

Phase Transition and Hybrid Star in a Nonlinear $\sigma - \omega$ Model

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The phase transition between the nuclear matter and the quark matter is examined. The relativistic mean field theory(RMF) is consider with interacting nucleons and mesons using TM1 parameter set for the nuclear matter equations of state. It is found that the trasion point depends on coupling constant α_s and bag pressure. From the study of the structure of a hybrid neutron star, it is observed that the star contains quark matter in the interior and neutron matter on the outer periphery.

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I. INTRODUCTION

It is widely believed that nuclear matter undergoes a phase transition to quark matter at high densities and/or high temperatures. The high temperature limit is expected to have interesting consequences in heavy ion collision and/or in cosmology, whereas high baryon density behaviour is important for the study of neutron stars.

It is expected that Quantum Chromodynamics (QCD) as the fundamental theory of the strong interaction should explain possible modifications of hadron properties in the nuclear medium. However, typical nuclear phenomena at intermediate and low energies cannot be analytically derived from QCD although one hopes that QCD will be solved numerically on the lattice in near future. Meanwhile we are left with the construction of phenomenological models in order to try to describe nuclear phenomena and its bulk properties. Walecka and others [1,2] used a kind of relativistic scalar-vector theory to describe the nucleon-nucleon properties of nuclear matter as well as of properties of finite nuclei. Some of the drawbacks of the original Walecka model are that the effective nucleon mass obtained at high densities is too small and its incompressibility at the energy density saturation is too large. To eliminate these difficulties Boguta and Bodmer [3] modified the original model by introducing selfcoupling terms to the scalar field. The inclusion of nonlinear terms to the scalar field surprisingly improved the results of nuclear matter as well as of finite nuclei. However, in most of the successful parameter sets, the last term of the self coupling constant is found to be negative [4]. This negative value of the last term gives an unphysical situation at high density, which is essential for a further modification of the model. This modification is done by Bodmer [5] by introducing a quartic nonlinear term to the vector potential to study the equation of state. Later on this suggestion was considered to study the properties of finite nuclei [6], which gives the nuclear matter and finite nuclei properties excellently well [7].

Many of the existing parametrisation is unable to reproduce the properties of finite nuclei, nuclear matter and that of the physical properties of accreting steller objects, like neutron star etc. Recently a detail calculation has been done with different models to study the properties of neutron stars [8]. Glendenning [9] has studied the properties of neutron star in the framework of nuclear relativistic field theory. This parameter set is not applicable directly to finite nuclei. Similarly many parameter sets which explain the properties of accreting matter is unable to explain the properties of finite nuclei as well as of normal nuclear matter properties. On the other hand, the parameter set which is able to explain the properties of finite nuclei and normal nuclear matter properties, failed to explain the properties of accreting matter. In this work, our aim is to see the applicability of the improved parametrisation of Sugahara and Toki. [6] to nuclear matter which explains well the properties of finite, including superheavy nuclei, infinite nuclear matter and the properties of negative energy bound states at normal as well as at high densities [10].

In a similar study [11], the equation of state for neutron matter is obtained in a nonperturbative method with pion dressing of neutron matter, an analysis similar to that of symmetric nuclear matter. The quark matter sector was treated perturbatively with bag constant $B^{\frac{1}{4}} = 148$ GeV. Stable solution for such a quark-neutron hybrid star was obtained with Chandrasekhar limit as $1.58M_{\odot}$ and radius around 10 km. However in the present calculation we have considered the effect of $\sigma - \omega$ mesons with the nonlinear interactions and have observed the increase in size and mass of such stars with mass about $2.3M_{\odot}$ and radius 13.5 km.

In a further study, Sugahara and Toki [12] have taken $\Lambda - \omega$ tensor coupling and found a heavier critical mass of neutron star beyond observational boarder but without tensor coupling they have shown that their result does not agree with observational result indicating that the tensor coupling is indispensible for meeting observational requirement.

