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Wave propagation in solar magnetic tubes

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Abstract. The work done on propagation of waves in magnetic tubes is reviewed. It is seen that majority of the results obtained so far are based on the thin tube approximation. It is shown that estimates of the mechanical energy flux based on such an approximation could be grossly in error. The need for studying the propagation of waves in finite tubes is emphasised. The possibility of non-linear effects altering the propagation characteristics of these waves is pointed out.

Key words : sun — small scale magnetic fields — wave propagation — heating of chromosphere and corona

1. Introduction

The solar magnetic field outside spots and active regions is concentrated into magnetic knots or flux tubes in the photosphere. The ratio of the fraction of the atmospheric volume enclosed within these tubes increases with height above the photosphere, approaching values close to unity in the solar corona (*e.g.* Gokhale & Venkatakrishnan 1978). Such hydrostatic considerations alone are sufficient to underline the necessity of incorporating these tubes as a major ingredient in atmospheric modelling. However, a fresh look at the problem of mechanical energy transport in the sun reveals compelling reasons to assign these tubes a more dynamic role.

One such reason is that the dominant power in photospheric motions exists at frequencies just below the acoustic cut-off frequency of the photosphere. Sound waves at higher frequencies where propagation is possible carry negligible power. In fact, an order of magnitude estimate by Athay & White (1978) based on recent OSO-8 data on microturbulence clearly rules out the possibility of even high frequency sound waves heating regions higher than 2000 km above the photosphere.

A second reason is that it is now becoming increasingly evident that magnetic tubes can support new kinds of wave modes which can propagate at those frequencies at which photospheric motions dominate. The study of these modes is still in its infancy. Considerable theoretical and observational efforts are needed before one can answer the question whether these modes do deliver sufficient energy at various heights to maintain the chromosphere and the corona. However, since each step forward in answering this question is bound to need substantial involvement of additional computational and experimental resources, it is imperative to consolidate our position at this stage by reviewing the work that has already gone into the study of waves in magnetic flux tubes, to draw, if possible, general conclusions about the behaviour of such waves and to identify gaps, if any, in the understanding of these new modes.

2. Historical development of the subject

The study of wave propagation in a magnetoplasma dates backs to the pioneering work of Alfven (for a comprehensive review see Alfven 1950). The effect of compressibility on the Alfven modes was first studied by Herlofson (1950). All these results are valid for a uniform plasma with homogeneous magnetic field and infinite conductivity. In this case we have what is known as the "pure" MHD modes, namely the incompressible Alfven mode and the compressible fast and slow magnetosonic modes. In the presence of gravity all these modes become modified. The characteristics of the resulting magneto-acoustic gravity waves (MAG waves) are described by Stein & Leibacher (1974). The presence of gravity introduces the complication of a complex wavenumber. The dispersion relation can be conveniently depicted in the space of the frequency ω and horizontal wavenumber k_x as a diagnostic diagram. Regions in $\omega - k_x$ space are labelled as vertically propagating or evanescent depending or whether the vertical component of the wavenumber is real or imaginary for a given frequency ω and horizontal wavenumber k_x.

The presence of radiative losses introduces further complications. Waves which were progressive in the absence of radiation are now spatially damped; and evanescent waves leak out with small group velocities and enormous phase velocities whenever radiative losses are included. Strictly speaking all these studies are the WKB limits of realistic situations. A more realistic approach, including finite radiative diffusivity and electrical resistivity, was followed by Antia (1979). He isolated several cases of overstability of the magneto-acoustic modes in the presence of thermal conductivity. The case of atmospheres with zero magnetic field has been the subject of intense study (Eckart 1950) and some very powerful methods exist in the fluid dynamical literature to tackle a variety of wave phenomena (see *e.g.* Whitham 1974).

