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A new star formation rate for disk galaxies

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Abstract. We suggest a general star formation law for disk galaxies. A real exponent n determines the timescale τ of star formation which however yields the corresponding star formation rate. Based on two physical dimensionless parameters : Q_A and f_P (defined in the text), we suggest a general equation for star formation rate. The relative significance of these parameters determines the star formation scenario in different interstellar environments. The star formation rate is calculated in the solar neighbourhood and is found to be in good agreement with values reported earlier in the literature.

Key words : galaxies : general-Galaxy : evolution-Galaxy : solar neighbourhood

1. Introduction

Star formation history of Galactic disk shows large variation which is revealed by stellar age markers, e.g. isochrone fitting method applied to stars near the mainsequence turn-off, chromospheric dating for a sample of late-type stars, and cooling ages of white dwarfs (Scalo 1986, Noh and Scalo 1990). However, the observations indicate about a constant star formation rate over disk age timescale.

Excellent CCD techniques and PSF fitting methods have enabled us to study similar populations (as above) in other galaxies too of the Local Group, e.g. Magellanic Clouds and several dwarf spheroidal galaxies. Comparison of the data reveals that star formation histories of these galaxies are altogether different from that of our own Galaxy. For example, CCD observations of the LMC star populations suggest an increase in the star formation rate over the last 4 Gyr (Bertelli et al. 1992) in conformity with earlier photographic surveys. Interestingly, an active star formation appears to peak in the last few Gyr. There is a signature of bi-modal age distribution : one population being younger than 3 Gyr and the other being older than 12 Gyr (Da Costa 1991). Metallicity also shows a bi-modal behaviour. Apparently, late-type galaxies are presumably rich in young clusters. It is noticeable that the SMC stellar samples also indicate a history different from the LMC or the Galaxy (Da Costa 1991, Gardiner and Hatzidimitriou 1993).

Studies of dwarf spheroidal galaxies in the Local Group shows a mixed population (Da Costa 1992). In these cases, evidence favours for bursts of star formation unlike continuous formation typical for disks of our Galaxy type. Dwarf spheroidal galaxies do not appear to form stars probably because they are devoid of sufficient interstellar gas to do so. Thus, the process of star formation varies from one member to the other in the Group. As discussed by Kennicutt (1996), color gradients, radial dependence of star formation, reddening and metal abundances are cumulative effect of many factors see also Ryder and Dopita 1994).

It is believed that star formation is usually driven by the large-scale self-gravity of the gas in a galaxy presumably supported by density waves (Larson 1988, 1992). A gas disk is gravitationally unstable if Toomre-parameter $Q \equiv \frac{\kappa v_g}{\pi G \sigma_g} < 1$, where κ = epicyclic frequency, v_g = velocity dispersion of random gas motions, σ_g = surface mass density of gas layer. Kennicutt (1989) has shown that star formation should occur if surface gas density σ_g exceeds a threshold value related to Q (Toomre 1964). The time required for the interstellar gas to form large cloud complexes is expected to be comparable to the growth rate time τ_{gi} of gravitational instability such that $\tau_{gi} = v_g/\pi G \sigma_g = Q/\kappa$, which is about 50 Gyr locally. Since, star formation efficiency is much less (~ 2%), timescale of converting gas into stars will be much larger than this. Rest of the gas is again dispersed into the ISM (Myers et al. 1986, Leisawitz et al. 1989 and Evans and Lada 1991). Standard IMF indicates that ~ 50% of the mass being converted to stars is locked up in low-mass stars, the rest being dispersed away to ISM by mass loss. One concludes that ~ 1% gas in a star forming complex is removed from ISM. Some negative feedback effects also occur as a result of formation of massive stars, e.g. ionization, winds and supernovae which lower the efficiency of star formation. It is because star-forming clouds are destroyed and returned to ISM before turning it into stars. In fact, ionization plays the significant role among all these effects. Thus, massive star formation can probably stretch the timescale of gas depletion from a gravitational timescale to the Hubble timescale.

In this communication, we aim to calculate the star formation rate in the solar neighbourhood. We, therefore, discuss it in the context of the spiral galaxies too. It is found that almost a large number of disk galaxies have spiral arms with different morphologies. A difference in velocity dispersion, mass column-density and epicyclic frequency, etc. might be the cause for different disk properties. Star formation processes are confined to spiral arms (as supported by theory and observations) displaying cumulative effects of gravitational, magnetic and macroscopic thermal instabilities. For a computer simulation image of different flocculent and regular galaxies one may refer to (Elmegreen 1989, Bertin et al. 1989, Thomasson and Donner 1993 and Elmegreen 1995). Bertin et al. (1989) have shown that arm amplitude variations in the spiral arms are the result of interference between inward and outward moving wave packets. They find a pattern speed ~ 26 km/s. By assuming a wave barrier near the center (outside the inner Lindblad resonance), Thomasson et al. (1990) studied wave patterns for M 81 (for numerical simulation, see Elmegreen and Thomasson 1993).