We consider here relativistic mean field theory with interacting nucleons and mesons, using a nonlinear version in both σ and ω mesons for the nuclear matter equation of states. Quark matter is treated perturbatively for high densities at short distances [13]. A first order phase transition between nuclear matter and quark matter seems to be indicated. Solutions of the Tolman - Oppenheimer - Volkoff (TOV) equations yield a hybrid star having a quark core with a crust of neutron matter.

The paper is organized as follows. In Sec. II, we present a brief theory for nuclear matter (neutron matter) equation of state. The quark matter equation of state is discussed in Sec. III. In section IV, we discuss the structure of hybrid neutron star. A summary and concluding remarks are given in Sec. V.

II. NUCLEAR MATTER EQUATION OF STATE

We start with the effective Lagrangian density for a nucleon-meson many-body system for nuclear matter. In this Lagrangian we have considered only the interaction of nucleons with σ , ω and ρ mesons.

The Lagrangian is given as [1,4,6]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - g_s\bar{\psi}\psi\sigma \\ & - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu - \frac{1}{4}R_{\mu\nu}^a R^{\mu\nu a} \\ & + \frac{1}{2}m_\rho^2 R_\mu^a R^{\mu a} - g_\rho\bar{\psi}\gamma^\mu\tau^a\psi R^{\mu a} \end{aligned} \quad (2.1)$$

The fields for the σ, ω and ρ -mesons are denoted by σ, ω_μ and R_μ respectively and ψ is the Dirac spinor for the nucleon. Here g_s, g_ω, g_ρ are the coupling constants for σ, ω and ρ -mesons and g_2, g_3 and c_3 are self coupling constants. M is the mass of the nucleon and m_σ, m_ω and m_ρ masses of the σ, ω and ρ -mesons respectively. The contribution of ρ -mesons to neutron matter is essential and has effect on the formation of hybrid stars.

In Ref. [7] it has been shown that RMF approach is successful to describe the result of Relativistic Dirac Bruckner Hartree-Fock (RDBHF) calculations in nuclear matter. It is found that although the RMF model with scalar self-interactions is able to describe effectively the binding energy of nuclear matter as well as the bulk properties of finite nuclei, this is not followed by a proper description of the effective nucleon mass and a time-like component of the vector self-energy. This is caused mainly by a too restrictive treatment of ω -meson term in the RMF approach, which does not take into account the density dependence produced by the relativistic Dirac-Brueckner approximations to all mesons involved in the theory. The study of finite nuclei and nuclear matter of Sugahara and Toki [6] shows that the vector potential of the RMF theory increases linearly with density and gets stronger, while RDBHF bends down with density. The scalar potential of the RMF theory seems to be overestimating the RDBHF results at high density in order to compensate for the strong repulsion in the vector channel. This is the reason for providing the wrong sign in σ^4 self-coupling constant in most of the successful parameter sets. Thus, Sugahara and Toki introduced a nonlinear term $(\omega_\mu\omega^\mu)^2$ into the ω vector meson potential to study the properties of finite nuclei.

In the mean field approximation the meson field operators are replaced by their expectation values. We also consider the isotropic system at rest. The equation of motions for meson and nucleon fields are

$$m_\sigma^2\sigma = -g_s\rho_s - g_2\sigma^2 - g_3\sigma^3 \quad (2.2a)$$

$$m_\omega^2\omega_0 = g_\omega\rho_B - c_3\omega_0^3 \quad (2.2b)$$

$$m_\rho^2 R_0^3 = \frac{1}{2}g_\rho \langle \bar{\psi}\gamma^0\tau_3\psi \rangle \quad (2.2c)$$

$$m_\rho^2 R_{03} = g_\rho\rho_{03} \quad (2.2d)$$

where

$$\rho_{03} = \frac{1}{2} \langle \bar{\psi}\gamma^0\tau_3\psi \rangle \quad (2.2e)$$

and

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta M^*)\psi = (E - g_\omega\omega_0)\psi \quad (2.2f)$$