3. Waves in magnetic flux sheaths

Before considering the problem of tubes (cylindrical geometry) one can consider a magnetic flux sheath which will include all the effects of lateral boundaries with the added advantage that one can use Cartesian coordinates. This was done by Cram & Wilson (1975). They considered the complementary aspects of examining the transmission of a given incident sound wave through the sheath and of investigating the modes of vibration of the sheath. The latter aspect is more interesting for the purposes of this study. Since the basic method of approach is the same for finite tubes, whether cylindrical or rectangular, the method will be described in detail. The basic equations used in this case are the linearised equations of continuity, momentum and magnetic induction:

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \mathbf{V}_1) = 0, \qquad \text{...(3.1a)}$$

$$\rho_0 \frac{\partial \mathbf{V}_1}{\partial t} = -\nabla p_1 + (\nabla \times \mathbf{b}_1) \times \mathbf{B}_0, \qquad \text{...(3.1b)}$$

$$\frac{\partial \mathbf{b}_1}{\partial t} = \nabla \times (\mathbf{V}_1 \times \mathbf{B}_0), \qquad \text{...(3.1c)}$$

where ρ_0 and $\mathbf{B}_0$ are the equilibrium density and magnetic field; and ρ_1, $\mathbf{V}_1$ and $\mathbf{b}_1$ are the perturbed density, velocity and magnetic field respectively. The solutions of these equations are subject to the following boundary conditions at the interface:

$$[\mathbf{V}_1 \cdot \mathbf{n}] = 0 \qquad \text{...(3.2a)}$$

and

$$[p_1 + (\mathbf{b}_1 \cdot \mathbf{B}_0)/4\pi] = 0, \qquad \text{.. (3.2b)}$$

where the parantheses indicate the jump in the enclosed quantities as one crosses the interface. Equations (3.1a) through (3.1c) can be reduced to a second order differential equation with coefficients that are constant on each side of the interface. This has in general two linearly independent solutions with two arbitrary constants of integration. There are three such regions separated by the two magnetic field discontinuities that make up the flux sheath. Hence there will be in general six constants of integration. The two boundary conditions at each interface along with two boundary conditions at infinity yield six linear homogeneous equations involving the six constants of integration. The condition that not all these constants are zero yields the "dispersion relation" between the frequency ω and the components of the wave vector parallel to the interface. For the particular case of equal temperatures inside and outside the tubes, Cram & Wilson (1975) obtained the dispersion relation:

$$\Omega^4 \left[(1 + \tfrac{1}{2}R^2) \begin{pmatrix} \csc^2 \\ \sec^2 \end{pmatrix} k_1 a - \tfrac{1}{4}R^4 \right] - 2R^2(1 + \tfrac{1}{2}R^2)\,\Omega^2 + R^4 = 0, \qquad \text{...(3.3)}$$

where $\Omega = \omega/k_3 S$, $R = A/S$, $\csc^2 k_1 a$ is for the 'taut wire' mode (see Piddington 1973), $\sec^2 k_1 a$ is for the contraction/dilation mode, A is the Alfven speed in the sheath, S the sound speed, k_1 the component of wave vector normal to the interface and k_3 the component parallel to the interface. The dispersion relation for magneto-acoustic waves in a homogeneous magnetic field yields the auxillary relation

$$\frac{k_1^2}{k_3^2} = \frac{(\Omega^2 - R^2)(\Omega^2 - 1)}{\Omega^2(R^2 + 1) - R^2}. \qquad \text{...(3.4)}$$

It must be noted that when $\Omega^2 \to R^2/(R^2 + 1)$, then $\left| \frac{k_1}{k_3} \right| \to \infty$. The condition $\Omega^2 = R^2/(R^2 + 1)$ is equivalent to $\omega^2/k_3^2 = c_T^2$, where $c_T = SA/(S^2 + A^2)^{1/2}$ is the phase velocity of the so called tube waves which we shall encounter quite frequently in what follows. More about this aspect later. Stability considerations led Cram & Wilson to conclude that only slow modes are allowed within the sheath. However, a recent extension of this work to the case of unequal temperature inside and outside the sheath by Roberts (1980) yields a richer variety of allowed modes in the sheath.

