# Centrifugal acceleration of plasma in pulsar magnetosphere* 

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#### Abstract

We present a relativistic model for the centrifugal acceleration of plasma bunches and the coherent radio emission in pulsar magnetosphere. We find that rotation broadens the width of leading component compared to the width of trailing component. We explain this difference in the component widths using the nested cone emission geometry. We estimate the effect of pulsar spin on the Stokes parameters, and find that the inclination between the rotation and magnetic axes can introduce an asymmetry in the circular polarization of the conal components. We analyse the single pulse polarization data of PSR B $0329+54$ at 606 MHz , and find that in its conal components, one sense of circular polarization dominates in the leading component while the other sense dominates in the trailing component. Our simulation shows that changing the sign of the impact parameter changes the sense of circular polarization as well as the swing of polarization angle.


Keywords. Pulsar - PSR 0329+54; rotation; pulse profiles.

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## 1. Introduction

To explain the coherent radio emission from pulsars, three major models have been proposed: curvature radiation by bunched plasma (Ruderman and Sutherland [1]), relativistic plasma emission (Melrose and Gedalin [2]), and maser mechanisms (Yihan et al [3]). In all these models rotation effects have hardly been taken into account. However, a later model by Blaskiewicz et al [4] has taken into account the effect of rotation on the particle motion. But they have assumed a constant emission height and estimated the effect of rotation only on the position angle swing. Next, Gangadhara [5] has proposed a model by taking into account the rotation effects over a range of emission heights, but the model is two dimensional and do not include the polarization

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Figure 1. Geometry of coordinate system with dipole axis $\hat{m}$ and rotation axis $\hat{\Omega}$.
estimates. In this paper, we relax the assumption of a fixed emission height and develop a mechanism of the curvature emission by including the rotation and coherency effects.

## 2. Coherent radiation by bunched plasma

Pulsars are highly magnetized with dipolar field structure. In $(r, \theta, \phi)$ polar coordinates centered on the magnetic axis, the magnetic field is given by

$$
\begin{equation*}
\mathbf{B}=\left(\frac{2 \mu \cos \theta}{r^{3}}, \frac{\mu \sin \theta}{r^{3}}, 0\right) \tag{1}
\end{equation*}
$$

Here $\mu=B_{0} R_{*}^{3}$ where $B_{0} \sim 10^{12} \mathrm{G}$ is the surface magnetic field and $R_{*}$ the neutron star radius ( $\sim 10 \mathrm{~km}$ ). Using the components of $\mathbf{B}$, we can derive an equation for the magnetic field line:

$$
\begin{equation*}
r=r_{e} \sin ^{2} \theta \tag{2}
\end{equation*}
$$

where $r_{e}$ is the field line constant. The magnetic axis at any time $t$ may be represented as

$$
\begin{equation*}
\hat{m}=\sin \alpha[\hat{x} \cos (\Omega t)+\hat{y} \sin (\Omega t)]+\hat{z} \cos \alpha \tag{3}
\end{equation*}
$$

where $\Omega$ is the angular velocity and $\alpha$ is the magnetic inclination angle (figure 1 ).
The spinning magnetic field induces a strong electric field near the polar cap. Then this induced electric field accelerates the plasma particles to relativistic velocities. Plasma particles are believed to be bunched by plasma instabilities such as oscillating two-stream instability. The guiding center velocity of the center of momentum of plasma bunch is

$$
\begin{equation*}
\mathbf{v}_{c} \cong v_{\|} \hat{b}+\Omega \times \mathbf{r} \tag{4}
\end{equation*}
$$

where $v_{\|}$is the velocity of the center of momentum parallel to $\hat{b}$, the unit vector tangent to the dipolar magnetic field lines. If $\mathbf{J}$ is the current density due to the flow of such a plasma, then the Fourier components of coherent radiation electric field is given by

$$
\begin{equation*}
\mathbf{E}(\omega)=-i \frac{\omega \mathrm{e}^{i \omega R / c}}{\sqrt{2 \pi} R c^{2}} \int_{-\infty}^{+\infty} \mathrm{d} t \int \hat{n} \times(\hat{n} \times \mathbf{J}) \mathrm{e}^{i \omega(t-(\hat{n} \cdot \mathbf{r} / c))} \mathrm{d}^{3} \mathbf{r} \tag{5}
\end{equation*}
$$

where $\hat{n}$ is the line of sight vector, $R$ the distance between source and observer, and $c$ the speed of light.

In the co-moving frame $x^{\prime} y^{\prime} z^{\prime}$, a plasma wave with frequency $\omega_{\mathrm{p}}^{\prime}$ and wave number $k_{\mathrm{p}}^{\prime}$ is excited in the plasma column by the plasma instabilities such as the oscillating two-stream instability [1]. Next by solving the integral in eq. (5), we obtain

$$
\begin{equation*}
\mathbf{E}(\omega)=-i \frac{\omega \mathrm{e}^{i \omega R / c}}{\sqrt{2 \pi} R c^{2}} \mathbf{A} \tag{6}
\end{equation*}
$$