A question arises what causes star formation in the spiral arm? High density of the gas in the arm generates gravitational instability and strong magnetic field. This causes turbulent energy to be dissipated quickly along the arm. Another important factor is the low shear causing the instability to grow almost parallel and uniform motion along arms. Lastly, galactic tidal forces facilitate the presence of low density clouds in the arms. Elmegreen (1995) has discussed that low density virialized clouds help the formation of massive virialized cloud. The self-gravity of the gas, eventually, opposes the usual tidal stretching due to the galaxy.

It has been concluded (Elmegreen 1993) that there are two important dimensionless parameters, namely, Q_A and f_P which determine star formation. Here, f_P is the square root of the ratio of cloud collision rate to the gravitational instability rate and therefore is a measure of the relative importance of cloud collisions. Star formation proceeds via energy dissipation to make a dense core. Low values of f_P ($f_P \sim 0.01$) imply that clouds are dense and self-gravitating. But if $f_P \sim 100$ in the inner Galaxy) where pressure becomes high, diffuse clouds collide and cool leading to large mass clouds. When Q_A and f_P both are large, either thermal instability triggers star formation or cooling reduces Q_A till gravitational instabilities switch over. However, when both parameters are small, gravitatioal instability forms clouds quickly but star formation is delayed in lack of the energy dissipation. When Q_A is large and f_P small, star formation occurs due to random cloud collisions triggered by thermal instability. Further, when Q_A is small and f_P large, gravitational instability is the main driving agent for cloud as well as star formation at all distances. It is to be remarked that our discussion does not take into account the galactic bulge component (Oort 1977) which might contribute the Galactic gas dynamics in the inner regions below 0.1 kpc. We present a general law of star formation for disk galaxies. Assuming neither infall nor radial flow in the disk, we obtain a general equation for star formation rate in Sect. 2. Star formation rate is calculated for the Galaxy in the solar neighbourhood in Sect. 3. Conclusions are given in the last section.

2. Star formation rate

Elmegreen and Elmegreen (1986) after analysing different star formation indicators, namely, Hα, UV and B-V found no systematic trend from one class to the other in spiral galaxies. CO column density per unit area appears constant for all arm classes (Stark et al. 1987). Type II supernovae rate per unit luminosity also does not show significant variation from grand design galaxies to flocculent galaxies (McCall and Schmidt 1986). Similarly, star formation efficiencies, measured from CO, H_I and H_α studies of SO and Sa galaxies have similar median values as that for late-type spirals (cf. Thronson et al. (1989), Wiklind and Henkel (1989, 1990), Devereux and Young (1991)).

Wyse and Silk (1989) have extended usual power law model of Schmidt (1959, 1963) with star formation rate R dependence on gas surface mass density σ_g and local angular frequency $\Omega(r)$ for both atomic and molecular gases respectively with exponent $n = 1$ and $n = 2$. A self-consistent model for star formation was given by Wang and Silk (1994, hereafter WS) for global star formation based on the gravitational instability parameter $Q < 1$. In the solar neighbourhood, the model agrees with (i) the observed star formation rate, (ii) the metallicity distribution among G-dwarfs, and (iii) the age-metallicity relation for F-dwarfs. A natural cut-off for $Q = 1$ in the star formation rate results. We note (see Fig. 4 and 6 in WS) that star formation occurs in the regions where $Q \geq 1$. One might ask as to how does star formation proceed when $Q \geq 1$ and the system is in the state of gravitational equilibrium?