Cram & Wilson, however, did not recognise the fact that their dispersion relation contained the relation for surface waves as well. Surface waves are waves which are confined close to the interface and are quite distinct from volume waves or bulk waves in that their propagation characteristics are a function of the discontinuities in the state variables like temperature, density and magnetic field intensity. As a matter of fact, they cease to exist when the discontinuities vanish. The subject of surface waves is well developed in non-astrophysical contexts. For the sun, only work of a limited nature exists. Wentzel (1979a) has attempted to examine the properties of surface waves relevant to the solar atmosphere. He lists those properties of surface waves which are of maximum interest:

(i) The dispersion relation.

(ii) The extent of the wave from the surface of discontinuity.

(iii) The degree by which the surface wave may be coupled to ordinary body MHD waves when the discontinuity is thin but finite.

(iv) The degree by which the gas is compressed.

To these one might add the following:

(v) The stability considerations that allow certain modes and forbid certain others.

Wentzel discusses results for a restricted range of the parameters and that too for fluids which are not stratified along the direction of the interface.

4. Wave propagation in cylindrical flux tubes

The analysis of a flux sheath can be extended to a flux tube of cylindrical cross-section. This was done by Roberts & Webb (1978) as part of a scheme to cheek their slender tube approximation. They obtained a dispersion relation of the form

$$\rho_0 \left[\frac{(k^2C_T^2 - \omega^2)(k^2V_A^2 - \omega^2)(C_0^2 + V_A^2)}{(k^2C_0^2 - \omega^2)}\right]^{1/2} \frac{I_0(m_0r_0)}{I_1(m_0r_0)}$$

$$= \rho_e\omega^2 \left[\frac{C_e^2}{k^2C_e^2 - \omega^2}\right]^{1/2} \frac{K_0(m_er_0)}{K_1(m_er_0)}, \qquad \ldots(4.1)$$

where $m_{0,e}^2 = (k^2C_{0,e}^2 - \omega^2)/C_{0,e}^2$, C_0, C_e are the sound speed within and exterior to the tube respectively, r_0 is the radius of the tube, $C_T = C_0V_A/(C_0^2 + V_A^2)^{1/2}$ is the so called tube velocity and I and K are the cylindrical Bessel functions of the first and second kind respectively. If we put $C_e = C_0$, we get

$$\omega^4 \left[\left(\frac{C_0^2 + V_A^2}{C_0^2}\right) - \left(\frac{\rho_e K_0 I_1}{\rho_0 K_1 I_0}\right)^2\right] - \omega^2 \left[\frac{(C_T^2 + V_A^2)(C_0^2 + V_A^2)}{C_0^2}\right] k^2$$

$$+ k^4 V_A^4 = 0 \qquad \ldots(4.2)$$

in a form which is strikingly similar to the dispersion relation (3.1) for a flux sheath. A conscious effort at understanding surface waves on cylindrical surfaces was made by Wentzel (1979b). The cylindrical geometry introduces some new features, for example, the presence of nodes in the variation of pressure perturbation with r, the radial distance from the axis of the tube. Moreover, in the case when the phase velocity of the surface wave exceeds the larger of the sound and the Alfven speeds in the exterior of the tube, Wentzel obtains an energy per unit length of the cylinder that diverges logarithmically when integrated towards $r \rightarrow \infty$. However such solutions cannot be excluded, since they are valid solutions of the differential equations for surface waves. Wentzel expects physical considerations like the swamping of surface wave pressure fluctuations at large distances by turbulence pressure in the background fluid to invalidate the integration beyond some finite r.

Wilson (1980) has obtained a general dispersion relation for the vibration modes of magnetic flux tubes. The relation is

$$Q^2(V^2 - R^2)\, \Phi_1(\tilde{m}_0) = (1 + \tfrac{1}{2}\gamma R^2)\; V^2\Phi_2(\tilde{m}_e), \qquad \ldots(4.3)$$

where

$$\tilde{m}_0^2 = \tilde{k}^2(1 - V^2)\,(R^2 - V^2)/[(T^2 - V^2)\,(1 + R^2)],$$

$$\tilde{m}_e^2 = \tilde{k}^2(Q^2 - V^2)/Q^2,$$

$$\phi_1(\tilde{m}_0) = I_n(\tilde{m}_0)/[\tilde{m}_0 I'_n(\tilde{m}_0)], \quad \tilde{m}_0^2 > 0$$