In the above we use the effective nucleon mass $M^* = M + g_\sigma\sigma$. The source terms for scalar and vector fields are the scalar density $\rho_s = \langle \bar{\psi}\psi \rangle$ and the vector(baryon) density $\rho_B = \langle \psi^\dagger\psi \rangle$ respectively. Using the standard positive energy solutions of the Dirac equation, we obtain

$$\rho_s = \frac{\gamma}{2\pi^2} \int_0^{k_f} k^2 \frac{M^*}{\sqrt{k^2 + M^{*2}}} dk \quad (2.3)$$

and

$$\rho_B = \frac{\gamma}{6\pi^2} k_f^3 \quad (2.4)$$

Here we assume that nuclear matter consists of filling nucleon levels up to the Fermi momentum k_f and γ is the spin-isospin degeneracy factor ($\gamma = 4$ for nuclear matter and $\gamma = 2$ for neutron matter). The effective nucleon mass M^* has to be determined self-consistently at each density by solving equation (2.2a) for the scalar field and (2.3) for the scalar density.

The energy density of the nuclear matter in the mean field approach is given by

$$\epsilon = \epsilon_N + \epsilon_\sigma + \epsilon_\omega + \epsilon_\rho \quad (2.5)$$

Here ϵ_N is the energy density of nucleons of mass M^*

$$\begin{aligned} \epsilon_N &= \frac{\gamma}{2\pi^2} \int_0^{k_f} k^2 \sqrt{k^2 + M^{*2}} dk \\ &= \frac{\gamma}{2\pi^2} \left[\frac{1}{4} k_f (k_f^2 + M^{*2})^{3/2} - \frac{1}{8} M^{*2} k_f \sqrt{k_f^2 + M^{*2}} - \frac{1}{8} M^{*4} \ln \left(\frac{k_f + \sqrt{k_f^2 + M^{*2}}}{M^*} \right) \right] \end{aligned} \quad (2.6a)$$

The energy density ϵ_σ is the sigma meson interaction term which may be written as

$$\epsilon_\sigma = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 \quad (2.6b)$$

The third term ϵ_ω is the omega-meson interaction term which is given by

$$\epsilon_\omega = -\frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{4} c_3 \omega_0^4 + g_\omega \omega_0 \rho_B \quad (2.6c)$$

and

$$\epsilon_\rho = \frac{1}{2} m_\rho^2 R_{03}^2 \quad (2.6d)$$

Similarly the pressure for the nuclear matter is given by

$$P = P_N + P_\sigma + P_\omega + P_\rho \quad (2.7)$$

$$\begin{aligned} P_N &= \frac{1}{3} \frac{\gamma}{2\pi^2} \int_0^{k_f} \frac{k^4}{\sqrt{k^2 + M^{*2}}} dk \\ &= \frac{1}{3} \frac{\gamma}{2\pi^2} \left[\frac{1}{4} k_f^3 \sqrt{k_f^2 + M^{*2}} - \frac{3}{8} M^{*2} k_f \sqrt{k_f^2 + M^{*2}} + \frac{3}{8} M^{*4} \ln \left(\frac{k_f + \sqrt{k_f^2 + M^{*2}}}{M^*} \right) \right] \end{aligned} \quad (2.8a)$$

$$P_\sigma = -\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \quad (2.8b)$$

$$\begin{aligned} P_\omega &= -\epsilon_\omega + \rho_B \frac{\partial \epsilon_\omega}{\partial \rho_B} \\ &= \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{4} c_3 \omega_0^4 + g_\omega \rho_B^2 \left[\frac{g_\omega}{m_\omega^2 + 3c_3 \omega_0^2} \right] - \rho_B (m_\omega^2 \omega_0 + c_3 \omega_0^3) \left[\frac{g_\omega}{m_\omega^2 + 3c_3 \omega_0^2} \right] \end{aligned} \quad (2.8c)$$