An alternative suggestion for cloud formation because of energy dissipation accompanied by shear instability thus leading to star formation (even if $Q > 1$) has been given by Elmegreen (1991a, 1993). Macrospic thermal instabilities and various cloud formation mechanisms are reviewed in Elmegreen (1991b). We express the star formation rate in the form

$$R^n = \alpha \ \sigma_g / \tau)^n, \quad (1)$$

where α = efficiency of star formation which also depends upon n, τ = timescale of star formation, σ_g = surface gas density composed of atomic and molecular components and n = an exponent. Timescale of star formation is related to the growth rate of instability (Goldreich and Lynden-Bell 1965). Gravitational instability of galactic disks has also been studied by Elmegreen (1979), Cowie (1981), Ikeuchi et al. (1984) and Bizyaev (1997). However, gravitational instabilities taking account of the turbulence are discussed in Bonazzola et al. (1987) and Leorat et al. (1990). A review of recent observations of the history of star formation and its relevance to galaxy formation and evolution has been discussed by Kennicutt (1996). For the evolution of global star formation history measured from the Hubble Deep Field one may refer to Connolly et al. (1997). Now, let us calculate the star formation rate in the solar neighbourhood assuming neither in fall nor radial flow in the disk.

Following WS, we write the maximum growth rate ω_{max} of instability occurring at

$$k_{max} = 2\sqrt{2}\, A/v_{eff}\, Q_A, \quad (2)$$

where A = Oort shear constant, v_{eff} = effective velocity dispersion in a magnetized gas (see Elmegreen 1993 for details), $Q_A \equiv 2\sqrt{2}\, A/v_{eff}/\pi G \sigma_g$, and κ_{max} = wave number corresponding to maximum growth rate of perturbation. Growth rate of perturbation (see WS for derivation) is written as

$$|\omega_{max}| = \frac{2\sqrt{2}\, A/(1-Q_A^2)^{1/2}}{Q_A} \quad (3)$$

Since, $\tau \simeq |\omega_{max}|^{-1}$, one gets from eqs. 1) and 3)

$$R^n = \frac{\alpha\, (2\sqrt{2}\, A)^n \sigma_g^n\, (1-Q_A^n)^{n/2}}{Q_A^n}. \quad (4)$$

Further, we define a function $f_c \equiv \sigma_g / \sigma_c$, σ_c = column density of individual molecular clouds. A relationship between individual cloud formation and star formation is complicated. If we assume that gravitational instability is predominant, small cloud collisions may form large molecular clouds wherein star formation ensues. Therefore, cloud formation timescale, cloud collision timescale, and growth rate of gravitational instability timescale are similar. WS obtain collision time between two clouds as

$$t^{-1}_{coll} = \frac{\sigma_g (2\sqrt{2}\,A)}{\sigma_c Q_A} \quad (5)$$

In view of the assumption, $t^{-1}_{coll} \sim \omega_{max}$, one gets

$$Q_A \sim (1-f^2_c)^{1/2} \quad (6)$$

It may be noted that this is not the general property of the interstellar medium as other types of instabilities namely thermal instability and Parker instability may also contribute and affect the timescale of star formation (and other related physical quantities). Substituting eq. (6) into eq. (4), the star formation rate is now expressed as

$$R^n = \frac{\alpha (2\sqrt{2}\,A)^n \sigma^n_g f^n_c}{(1-f^n_c)^{1/2}} \quad (7)$$

In this form, eq. (7) inherits the conversion from column density to density using galactic scale heights. We write the general equation for star formation rate as

$$R^n = \alpha a^n + \beta\,(afp)^n, \quad (8)$$

where $a \equiv 2\sqrt{2}\,A\sigma^2_g / Q_A\sigma_c$, β is another efficiency parameter for star formation as a result of energy dissipation. Contributions to R due to Q_A and f_p are summed up as linear combination. Since, star formation rate may also be written as

$$R^n = \left(\frac{1}{1-\delta}\right)^n \frac{d\sigma^n_g}{dt^n}, \quad (9)$$

where δ is the fraction of mass returned to the interstellar medium from the stellar content. From eqs. (8) and (9), we get

$$\frac{\alpha (2\sqrt{2}\,A)^n \sigma^{2n}_g}{Q^n_A \sigma^n_c} + \frac{\beta (2\sqrt{2}\,A)^n \sigma^{2n}_g}{Q^n_A \sigma^n_c} fp^n = \left(\frac{1}{1-\delta}\right)^n \frac{d\sigma^n_g}{dt^n} \quad (10)$$