$$= J_n\,(\tilde{k}_0)/[\tilde{k}_0 J'_n\,(\tilde{k}_0)], \quad \tilde{m}_0^2 = -\,\tilde{k}_0^2 < 0,$$

$$\phi_2(\tilde{m}_e) = K_n(\tilde{m}_e)/[\tilde{m}_e K'_n\,(\tilde{m}_e)], \quad \tilde{m}_e^2 > 0$$

$$= \frac{[A_{ne} J_n(\tilde{k}_e) + B_{ne} Y_n(\tilde{k}_e)]}{\tilde{k}_e\,[A_{ne}\, J'_n\,(\tilde{k}_e) + B_{ne} Y'_n\,(\tilde{k}_e)]}\,, \quad m_e^2 = -k_e^2 < 0$$

and A_{ne} and B_{ne} are arbitrary constants, in terms of the dimensionless variables

$$\tilde{k} = kr_0,\ \tilde{m}_0 = m_0 r_0,$$

$$R = A/C_i,\ Q = C_0/C_i,\ V = \omega/kC_i \text{ and } T^2 = R^2/(1 + R^2) = C_T^2\,/C_1^2\,,$$

r_0 being the radius of the tube.

However, Wilson (1980) confined his study to the case of standing wave modes in the exterior where one can set $B_{ne} = 0$. He obtained the dispersion relation for some specific values of the parameters Q, R and T. For outwardly propagating waves, however, he expects complex ω. One can then think of the outwardly propagating waves acting as a source of damping for the surface wave.

Ionson (1978) and Wentzel (1979c) have examined the possibility of heating the solar atmosphere by resonant absorption of surface Alfven waves. In their model, the magnetic field is not discontinuous but changes over a finite thickness. In such a case, the surface Alfven waves (which would have been confined to the boundaries of the tubes if the field were discontinuous) excite kinetic Alfven waves which could be easily dissipated. In fact, they find that the surface wave is indeed damped once its phase velocity equals the local Alfven velocity. They interpret this damping as evidence of plasma heating. However, as demonstrated by Lee (1980), the ideal MHD equations used by both Ionson (1978) and Wentzel (1979c) do not provide any information on dissipation that might result from a kinetic treatment. Lee (1980) has pointed out that the decay rate of the surface Alfven wave may not, therefore, be interpreted directly as a plasma heating rate. In the presence of a twisted magnetic field, however, the spatial resonance of the surface Alfven wave takes place over a finite width (Krishan 1981) as opposed to the zero absorption width for an untwisted field. The width of this resonance was found by Krishan (1981) to depend on the amount of twist in the field.

A detailed exposition of the properties of Alfven surface waves can be obtained from the monograph of Hasegawa & Uberoi (1981). Applications to the sun specially in the context of coronal heating are indicated in this monograph and an extensive bibliography is given. One can see that even in the case of simple non-stratified tubes the characteristics of surface waves are extremely complicated. Eventually of course one would have to include radiative losses and finite electrical resistivity in stratified tubes if one wished to answer in a realistic manner the questions of energy transport and dissipation.

On the other hand considerable simplification of the equations can be achieved if one carries out a perturbation analysis by expanding the equations in ascending powers of r_0/L, where r_0 is the radius of the tube and L is the scale length of variation of tube radius. This "slender flux tube approximation" had been the subject of numerous recent investigations and the quantity of results alone, if not their applicability to the sun, warrants a separate discussion:

5. The slender flux tube approximation

From purely intuitive considerations, Defouw (1976) obtained the following dispersion relation for waves propagating along tubes of slender dimensions :

$$(1 + \mu) V^4 - (C_e^2 + \mu C^2 + C_T^2) V^2 + C_e^2 C_T^2 = 0. \quad \ldots(5.1)$$

Here

$$C_T^2 = C^2 C_A^2/(C^2 + C_A^2), \quad C_A^2 = B^2/4\pi\rho,$$

$$\mu = \frac{\sigma\rho_e}{\sigma_e\rho} \frac{C_e^2}{(C^2 + C_A^2)} = \frac{\sigma}{\sigma_e}\left(\frac{C^2 + \frac{1}{2}\gamma C_A^2}{C^2 + C_A^2}\right),$$

where C^2 is the sound speed inside the tube, C_e^2 is the sound speed outside the tube and σ and σ_e are the cross sections of the tube and its surroundings respectively. As $\mu \to 0$, we get $V^2 = C_e^2, C_T^2$ and as $\mu \to \infty$ we get $V^2 = C^2, 0$. Thus it is clearly seen that C_T^2 is purely a result of the tube and the corresponding mode resembles surface waves (*cf.* section 2).

