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Some thoughts on Bhadra Chowkon and Ankapasha in Indian mathematics

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Abstract. It is shown how the Indian concepts about Bhadra Chowkon (Magic Squares) can be utilized for developing new computer algorithms and how Indian Ankapasha (series) technique can be used for computing trgonometric functions.

Key words : Indian mathematics, compur applications

1. Introduction

The most neglected field in Mathematics as of today is that of magical squares. Contemporary treament given to this branch of mathematics is not serious. The reasons are a plenty for this, but the most striking one could be due to its respectability in the dark ages of Europe. Most of the mathematicians treat the subject as a matter of mathematical recreation. No serious thought is given to evolve any theory of magical squares, in particular whether they could be useful for solving todays problems of mathematics. This neglect is inspite of their well known applications in statistics, agriculture, marketing and sociology.

It is an accepted fact that the Indians know how to construct magical squares (Bhadra Chowkons), but nobody has any idea regarding their relevance in Indian mathematics. The fact that the tantrik texts refer to these squares as *Yantras* might be the reason for the ill treatment given by Indian scholars to magical squares. The only authoritative text that is available on the subject with the mathematical point of view is that of Narayan Pandit's Ganit Koumadi of 1300 AD. This book is a treatise on algebra and geometry which culminates in the study of magical squares. Narayan Pandit says that this is the ultimate mathematics that human beings know, because it is directly given by Lord Shiva. This must be a frantic effort on his part to snatch away the beautiful magical squares from the Tantrik disciples. He says that the mathematics of magical squares is *Bhadra Ganit.* The term *Bhadra* used by him is quite intelligent. A wide spread distrust in tantrik practices was prevalent in his times, so to nullify it he used the term *Bhadra* which means all that is good.

References of widespread use of *Bhadra Ganit* can be found in Vastushastra (architecture), Shilpashastra (sculpture) and Jyotish (astronomy). The chapter that deals with the magical squares comes last in the treatise of Narayan Pandit. This chapter on *Bhadra Ganit* is given immediately after the chapter on Ankapash i.e. series, permutations and combinations. It is necessary to see how the sequences and series are handled in the text. Many problems that are treated by Ganesh Daivednya in his Grahalaghava can be solved by using the techniques given in Ganit Koumadi. Bhaskarcharya has referred to Ankapash, but he does not give any elaborate treatment of it. This is corrected by Narayan Pandit. His treatment goes hand in hand with that done using the modern group theory. The work is more advanced than that of Sangit Ratnakar.

2. Indian concepts about Bhadra Chowkon

The subject of Bhadra Chowkon is vast, so we propose to give some glimpses to create awareness and enthusiasm for further work. When one goes through various treatises of ancient Indian Mathematics, one is sure to find out that the treatment that is given in the various treatises is quite thought provoking. Provided an open approach is taken and one tries to learn Indian Mathematics without any of the western influence while dealing (learning) with this mathematics, one finds various salient features that tempts one to design even new architectures say in computer designings, new approaches and a rather different view about the whole mathematics known today.

This work is included in the treatise on Algebra and its last chapter deals with the' Bhadra Chowkon'. The basic approach to solve any mathematical problem in ancient India is to treat the numbers as 'Strings' to use as a concept analogous to modern mathematics or physics. This basic difference makes available to us various techniques that are not otherwise available in contemporary mathematics. The numbers are also supposed to be carrying unique properties with them, besides the conventional numerical ones, or the representations of collections of items. They are treated as manifestations of various physical properties. This treatment justifies that the numbers are strings as strings ought to have properties of their own.

This approach, regarding Indian Mathematics, gets confirmed when one happens to see a typical temple in a remote place like 'Pimple Shev' in Daulatabad taluk. This temple has a "Magical Square" carved on the entrance. The numbers used for this represent 'Indian Time'. It looked like as if it was realised suddenly that a series of numbers might be divergent or convergent, but the series of properties (string) cannot be and must not be divergent. The magic squares is such a branch of mathematics that is treated by Indians as the mathematics of series. The series approach to algebra paves way to various techniques. The 'Magic Squares' can be treated as the mathematics of a series which converges. This approach gives rise to various applications of these squares in the field of Computing , Astronomy, Statistics and Fine Arts.

The magical squares are the arrangement of elements of a series in a specific manner which is able to give a constant solution if a mathematical function is applied to all its elements. This technique offers us ways to solve simultaneous and transcendental equations. The interesting property of these chowkons when constructed gives us matrices which have the same determinant inspite of rotations. This technique can be used to solve the simultaneous equations of N variables. Another point mentioned by Narayan Pandit is that of conversion of a square into cirlce. He says that even though it is not possible to convert a square to a circle a Bhadra Chowkon can be converted into N sided Magical shape by realignment of its member elements and he describe various techniques to do the same.