$$P_\rho = \frac{1}{2} m_\rho^2 R_{03} \quad (2.8d)$$

Here we use TM1 parameter set. The values of the parameter set are [6] $M = 938.0$, $m_\sigma = 511.198$, $m_\omega = 783.0$, $m_\rho = 770$, $g_s = 10.0289$, $g_\omega = 12.6139$, $g_\rho = 4.6322$, $g_2 = -7.2325 \text{ fm}^{-1}$, $g_3 = 0.6183$ and $c_3 = 71.3075$ where masses are in MeV. The corresponding nuclear matter properties obtained from the parameter set are $\rho_0 = 0.145 \text{ fm}^{-3}$, $E/A = -16.3 \text{ MeV}$, $K = 281 \text{ MeV}$, $M^*/M = 0.634$ and $a_{asy} = 36.9 \text{ MeV}$, where ρ_0 , a_{asy} , K , and E/A are the density, asymmetric parameter, compressibility modulus and the binding energy per particle, respectively. Sugahara and Toki [6] show that the TM1 parameter set gives more closer results with the RDBHF than the NL1 and NL-SH parameter sets. Also, we know earlier [2] that the linear set of Horowitz and Serot gives stiff equation of states and predicts a too high value of compressibility modulus of about 560 MeV whereas the empirical value is $210 \pm 30 \text{ MeV}$ [14]. The value obtained by TM1 parameter set is more convincing. In figure 1, we have shown the behaviour of pressure against density in TM1 and NL-SH parameter set.

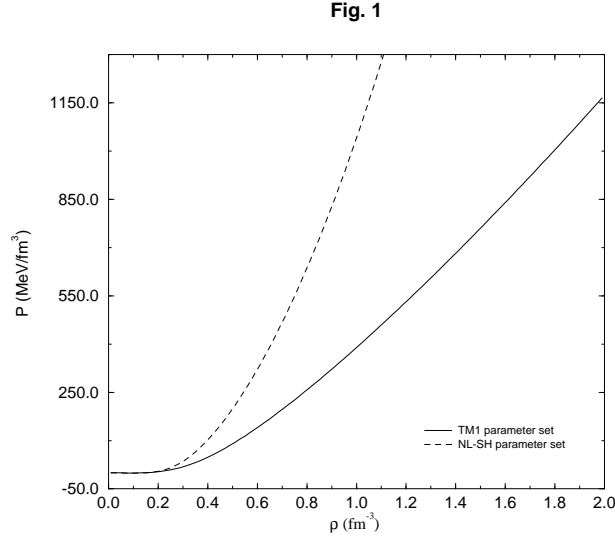


FIG. 1. Pressure (P) versus density (ρ) of nuclear matter is shown with TM1 parameter set.

III. QUARK MATTER EQUATION OF STATE AND PHASE TRANSITION

Existence of quark matter in the core of neutron stars/pulsars is an exciting possibility [15]. Densities of these stars are expected to be high enough to force the hadron constituents or nucleons to overlap thereby yielding quark matter. Since the distance involved is small, perturbative QCD is used to derive quark matter equation of state. We take the quark matter equation of state as in Refs. [16,17] in which u,d and s quark degrees of freedom are included in addition to electrons. Here we set the electron, up and down quark masses to zero [16] and the strange quark mass is taken to be 180 MeV. In chemical equilibrium $\mu_d = \mu_s = \mu_u + \mu_e$. In terms of baryon and electric charge chemical potentials μ_B and μ_E , one has

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_E, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E, \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E. \quad (3.1)$$

The pressure contributed by the quarks is computed to order $\alpha_s = g^2/4\pi$ where g is the QCD coupling constant. Confinement is simulated by a bag constant B . The electron pressure is [16]

$$P_e = \frac{\mu_e^4}{12\pi^2}. \quad (3.2)$$

The pressure for quark flavor f , with $f=u,d$ or s is [13,16,17]

$$P_f = \frac{1}{4\pi^2} \left[\mu_f k_f (\mu_f^2 - 2.5m_f^2) + 1.5m_f^4 \ln \left(\frac{\mu_f + k_f}{m_f} \right) \right] - \frac{\alpha_s}{\pi^3} \left[\frac{3}{2} \left(\mu_f k_f - m_f^2 \ln \left(\frac{\mu_f + k_f}{m_f} \right) \right)^2 - k_f^4 \right]. \quad (3.3)$$

The Fermi momentum is $k_f = (\mu_f^2 - m_f^2)^{1/2}$. The total pressure, including the bag constant B is

$$P = P_e + \sum_f P_f - B. \quad (3.4)$$

There are only two independent chemical potentials μ_B and μ_E . μ_E is adjusted so that the matter is electrically neutral, i.e. $\partial P / \partial \mu_E = 0$. The baryon number density is given by $\rho = \partial P / \partial \mu_B$.

