

## Magnetohydrodynamic turbulence in solar wind

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**Abstract.** Alfvénic fluctuations in the solar wind is known to exhibit turbulent behaviour. In this paper we state the existing phenomenologies. After this we show the numerical results from pseudo-spectral method. The numerical results appear to favour the Kolmogorov-like MHD turbulence phenomenology over the Kraichnan phenomenology atleast in some regime.

*Key words:* solar wind, turbulence, dissipation

In statistical theory of MHD turbulence, the energy spectra of velocity and magnetic field, or of the Elsässer variables are still under debate. As in fluid turbulence, the intermediate range, that is between the large-scale (forcing) and small-scale (dissipative), the energy spectra  $E(k)$  of velocity ( $\mathbf{u}$ ), magnetic field ( $\mathbf{b}$ ), and Elsässer variables ( $\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}$ ) have been conjectured to follow powerlaws. According to Kraichnan (1965), the magnetic energy and fluid energy are equipartitioned, and their energy spectra are

$$E(k) = A(\Pi B_0)^{1/2} k^{-5/3} \quad (1)$$

where  $k$  is the wave number,  $\Pi$  is the energy cascade rate,  $B_0$  is the mean magnetic field, and  $A$  is a universal constant of the order of 1. Later on Kolmogorov-like MHD turbulence phenomenology was constructed (Marsch 1989; Matthaeus & Zhou 1989; Zhou & Matthaeus 1990), according to which

$$E^\pm(k) = K_{k_0}^\pm \frac{(\Pi^\pm)^{4/3}}{(\Pi^\pm)^{2/3}} k^{-5/3} \quad (2)$$

where  $E^\pm(k)$  are the energy spectra of  $\mathbf{z}^\pm$ , and  $\Pi^\pm$  are cascade rates of  $\mathbf{z}^\pm$ , and  $K_{k_0}^\pm$  are constants. An implicit assumption here is that Kraichnan assumes that the mean magnetic field or magnetic field of the large eddies ( $B_0$ ) is large as compared to the fluctuation, while the Kolmogorov-like phenomenology assumes that  $B_0$  is small. Matthaeus & Zhou (1989), Zhou & Matthaeus (1990), and Dobrowolny et al. (1980) have generalized these phenomenologies.

Solar wind observations show that the exponents for the total and magnetic energy spectra are 1.69 and 1.73 respectively. They are closer to 5/3 rather than 3/2. This is surprising

because the mean magnetic field in the solar wind at 1 AU is of the order of 5 nanotesla that is around 5 times larger than the fluctuations, hence Kraichnan's (1965) phenomenology should have been applicable here.

We (Verma et al. 1996) investigated the energy spectra and cascade rates using direct numerical simulation of MHD turbulence. Most of our simulations were done in two dimensions. Fortunately, the statistical behaviour of two and three dimensional MHD turbulence is expected to be the same. The exponents  $5/3$  and  $3/2$  are too close to be differentiated in our simulation (see Figure 1 of Verma et al. 1996). Therefore, we used an indirect test. According to Kolmogorov-like phenomenology the cascade rates are related by the expression  $E^-(k)/E^+(k) = (K_{k_0}^-/K_{k_0}^+)(\Pi/\Pi^+)^2$  whereas in Kraichnan's and Dobrowolny et al.'s phenomenology  $\Pi^+ = \Pi^-$  irrespective of  $E^-/E^+$  ratio. We found that for  $E^-/E^+$  between 0 and 0.6, Kolmogorov-like MHD turbulence phenomenology holds with  $K_{k_0}^+ \approx K_{k_0}^-$  (see Figure 2 of Verma et al. 1996). This behaviour was observed for large  $B_0$  as well. In our simulations we also find that for small  $E^-/E^+$ , the numerical results are not consistent with Kolmogorov-like phenomenology with  $K_{k_0}^+ \approx K_{k_0}^-$ . These simulation results show that Kolmogorov-like MHD phenomenology appears to be more appropriate for MHD turbulence than Kraichnan's or Dobrowolny et al.'s phenomenology when  $E^+/E^-$  close to 1, but for small  $E^-/E^+$ , the phenomenological models are only partially correct and need modifications. Attempts are being made to get a consistent theory by making the Kolmogorov's constants depend on  $E^+/E^-$  ratio.

The MHD turbulence phenomenologies have been used for estimating turbulence dissipation in the solar wind (Tu et al. 1984; Tu 1988; Verma et al. 1995; Verma 1996a) and calculation of nonclassical viscosity and resistivity of the solar wind at the dissipative scale (Verma, 1996b).

In summary, we notice that in statistical theory of MHD turbulence, the phenomenologies are not on strong footing. Our preliminary numerical results show that at least for  $E^+ \approx E^-$ , Kolmogorov-like MHD turbulence phenomenology rather than Kraichnan's phenomenology appears to be more appropriate for MHD fluids. The solar wind observations, numerical and analytical results appear to be consistent with this conjecture. However, we need to perform further numerical, analytical, and observational studies for conclusive theories.

## References

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