3. Computer applications of Bhadra Chowkon

The various experiments using the techniques employed by Ancient Indian mathematics to construct 'Magical Squares' leads to some interesting observations, as listed below.

1. Properties can not have proofs hence sometimes it is difficult to prove a particular statement by mathematical logic. The proof can be obtained by using linguistic expression e.g., Division by zero.

2. Magic squares represent a dynamically balanced system.

3. Models can be created by using a Magical Square to describe a phenomenon or structure. Description of various buildings are found out in Kashyapa Vastushastra.

4. Determinant of a matrix can be judged quite accurately, it is given directly in the Samhita.

5. If a programming problem can be divided into its qualitative representation then load balancing can be achieved.

6. Learning machine can be divised by using these methods as recursion and iteration in the closed space of a magical square which is always convergent, because magical square represents a function in a given space which is true.

7. Music can be created by using the magical squares as the seed required for a performing art which should be logically asymmetric or symmetric, and the randomness of the machine can be controlled due to the topological insideness. The performance would be always inside. We have developed algorithms for real time music creation by a machine and have created notations for real time performance lasting for 15 minutes and it was played by a harmonium player.

8. A very simple machine of interest can be constructed for resolving the mathematics of this type by using shift registers and clocks which can represent a new type of computing machine in which there would be no processor and separate memory but the intelligence can be derived by using probing for states at various points at various clock cycles. The problem of resolving the past can be solved by selecting various outputs of shift registers at various linear times. As soon as the system balances itself it will be static till a new stimulus arrives. The problem of direction or vector space is resolved due to the exactness of the structure. We have constructed a 8×8 machine which converged but could not continue due to non-availability of a Data Logger at that instant.

9. The scheduling type problems can be handled efficiently. Construct of algorithms and programs has been suggested to a promising student.

10. New types of logical operations could be devised regarding direction. We have created TTL logic for directional boolean logic.

4. Use of Ankapasha

Let us now try to find out what are the offerings of Ankapasha. The theory of Ankapasha directly leads us to find out the solutions for many mathematical series and sequences. The various *Panktis* and *Merus* described there allow us to find values of functions like sin x, cos x etc., and to solve transcendental equations.

(a) Calculation of cosine

The series expansion of cos x is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots \tag{1}$$

where x is in radians. Now, if we write it as

$$\cos x = 1 + A + B + C + D + \ldots\ldots \tag{2}$$

then

$$B = \frac{A^2}{6}, \quad C = \frac{AB}{15}, \quad D = \frac{AC}{28}, \text{ etc.}$$

This means if we have value of $x^2/2$ the very next term can be obtained by squaring this and dividing by 6. The next term can be obtained by multiplying the two previous terms and dividing by 15 and so on. For illustration let us calculate cos 0.2. We shall use the notation $\bar{x} = -x$ and a_b when a is quotient and b is remainder. Now in the first row we write $1 - x^2/2$ as $1.0\bar{2}$. In order to obtain the second row we note that $0.0\bar{2} = 0.0004$ and dividing it by 6 we get $0.0006_4 6_4 6_4 \ldots$. Then multiplying $0.0\bar{2}$ by 6 we get $\bar{2} \times 6 = \overline{12}$ and $\overline{12}/15 = (\overline{15} + 3)/15 = \bar{1}_3$. Thus we have

Row	Term	Divisor	Value
1.	$1 - x^2/2$		$1.0\bar{2}$
2.	$x^4/24$	6	$0.00006_4 6_4 6_4 6_4 6_4 6_4$
3.	$-x^6/720$	15	$\bar{1}_3 1 \bar{1}\ 1$
Total	cos0.2.		$i.\underline{0}200665757 = 0.980066576$

b) Solution of a transcendental equation

Consider

$$x + \sin x = 1. \tag{3}$$

It can be written as

$$x + x \quad \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \quad = 1 \tag{4}$$

or

$$x = 0.5 + \frac{x^3}{12} - \frac{x^5}{240} + \ldots \tag{5}$$

We solve Eq. (5) by successive iterations by adding one term at a time. Thus, we have

Row 1.	0.5	0.5		
Row 2.	$+ x^3 / 12$	$0.01113_{\bar{2}}$		
Row 3.	$- x^5 / 240$	$0.000\bar{1}\bar{5}$		
	Total	$0.5110\bar{2}$	=	0.51098

5. Conclusion

These are the preliminary observations regarding the Indian treatment of Magical Squares and series. The efforts at present are quite meagre with respect to the vastness of the subject. The subject needs a team effort for further explorations. If anybody is interested in continuing or participating it would be a quite fruitful exercise. The original manuscript of Narayan Pandit is worth reading. All necessary help that is needed can be provided with respect to reading the Sanskrit Samhita and having brain storming sessions.