We now consider the scenario of phase transition from nuclear matter to quark matter. As usual, the phase boundary of the coexistence region between the nuclear and quark phase is determined by the Gibbs criteria. The critical pressure and critical chemical potential are determined by the condition

$$P_{nm}(\mu_B) = P_{qm}(\mu_B). \quad (3.5)$$

We take $\alpha_s = 0.5, 0.6$ and the bag constant $B = (150 \text{ MeV})^4, (155 \text{ MeV})^4$, which is a reasonable value to calculate pressure in the quark sector. Schaab et al. [18] used this value to be $B^{1/4} = 145 \text{ MeV}$. However in a calculations of Glendinning [19] the bag pressure was taken as $B^{1/4} = 180 \text{ MeV}$. In this calculations [19] the transition was determined for the above bag constant which places the energy per baryon of strange quark matter 1100 MeV, well above the energy per nucleon in infinite nuclear matter as well as the most stable nucleus, ^{56}Fe ($E/A \approx 930 \text{ MeV}$). In figure 2, we have plotted pressure versus chemical potential for nuclear matter and quark matter. The solid line is shown for nuclear matter, with TM1 parameter set. The dash and the dotted lines are shown for quark matter with $\alpha_s = 0.5$ and $\alpha_s = 0.6$ with bag pressure $B = (155 \text{ MeV})^4$ respectively. A remarkable feature of the state of affair is that there exist transition points for nuclear matter to quark at different pressures and chemical potentials. These transition points (P_{crit}, μ_{crit}) are at $\alpha_s = 0.6$ with $(150 \text{ MeV}/\text{fm}^3, 1280 \text{ MeV})$ and at $\alpha_s = 0.5$ with $(260 \text{ MeV}/\text{fm}^3, 1445 \text{ MeV})$ showing dependance on α_s and these also indicate the first order phase transition from nuclear matter to quark matter at different thermodynamical conditions. We also note that the phase transition seems to occur around the number density of about 5 times the nuclear matter density. These points also change under different bag pressure. Figure 3 shows the phase diagram with a different bag pressure $B = (150 \text{ MeV})^4$. Here we found that transition point decreases with decrease of bag pressure, whereas the transition point shifts to a higher value with an increase in coupling constant α_s . The early phase transition from nuclear matter to quark matter obviously implies that the interior of "neutron star" will usually consists of quark matter. We investigate this possibility in the next section.