and

$$
\begin{align*}
\mathbf{A}= & \frac{J_{0} N s_{0} \xi_{0} \eta_{0}}{2 i} \frac{\sin \left[\left(k-k_{\mathrm{p}}\right) s_{0} / 2\right]}{\left(k-k_{\mathrm{p}}\right) s_{0} / 2} \frac{\sin \left(k \eta_{0} \theta / 2\right)}{k \eta_{0} \theta / 2}  \tag{7}\\
& \times\left\{\hat{\varepsilon} \theta\left(\frac{6 \rho^{2}}{\omega c^{2}}\right)^{1 / 3} L_{1}(z)-\hat{y}\left(\frac{36 \rho}{\omega^{2} c}\right)^{1 / 3} L_{2}(z)\right\}, \tag{8}
\end{align*}
$$

where $\hat{\varepsilon}=\hat{n} \times \hat{y}^{\prime}$, and $(\omega, k)$ are the radiation frequency and wave number, and

$$
\begin{equation*}
\frac{\omega_{\mathrm{p}}^{\prime}}{\omega_{L}}=\frac{\kappa \gamma}{6}+\frac{1}{2 \gamma}, \tag{9}
\end{equation*}
$$

and the constant $\kappa$ is of the order $10^{-3}$ [6]. The parameter

$$
\begin{equation*}
z=\left(\frac{6 \omega^{2} \rho^{2}}{c^{2}}\right)^{1 / 3}\left[\frac{1}{2 \gamma^{2}}+\frac{\theta^{2}}{2}-\frac{\omega_{\mathrm{p}}^{\prime}}{\gamma \omega}-\frac{\xi_{0}}{\rho}\right] \tag{10}
\end{equation*}
$$

and for positive $z$, we have

$$
\begin{align*}
& L_{1}(z)=\frac{2}{3} z^{1 / 2} K_{1 / 3}\left[2(z / 3)^{3 / 2}\right]  \tag{11}\\
& L_{2}(z)=i \frac{2}{3^{3 / 2}} z K_{2 / 3}\left[2(z / 3)^{3 / 2}\right] \tag{12}
\end{align*}
$$

The functions $K_{1 / 3}$ and $K_{2 / 3}$ are the modified Bessel functions.
The instantaneous radius of curvature of a particle orbit is given by

$$
\begin{equation*}
\rho=\frac{v_{c}^{2}}{a} \cong \frac{2 r}{\left[\varepsilon^{2}-(6 r \Omega / c) \sin \alpha \sin \beta \sin (\Omega t-\phi)\right]^{1 / 2}} \tag{13}
\end{equation*}
$$

where $\phi$ is the angle between $\mathbf{v}_{c}$ and $\hat{n}$ and $a$ the particle acceleration.
If $E_{y}(\omega)$ and $E_{\varepsilon}(\omega)$ are the components of radiation electric field $\mathbf{E}(\omega)$ given by eq. (6), then the Stokes parameters are given by


Figure 2. (a) Normalized Stokes parameters, $I, L, V$ and (b) polarization angle for a single pulse simulated using $\alpha=30^{\circ}, \sigma=2.5^{\circ}$.

$$
\begin{align*}
I & =E_{y} E_{y}^{*}+E_{\varepsilon} E_{\varepsilon}^{*}  \tag{14}\\
Q & =E_{y} E_{y}^{*}-E_{\varepsilon}  \tag{15}\\
U & =2 \operatorname{Re}\left(E_{y} E_{\varepsilon}^{*}\right)=0,  \tag{16}\\
V & =2 \operatorname{Im}\left(E_{y} E_{\varepsilon}^{*}\right) \tag{17}
\end{align*}
$$

The polarization position angle is given by

$$
\begin{equation*}
\psi=\frac{1}{2} \tan ^{-1}\left(\frac{U}{Q}\right) \tag{18}
\end{equation*}
$$

For numerical calculations we adopt the values of emission heights and component locations provided by Gangadhara and Gupta [7] for PSR B0329+54 at 606 MHz . The simulated profile of PSR B0329+54 (figure 2) shows that (i) the conal components are asymmetrically located with respect to the core, (ii) width of the leading component is broader than the trailing component and (iii) circular polarization $V$ of one sense dominates in conal components.

## 3. Discussion

The emission region on the polar cap is believed to be organized in the form of concentric hollow cones. Our simulation shows that the leading component becomes broader than its trailing counterpart. This broadening is induced by the rotation through a change in curvature of particle trajectories. By analyzing 24 pulsar pulse profiles, we find that 19 of them have leading components broader compared to trailing ones, and thereby confirm the possibility of detecting such an effect through observations. We note that such a broadening becomes observable only when the emission components are organized in the form of nested cones. A more detailed presentation of this work is given by Peyman and Gangadhara [8].

## 4. Conclusion

Pulsar spin plays an important role in the dynamics of plasma, which in tern reflected in the morphology of pulsar profiles. The phase width of leading component becomes broader than its trailing counterpart. Due to the inclination of magnetic axis with respect to the rotation axis and alteration of the particles trajectory by rotation, one sense of circular polarization becomes stronger in conal components of single pulses. As a result, one sense of circular polarization dominates in the conal components of average pulse profile. It is worth mentioning that the inclination of magnetic field line planes with respect to the line-of-sight is mainly responsible for the enhancement of one sense of circular polarization in conal component. But in the case of core component this effect vanishes and consequently leads to the circular polarization with an antisymmetric profile.

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