For a stratified tube with isothermal stratification Defouw obtains the analogue of the acoustic cut-off frequency of a homogeneous atmosphere. This tube cut-off frequency turns out to be

$$\omega^2 = \frac{C_T^2}{H^2}\left[\frac{9}{16} - \frac{1}{2\gamma} + \frac{C^2}{C_A^2}\frac{(\gamma - 1)}{\gamma^2}\right], \qquad \text{...(5.2)}$$

which tends to $C_T^2/16H^2$ as $\gamma \rightarrow 1$. Defouw sees a close analogy for this cut-off frequency in the case of acoustic wave propagation in a uniform gas confined by a rigid tube of exponentially varying cross-section with the critical frequency given by $C/4H$. Thus, a knowledge of the local scale height of variation of the tube radius is essential before one can determine local cut-off frequencies.

A more rigorous approach was followed by Roberts & Webb (1978) where they formally expanded the equations as powers of r_0/H and equated terms of formal order in r_0/H. By this procedure they were able to retain magnetic fields, compressibility and stratification of density and temperature. For a general variation of temperature and density with height they obtain the critical cut-off frequency as

$$\omega_C^2 = \frac{C_T^2(z)}{\Lambda_0^2(z)}\left\{\frac{3}{4}\Lambda_1'(z) + \frac{9}{16} - \frac{1}{2\gamma} + \frac{(\gamma - 1)}{\gamma^2} + \frac{\Lambda_0'(z)\, C_0^2}{V_A^2}\right\}, \qquad \text{...(5.3)}$$

which reduces to Defouw's equation for the critical cut-off frequency in the limit of constant Λ_0. Here Λ_0 is the scale height of temperature variation, and C_0^2, V_A^2 and C_T^2 are the speeds of acoustic waves, Alfven waves and tube waves respectively.

In the general case of variable Λ_0, Roberts & Webb find a singularity at $\omega^2 = N_0^2$, where N_0 is the Brunt-Vaisala frequency. They associate this with critical levels in wave propagation as has been discussed in fluid mechanics (*e.g.* Lin 1955) magnetohydrodynamics (Acheson 1972; Rudraiah & Venkatachalappa 1972) and in solar physics (*e.g.* Adam 1977; Thomas 1976). However they find the functions to be well behaved at this "singular point" contrary to the results of Acheson (1972) and Adam (1977). This is not so puzzling since their "singular point" is a spurious singularity. In reality it is that point where the solution for p, the pressure perturbation, has a turning point in $p - \upsilon$ space (phase diagram). Such a phenomenon is quite normal in wave propagation along stratified fluids (see Eckart 1950).

Roberts & Webb have, as an application to the sun, calculated the values of the cut-off frequency as a function of height in the solar atmosphere. They find that this frequency has a peak at a depth of 9.2 km corresponding to a period of 62 s. These results are in contrast to the results obtained for an isothermal atmosphere which differ by a factor of 3. Roberts & Webb therefore advocate caution while applying isothermal models to the non-isothermal atmosphere of the sun.