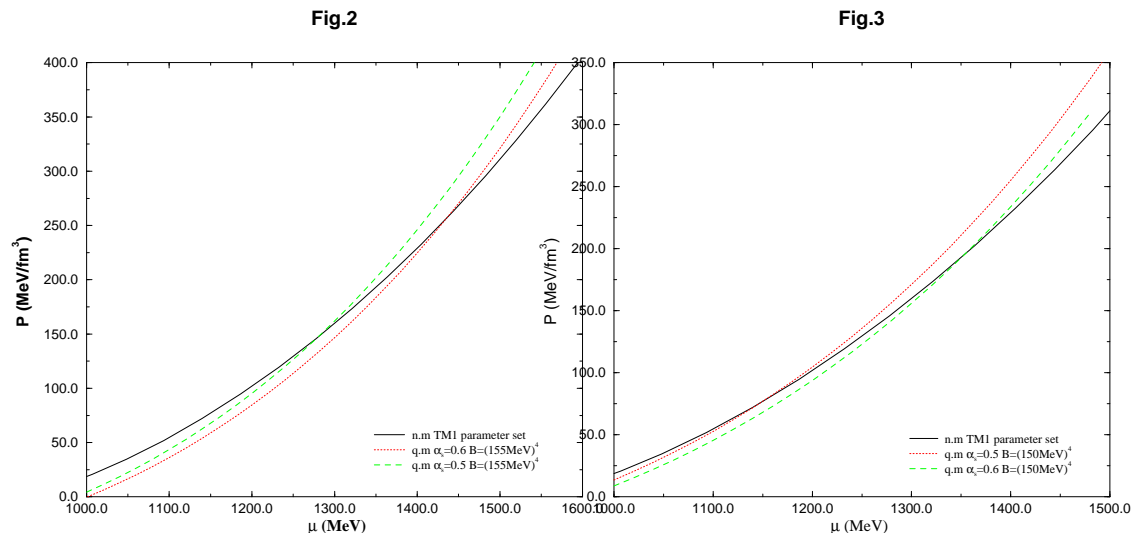


FIG. 2. Pressure (P) versus chemical potential (μ) for nuclear matter and for quark matter pressure (B) with various α_s at constant bag pressure (B). FIG. 3. Same as Fig.2 with a different bag

IV. HYBRID STARS

For the description of neutron star, which is highly concentrated matter so that the metric of space-time geometry is curved and one has to apply Einstein's general theory of relativity. The space-time geometry of a spherical neutron

star described by a metric in Schwarzschild coordinates has the form [20]

$$ds^2 = -e^{\nu(r)}dt^2 + [1 - 2M(r)/r]^{-1}dr^2 + r^2[d\Theta^2 + \sin^2\Theta d\phi^2] \quad (4.1)$$

The equations which determine the star structure and the geometry are, in dimensionless forms [20,21]

$$\frac{d\hat{P}(\hat{r}r_0)}{d\hat{r}} = -\hat{G} \frac{[\hat{\epsilon}(\hat{r}r_0) + \hat{P}(\hat{r}r_0)][\hat{M}(\hat{r}r_0) + 4\pi a \hat{r}^3 \hat{P}(\hat{r}r_0)]}{\hat{r}^2 [1 - 2\hat{G}\hat{M}(\hat{r}r_0)/\hat{r}]}, \quad (4.2a)$$

$$\hat{M}(\hat{r}r_0) = 4\pi a \int_0^{\hat{r}r_0} d\hat{r}' \hat{r}'^2 \hat{\epsilon}(\hat{r}'r_0), \quad (4.2b)$$

and the metric function, $\nu(r)$ is given by

$$\frac{d\nu(\hat{r}r_0)}{d\hat{r}} = 2\hat{G} \frac{[\hat{M}(\hat{r}r_0) + 4\pi a \hat{r}^3 \hat{P}(\hat{r}r_0)]}{\hat{r}^2 [1 - 2\hat{G}\hat{M}(\hat{r}r_0)/\hat{r}]}. \quad (4.2c)$$

In equations (4.2) the following substitutions have been made.

$$\hat{\epsilon} \equiv \epsilon/\epsilon_c, \quad \hat{P} \equiv P/\epsilon_c, \quad \hat{r} \equiv r/r_0, \quad \hat{M} \equiv M/M_\odot, \quad (4.3a)$$

where, with $f_1 = 197.327$ MeV fm and $r_0 = 3 \times 10^{19}$ fm, we have

$$a \equiv \epsilon_c r_0^3 / M_\odot, \quad \hat{G} \equiv \frac{G f_1 M_\odot}{r_0} \quad (4.3b)$$

In the above, quantities with hats are dimensionless. G in equation (4.3b) denotes the gravitational constant with $G = 6.707934 \times 10^{-45}$ MeV⁻².

In order to construct a stellar model, one has to integrate equations (4.2a) to (4.2c) from the star's center at $r = 0$ with a given central energy density ϵ_c as input until the pressure $P(r)$ at the surface vanishes. As stated in the last section, with any reasonable central density, we expect that at the center we shall have only quark matter. Hence we shall be using here the equation of state for quark matter through equation (3.4) with $\hat{P}(0) = P(\epsilon_c)$. We then integrate the TOV equations until the pressure and density decrease to their critical values at radius $r = r_c$. For $r > r_c$, we shall have equation of state for neutron matter where pressure will change continuously but the energy density will have a discontinuity at $r = r_c$. TOV equations with equation of state for neutron matter shall be continued until the pressure vanishes. This will complete the calculations for stellar model for hybrid "neutron" star, whose mass and radius can be calculated for different central densities.