Assuming that parameter fp and σ_c are independent of time, we integrate eq. 10)

$$\frac{t^n}{\tau^n_\alpha} + \frac{t^n}{\tau^n_\alpha} fp^n = -\frac{(1-f^2_c)^{1/2}}{f^n_c} - \sin^{-1} f^n_c + \text{const.}, \quad (11)$$

where we put

$$\tau_\alpha{}^n = \alpha\,(1-\delta)^n\,(2\sqrt{2}\,A)^n$$

$$\tau_\beta{}^n = \beta\,(1-\delta)^n\,(2\sqrt{2}\,A)^n \quad (12)$$

If we express $f_g = \sigma_g / \sigma_c$ and $f_{ci} = \sigma_i / \sigma_c$, where subscript 'i' denotes initial values, we can write eq. (11) as

$$\frac{t^n}{\tau_\alpha^n} + \frac{t^n}{\tau_\beta^n} fp^n = -\frac{(1 - f_g^2 f_{ci}^2)^{n/2}}{f_c^n f_{ci}^n} - \sin^{-1}(f_g^n f_{ci}^n) + \text{const.} \tag{13}$$

The value of const. in eq. (13) is determined from the requirement, $t = 0$, $\sigma_g = \sigma_i f_g = 1$). We get from eq. (13)

$$(1 + fp^n)\frac{t^n}{\tau^n} = -\frac{(1 - f_g^2 f_{ci}^2)^{n/2}}{f_g^2 f_{ci}^2} + \frac{(1 - f_{ci}^2)^{n/2}}{f_{ci}^2} - \sin^{-1} f_g^n f_{ci}^n + \sin^{-1}(f_{ci}^n) , \tag{14}$$

where we have put $\tau_\alpha^n = \tau_\beta^n = \tau^n$. When $fp = 0, n = 1$, eq. (14) reduces to (except for a minus sign) the equation derived by WS (cf. eq. (23) in WS).

3. Star formation rate in the solar neighbourhood

Assuming a constant IMF in the solar neighbourhood Miller and Scalo 1979, Scalo 1986), we take the following input parameters : $\sigma_{i,\odot} \simeq 50\ M_\odot pc^{-2}$ (Kuijken and Gilmore 1989, Bahcall et al. 1992), $\sigma_g \simeq 10 M_\odot pc^{-2}$ McKee 1990), $f_g \sim 0.2$, $fp \sim 0.05$ (Elmegreen 1993), $t = 15$ Gyr (age of the Galaxy), $\alpha = 0.1$ Myers et al. 1986), Oort shear constant $A = 15$ km/s/kpc (Kerr and Lynden-Bell 1986), $\delta = 0.3$ Miller and Scalo 1979, Scalo (1986). For $n = 1$, we get timescale of star formation as $\tau = 0.38$ Gyr. We calculate $f_{ci,\odot}$ using eq. (14) to obtain $f_{ci,\odot} \sim 0.10$. Substituting the required values into eq. (7), we get star formation rate as $R = 3.8 M_\odot pc^{-2}\ Gyr^{-1}$, being in agreement with Scalo (1986). For $n = 2$. we get t = 0.12 Gyr, $f_{ci,\odot} \sim 0.05$ and star formation rate $R = 6.0\ M_\odot\ pc^{-2}\ Gyr^{-1}$.

We note that the model with $n = 1$ provides star formation rate which is in agreement with the inferred rate in the solar neighbourhood. Even if the efficiency drops by 10% (i.e. $\alpha \sim 0.01$), the model with $n = 2$ gives the same star formation rate as model $n = 1$ with $\alpha = 0.1$. Thus, parametric freedom for α and fp even when $Q_A \geq 1$ (i.e. when non-gravitational instabilities are dominant) provide a general star formation scenario. This model may be regarded as a generalisation of WS model with a dependence of star formation rate on Oort shear constant A. As opposed to WS, one finds continuous (in the sense of Q-values) star formation rate obeying a similar criterion for gravitational instability to set in the disk. Competitive nature of the two terms in eq. (8) helps one to visualize the essence of continuity in the star formation process.

4. Conclusions

We have obtained generalised version of WS equation in the sense that in view of eq. (8), one comes across a natural criterion which eliminates the cut-off condition providing the star formation even when $Q_A \geq 1$. The relative competence of either of these terms determines star formation as discussed in the text at all radial distances. For our model with $n = 1$, we obtain star formation rates in the solar neighbourhood in agreement with values inferred earlier by Scalo (1986). Our models are sensitive enough to efficiency α and timescale τ of star formation. A given exponent n determines τ which however yields the corresponding star formation rate R. A simple model discussed as above provides some important characteristics of our Galactic disk. Star formation is a complicated function of several parameters Tinsley (1980) namely gas density, gas sound speed, shock frequency, shock strength, gas rotation, shear constant A, magnetic field, gas metal abundance, and background star density. It is thus hard to predict the exact dependence of R on them. Usually, one studies some from of R and calculates its immediate effect on chemical and photometric evolution. Finally, the model predictions are put to a test of observations.

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