Inclusion of radiative relaxation (in the optically thin approximation) makes the evanescent waves propagate and damp the progressive waves (Roberts & Webb 1980b). If τ_S and τ_R are the acoustic and radiative relaxation times respectively, then in the limit $\tau_S/\tau_R \ll 1$, Roberts & Webb (1980a) obtain five modes for a homogeneous magneto-plasma. These can be represented as

$$\omega_{1,2,3,4} \approx \omega_* + \frac{i}{2} kC_0 \frac{\tau_S}{\tau_R} \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{m^2 V_A^2 - \omega_*^2}{m^2(C_0^2 + V_A^2) - 2\omega_*^2}\right) \quad \text{...(5.4a)}$$

and

$$\omega_5 \approx \frac{i}{\gamma} kC_0(\tau_S/\tau_R), \quad \text{...(5.4b)}$$

where ω_* satisfies the relation

$$\omega_*^2 = \tfrac{1}{2}m^2 [(C_0^2 + V_A^2) \pm \{(C_0^2 + V_A^2)^2 - 4C_0^2 V_A^2 \cos^2\theta\}^{1/2}]. \quad \text{...(5.4c)}$$

In the limit $\tau_R/\tau_S \ll 1$,

$$\omega_{1,2,3,4} = \omega_* + \frac{i(\gamma - 1)\omega_*^2 \; (m^2 V_A^2 - \omega_*^2)\, mC_0\, \tau_R/\tau_S}{2m^2 V_A^2 [m^2(C_0^2\gamma^{-1} + V_A^2) - 2\omega_*^2] \cos\theta} \quad \text{...(5.5a)}$$

and

$$\omega_5 = ikC_0(\tau_S/\tau_R), \quad \text{...(5.5b)}$$

where

$$\omega_*^2 = \tfrac{1}{2}m^2\left[\left(\frac{C_0^2}{\gamma} + V_A^2\right) \pm \left\{\left(\frac{C_0^2}{\gamma} + V_A^2\right)^2 - 4\frac{C_0^2}{\gamma} V_A^2 \cos^2\theta\right\}^{1/2}\right]. \quad \text{...(5.5c)}$$

Here k and m are the wavenumbers parallel and normal to the magnetic field respectively and θ is the angle between the direction of wave propagation and magnetic field. In the case of a slender tube the dispersion relation becomes

$$(C_0^2 + V_A^2)\,\omega^3 - \frac{i}{\tau_R}\left(\frac{C_0^2}{\gamma} + V_A^2\right)\omega^2 - k^2C_0^2 V_A^2\, \omega + i\frac{k^2C_0^2 V_A^2}{\gamma\tau_R} = 0. \quad \text{...(5.6)}$$

Thus the equation is of third degree in ω now as compared to the fifth degree for a homogeneous fluid. The disappearance of two modes is reminiscent of the disappearance of modes in a flux sheath (see discussion in Cram & Wilson 1975). Maximum damping occurs at those wavelengths λ_M where

$$\lambda_M \approx \tau_R C_0 \qquad \ldots(5.7)$$

Roberts & Webb (1980b) extended this analysis to a stratified tube. As in the case of homogeneous media the modes which were progressive in the absence of dissipation are now attenuated and waves which were formerly evanescent now propagate. Roberts & Webb classify these mixed modes as mainly damped or mainly progressive according as $\omega^2 < \omega^2_{VT}$ or $\omega^2 > \omega^2_{VT}$ respectively where ω^2_{VT} is the root of the equation

$$\omega^2(\omega^2 - \omega^2_V) + \frac{1}{\gamma^2 \tau^2_R}\left(\omega^2 + \overline{\gamma - 1}\,\omega^2 \frac{C^2_T}{C^2_0} - \frac{C^2_T}{16\Lambda^2_0}\right) = 0, \qquad \ldots(5.8)$$

and ω_V is the cut-off frequency in the absence of radiation. By a 'local' calculation Roberts & Webb show that radiative dissipation considerably modifies the waves for the first few scale heights above $\tau_{5000} = 1$, but thereafter has negligible effect on the wave propagation.