In Fig.4a, we plotted the mass of the star as a function of central energy density to examine the stability of such stars. As may be seen from the figure, $dM/d\epsilon_c$ starts becoming negative around 1500 MeV/fm³ after which it becomes unstable and may collapse into black holes [20,22] with the Chandrasekhar limit as $2.3M_\odot$. This yields stable hybrid star of mass $M \sim 2.3M_\odot$ with radius $R \sim 13.5$ km with a quark core around 4.2 km as seen from Fig.4b.

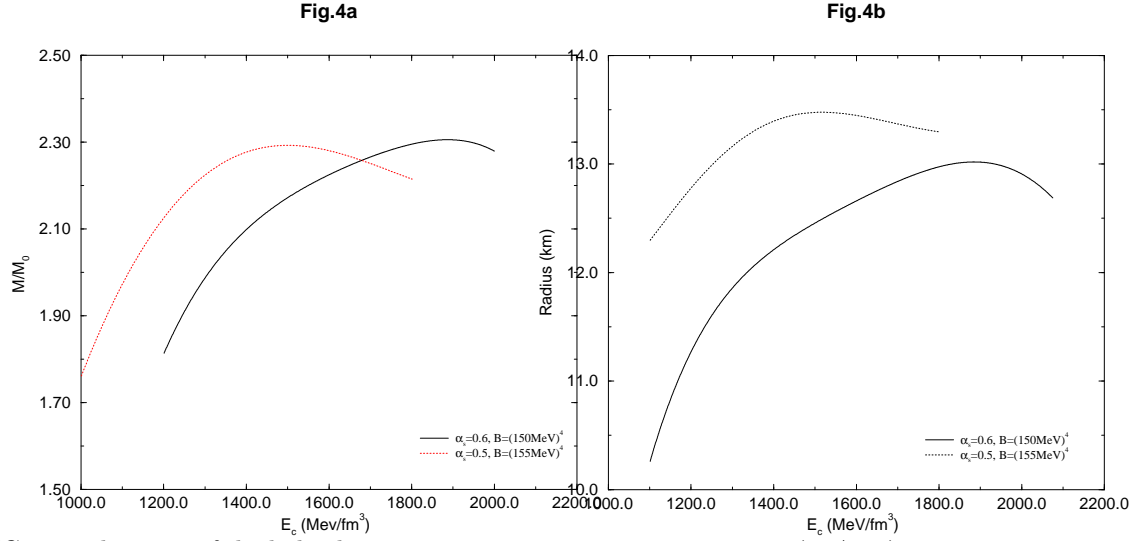


FIG. 4a. The mass of the hybrid neutron star (M/M_\odot) as a function of central energy density (ϵ_c). FIG. 4b. The mass ratio (M/M_\odot) versus radius (R) of the hybrid stars.

We also calculate the surface gravitational red shift Z_s of photons which is given by [20,23]

$$Z_s = \frac{1}{\sqrt{[1 - 2GM/R]}} - 1. \quad (4.4)$$

In Fig 5 we plot Z_s as a function of M/M_\odot . It is however possible that the discrete slowing down of pulsars due to the presence of two states of matter with various mass throwing some light on the above structure. Our graph shows there is a discontinuity of Z_s around $M/M_\odot = 0.4$ indicating a peculiar behaviour of redshift with Mass of the hybrid star.

