda_0^2}\left[\frac{\alpha\lambda_0^2 + \beta\lambda_0}{A} + \frac{\delta^2\rho_0^2\gamma^2\lambda_0^2}{8\alpha A^2}\right]$$

and

$$A = \alpha\lambda_0^2 + \beta\lambda_0 - \frac{\gamma^2\lambda_0^2}{4\alpha}$$

Now, the procedure is to determine the Lagrange multipliers in terms of $<E>$, $<H_m>$, $<\psi_t>$ and $<H_c>$. We can express the correlation coefficient in terms of the ensemble averaged invariants as:

$$\rho_c = \frac{<(\eta_0 - <\eta_0>)(\xi_0 - <\xi_0>)>}{[<(\eta_0 - <\eta_0>)^2><(\xi_0 - <\xi_0>)^2>]^{1/2}}$$

$$= \frac{<\eta_0\xi_0> - <\eta_0><\xi_0>}{[<\xi_0^2> - <\xi_0>^2]^{1/2}[<\eta_0^2> - <\eta_0>^2]^{1/2}}$$

$$= \frac{\frac{<H_c>}{\lambda_0^2} - \frac{<H_c><\psi_t>^2}{2\lambda_0\rho_0^2<H_m>}}{\left(\frac{<H_m>}{\lambda_0} - \frac{<\psi_t>^2}{\rho_0^2}\right)^{1/2}\left[\frac{<E> - \lambda_0<H_m>}{\lambda_0^2} - \frac{<H_c>^2<\psi_t>^2}{4\lambda_0^2\rho_0^2<H_m>^2}\right]^{1/2}} \tag{24}$$

Thus the correlation coefficient ρ_c has been expressed in terms of the measurable quantities. For the case of Alfvenic type fluctuations one has for $\delta\underset{\sim}{V} = \delta\underset{\sim}{b}$

$$\langle \eta_0^2 \rangle - \langle \eta_0 \rangle^2 = \langle \xi_0^2 \rangle - \langle \xi_0 \rangle^2$$

which describes the equipartition of energy in the velocity and magnetic fluctuations and in this case one gets:

$$\rho_c = \frac{\frac{\langle H_c \rangle}{\lambda_0^2}\left[1 - \frac{\langle \psi_t \rangle^2 \lambda_0}{2\rho_0^2 \langle H_m \rangle}\right]}{\frac{\langle H_m \rangle}{\lambda_0}\left[1 - \frac{\langle \psi_t \rangle^2 \lambda_0}{\rho_0^2 \langle H_m \rangle}\right]} = 1$$

The maximum value of the correlation coefficient ρ_c is ± 1. We can estimate a value of cross helicity $\langle H_c \rangle$ compatible with the values of other invariants indicated in earlier work (Krishan 1983, 1984, 1985a). For $\langle E \rangle \sim 10^{28}$ ergs., $\lambda_0 R \sim 1$, $R = 10^9$ cm, $\langle \psi_t \rangle \sim 10^{17}$ Maxwell and $\langle H_m \rangle \sim 10^{36}$ ergs.cm. one finds $\langle H_c \rangle \sim 10^{27}$ ergs. from Equation (25). The temporal evolution of the correlation coefficient in the case of solar wind has been studied by Grapin et al. (1982) where this study reflects on the origin of Alfvenic type velocity and magnetic field fluctuations. We plan to study the temporal behaviour of the correlation coefficient for a time dependent steady state of the loop plasma where one expects a time evolution in the Elsasser variables and a possible link with the waves responsible for heating the plasma.

3. Conclusion

It is found that the velocity and magnetic field fluctuations balancing the mechanical pressure of plasma in a coronal loop are of Alfvenic type. Using the statistical description of this MHD turbulent state, the correlation coefficient of the velocity and magnetic field has been expressed in terms of the measurable quantities.

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