The work of Roberts & Webb dealt mainly with longitudinal waves. However, the longitudinal waves have cut-off frequencies close to the acoustic cut-off frequency and therefore suffer from the same limitation as sound waves. This prevents them from being potential candidates for heating the chromosphere and corona (Spruit 1981a). However, a similar analysis by Spruit (1981a) for transversal waves yields cut-off frequencies well below the acoustic cut-off. To obtain this result Spruit (1981a) first writes down the equation of motion normal to the surface of a flux tube which has arbitrary shape as

$$\rho\left(\frac{d\mathbf{V}}{dt}\right)_\perp = -\left[\hat{\mathbf{1}} \times \nabla\left(p + \frac{B^2}{8\pi}\right)\right] \times \hat{\mathbf{1}} + \frac{B^2}{4\pi}\mathbf{k} + \rho(\hat{\mathbf{1}} \times \mathbf{g}) \times \hat{\mathbf{1}}, \qquad \ldots(5.9)$$

where $\hat{\mathbf{1}}$ is a unit vector along the axis of the tube, **k** is the curvature vector which is normal to the tube and possesses a magnitude equal to R^{-1} (where R is the local radius of curvature of the tube), **g** is the acceleration due to gravity and p and B are the pressure and magnetic field inside the tube. For an isothermal atmosphere in a vertical flux tube Spruit (1981a) obtains the dispersion relation

$$\omega^2 = (16k^2H^2 + 1)\,\omega^2_c\,. \qquad \ldots(5.10a)$$

Here

$$\omega^2_c = \frac{g}{8H}\left(\frac{1}{2\beta + 1}\right) = \frac{\omega^2_a}{4r^2}\left(\frac{1}{2\beta + 1}\right), \qquad \ldots(5.10b)$$

where ω_a is the acoustic cut-off frequency and β is the ratio of gas to magnetic pressure of the equilibrium state. Thus it is seen that ω_c can be much less than ω_a for moderate fields. A further extension of this work can be seen in Spruit (1981b) where one more wave, *viz.* the torsional Alfven wave, is described. The torsional Alfven waves cannot be excited by motions in the fluid surrounding the tube. If they are indeed present, they represent unwinding of twists stored in the field before its emergence. The transversal waves are easily excited. These would be damped by MHD radiation emanating from the tube, if the tube were less dense than the surroundings, but not if it were denser.

The transversal waves of Spruit (1981a, b) may also have their counterparts in surface waves, for example the Alfven surface waves of Uberoi and Somasundaram (1980). Hence a study of such waves is all the more important if indeed the cut-off frequency is well below the acoustic cut-off frequency. However, Spruit's estimates of the energy delivered to the chromosphere and the corona must await the results of calculations based on wave propagation in finite tubes before they can be accepted. The reason is that finite tube analysis for longitudinal waves shows that the tube waves correspond to a limiting case of interface waves confined to the rim of finite tubes [Wilson 1980; see also equation (3.4) of this article]. In this case the energy is delivered only at a vanishingly small fraction of the tubes' cross-section. The same kind of behaviour may hold true for transversal tube waves as well. Thus the estimates based on the thin tube approximation may be grossly in error.

For the sake of completeness, mention must be made here of the possibility of convective instability in thin flux tubes leading to either intensification of the field or it dispersal. This was independently arrived at by Webb & Roberts (1978) and Spruit (1979). Spruit & Zweibel (1979) have some numerical calculations of the growth rates. Even here the criteria for onset of instability in finite tubes may be different from that for a thin tube.

6. Non-linear calculations

The work described so far was based on linear analysis. The quantitative estimation of power delivered can be obtained only through non-linear calculations. In the case of stratified fluid without discontinuities, several such calculations are available (see *e.g.* Ulmschneider 1971; Ulmschneider *et al.* 1977, 1978) and are in good agreement with the gross structure of the solar atmosphere. However, the effect of discrete magnetic inhomogeneities on these calculations is still unknown. From a totally different angle, Hasan & Venkatakrishnan (1980, 1981) and Venkatakrishnan & Hasan (1981) have considered the non-linear effects of tube motions on the gas within the tubes. They have essentially studied the flow of gas along tubes whose shape is a given function of time. One result that emerges from these calculations is that a steady flow is generated along the field in the direction of increasing amplitude of the lateral motions. Such a flow is induced because of the centrifugal acceleration on the gas associated with the field line when it is constrained to move on a curved path. The curvature depends on the gradient, along the field, in the amplitude of motions normal to the field. Such an acceleration is a purely non-linear effect. The unidirectional nature of the force even in the case of oscillatory motion of the tube walls makes it important dynamically since even a small acceleration can produce significant flows if it persists long enough. The effect of such forces on wave propagation in and stability of the tubes is yet to be considered.