Since the stars rotate about a centre, the relation between relativistic Kepler frequency and Newtonian Kepler frequency is given by

$$\Omega_k \simeq 0.65\Omega_c, \quad (4.5a)$$

where

$$\Omega_c = \sqrt{M/R^3} \quad (4.5b)$$

$$= 3.7 \times 10^3 \sqrt{\frac{M/M_\odot}{(R/km)^3}} \text{ s}^{-1}. \quad (4.5c)$$

Ω_k is newtonian Kepler frequency balancing gravity with centrifugal force . The factor 0.65 is empirical and approximate. The figures Fig.6 shows that

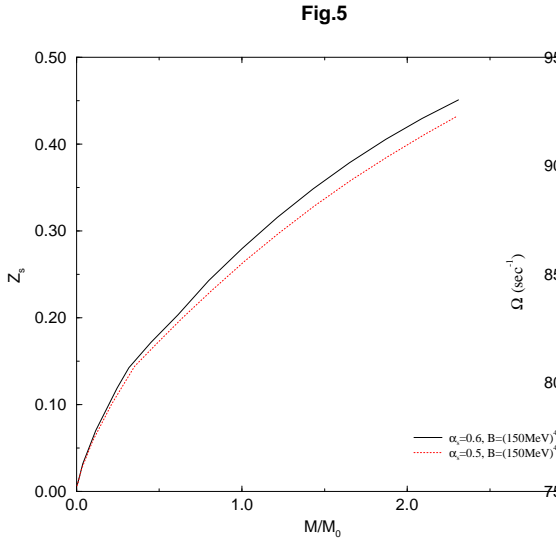


FIG. 5. The surface gravitational redshift (Z_s) as a function of star mass (M/M_\odot)

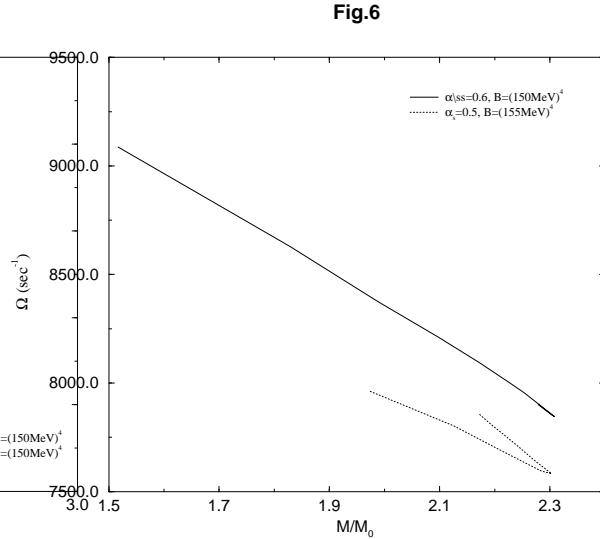


FIG. 6. The mass ratio (M/M_\odot) versus keplerian frequency (Ω_c) showing the mass ratio restricted to 2.3

the frequency is higher for lower mass and decreases for higher mass i.e. M/M_\odot is about 2.3 . It clearly shows that we can not have mass of the hybrid star more than about 2.3 times that of sun . That also indicates (e.g. Fig.4b) the radius of the star can not increase beyond 13.25 km .

V. SUMMARY AND CONCLUSIONS

We considered the equation of states taking into account the self-coupling interactions of σ and ω - mesons. The inclusion of the quartic term to the ω meson field gives a soft equation of state. In our calculations, we used the TM1 parameter set, which has a capability to reproduce the known results of finite nuclei as well as of normal nuclear matter. Here also the TM1 parameter set gives a phase transition for hadronic matter and quark matter. The same parameter set predict the Chandrasekhar limit for Hybrid stars to be $2.3 M_\odot$. In our calculation, we predict the inner quark core having a radius of about 4.2 kilometers, whereas the total radius of the hybrid neutron star is found to be 13.5 kilometers as compared to the earlier result where Chandrasekhar limit is $1.58 M_\odot$ and radius around 10 km [11], where the nuclear matter equation of state was calculated through the dressing of pion pairs. This is due to the contribution from $\sigma - \omega$ mesons. One also notes that the redshift has discontinuity around $M/M_\odot = 0.4$, a peculiarity of hybrid stars. It is also observed that the Newtonian Kepler frequency of the hybrid stars can not increase beyond $M/M_\odot = 2.3$, showing a decrease with increase in M/M_\odot . Pulsars are expected to be stars of this type but the gross properties appear to be similar to what we believe regarding neutron stars.

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