7. Observations

Observations of waves on the sun date back to the observations of "wiggly line" spectra (Richardson & Schwarzschild 1950). However, direct observations of waves within magnetic elements are extremely demanding of instrumental stability and resolution. Since magnetic elements cannot be resolved spatially at present their oscillation properties may provide a valuable diagnostic to physical conditions within the elements. One such diagnostic technique to measure the size of unresolved magnetic elements was suggested by Venkatakrishnan (1979). For this method to work, one requires a large abundance of high frequency sound waves to be incident on the tubes for large times ($\sim$ 30 min). The "seeing" must also be excellent during these 30 min.

A method to measure gas velocities within tubes has been devised (Giovanelli & Ramsay 1976) which was used by Giovanelli & Brown (1977) and subsequently by Giovanelli *et al.* (1978) to study motions in isolated magnetic elements as well as in plage region magnetic points. This method is based on the Zeeman splitting of a spectral line into two circularly polarised components in a longitudinal magnetic field. The separation depends on the field strength but there is one wavelength in the line profile where the intensities of the two components are identical *viz.* the centre of the Zeeman pattern. The latter will be displaced by a Doppler shift if there happens to be a longitudinal motion of the gases in the tube. If we can locate this wavelength, we have a measure of the gas velocity averaged over the magnetic elements. The main results of such observations were as follows:

1. Magnetic elements in each window studied within plage regions and in quiet non-plage regions exhibited velocity oscillations. The general pattern was the same as that of the well known oscillations in non-magnetic regions. In all the spectral lines observed, stronger oscillations occurred in pulses between which the amplitude was smaller. The period of the oscillation in all lines except H_α was 5 min. In H_α the period was 3 min. No similar oscillations in magnetic field strength were seen.

2. All the waves observed seemed to be propagating with a widely scattered range of time delays. There was even a time delay between a disturbance within the tube and exterior to the tube at approximately similar heights. There was a systematic increase of amplitude with height but this amplitude did not reach shock strengths.

3. There was no precise one-to-one relation between motions within and exterior to the tube. However, the motions behaved more or less in the same way. Giovanelli *et al.* (1979) attribute this to either strong interaction of tubes with their environment or to a common mechanism for the excitation of these waves.

The absence of magnetic oscillations around 5 min period is contrary to the observations of Severny (1971). The absence of observations of shocks might be due to either radiative damping or presence of surface shocks in thin rims near the boundary of the tube which cannot be detected in the data averaged out over the cross-section.

In a different observation, Giovanelli (1975) saw two kinds of wave patterns along fibrils. One consisted of velocity disturbances accompanied by intensity changes propagating at a phase speed of 70 km s^{-1}. The other consisted of a drift at sonic speeds of intensity patterns unaccompanied by line-of-sight velocity variations. It would be tempting to identify the first mode with transversal tube waves and the other with longitudinal tube waves. However, it would be premature to contemplate such identifications before one understands wave propagation along finite tubes.

An indirect way of estimating unresolved velocity fluctuations within tubes is to study the broadening of spectral lines. In fact, the width of the Ca II K line seen within network regions was found to be narrower than that seen outside the network (Bappu & Sivaraman 1971). Though one cannot infer anything about the dynamical state of the gas within these regions until line formation mechanisms are fully understood, spectroscopy of these tubes may well open a promising line of enquiry.

8. Summary

It is seen that tubes—or more precisely discontinuties in magnetic field—can support various new wave modes that may be able to propagate at frequencies at which sound waves cannot. However, the majority of work reported in the literature is based on the thin tube approximation which may give results that are misleading both qualitatively and quantitatively. There is great scope for further study of wave propagation within tubes, especially within finite tubes. To understand the signatures of such waves more attention must be paid to line formation within tubes. Observationally, spectroscopy of these tubes would greatly aid the construction of realistic thermodynamic models. Needless to say, more observations of the type attempted by Giovanelli *et al.* (1978) are necessary.

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