Origin of quantum-mechanical complementarity without momentum back action in atom-interferometry experiments

C. S. Unnikrishnan*

Gravitation Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India, and NAPP Group, Indian Institute of Astrophysics, Bangalore 560 034, India

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We identify the physical origin of the loss of interference pattern in the which-path atom-interferometery experiments that have been discussed widely. The origin of complementarity between the which-path information and the interference pattern is a discrete spinor phase with random sign. This clarifies how complementarity can arise without the Heisenberg back action in momentum.

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In the early days of quantum mechanics, complementarity principle was discussed based on the Heisenberg uncertainty principle connecting position and momentum and the famous Heisenberg microscope experiment was a popular device to demonstrate the inevitability of complementarity in quantum physics. In recent times there have a been a different class of experiments or proposals in which the Heisenberg back action in momentum is not the cause of quantum complementarity. This is the case, for example, in the atom-interferometry experiment that was discussed by Scully, Englert, and Walther [1]. There have been considerable discussions [2–6] on the issue and the fact that there is no momentum back action was confirmed in a recent experiment by Durr, Nonn, and Rempe [6]. Since the detection of the path is done by the presence of a very-low-energy photon in the tuned cavity, the resulting momentum kick is too small to account for the loss of interference. The loss of interference is attributed to the correlation established between the which-path detector and the atomic wave function. There have been detailed general analyses of loss of interference in which-path experiments [7,8]. Formally, there are two equivalent views to analyze which-path experiments [7]. In one view, the phase accumulated by the wave function of the interfering particle, if random in its relative values, washes out the interference pattern. In the other view, interaction of the quantum system with the detector (or the environment) and the resulting correlations imply loss of interference. In Ref. [7], the authors discuss the idea of loss of interference due to spin rotation for the electron in a magnetic field. Our own analysis that follows is similar in spirit, in the sense that we use the equivalence in description of the two-level atom and the spin-1/2 particle to derive the loss of interference in the atom-interferometry experiments. Reference [8] describes the loss of interference by a random-average model over basis states of the detectors.

The view that complementarity can arise due to correlations is very significant. But the question remains as to whether there is any physical mechanism in the atom-interferometry experiments that acts on the wave function to scramble the phase without changing the momentum. Also, the correlation with a detector takes a finite time to be established and it is physically more reasonable to think that the phases of the amplitudes are altered even before the final correlation is established with a cavity as in the experiments described in Refs. [1] and [6]. In this paper, we focus on the physical mechanism of phase scrambling without momentum back action and present a simple view that is very useful for the correct analysis of a class of experiments.

It is possible to change the phase without introducing a momentum kick—by a change in the geometric phase or, in the specific cases we consider, by the phase change of the spinor wave function during rotation. If the quantum state changes by a process equivalent to a spin flip in the interferometry experiment, the wave function picks up a phase of $\pi/2$. This is of course demonstrated in neutron interferometry experiments [9] and [10] as well as in the recent remarkable experiment of the nondemolition detection of the photon [11]. In the latter experiment, since an absorption and emission cycle takes place, the total effect is equivalent to the rotation of the spin through 2π and the resulting phase change is π. In an atom-interferometry experiment, *if the which-path detectors work on the principle of spontaneous emission into a tuned cavity, then a change of state is equivalent to a rotation of the a spinor through* π *and the interference pattern should shift by* $\pi/2$. Since the direction of the rotation is unspecified in the case of spontaneous emission and since the phase change can be in any of the two interfering paths, there are two distinct sets of interference patterns shifted from a mean position by $\pm\pi/2$. This results in two overlapping patterns shifted with respect to each other by π. The result is the apparent absence of the interference pattern. All which-path experiments in which the change in the internal state can be described as a rotation of the spinor can be analyzed correctly in this simple picture. There is no momentum back action. It is really remarkable that the spinor phase is at the root of complementarity in this class of experiments.

Consider the much debated example of the atom interferometry experiment employing excited-state atoms and the micromaser which-path detectors [1]. The atomic wave function splits into two amplitudes at a double slit and then there are two resonant cavities in line with each slit through which the atoms pass. The high finesse cavity is tuned to ensure

*Electronic address: unni@tifr.res.in

emission of a low-energy photon into the cavity and the presence of this photon after the atom passes to the detector provides the means for which-path detection. The initial state, before entering the double slit and cavities, can be represented as $\psi_0(r,i)=\chi(r)|e\rangle$, where the spatial part and the internal state (denoted by $i=e$ or g, for excited state or ground state) are explicitly written. If there are no cavities, the wave function after passing through the double slit is

$$\psi(r)=\frac{1}{\sqrt{2}}[\chi_1(r)+\chi_2(r)]|e\rangle. \tag{1}$$

This coherent superposition gives the interference pattern at the detector plane. After passing through the cavities, the probability of the spontaneous emission is unity by design, and one could write a combined wave function

$$\psi(r)=\frac{1}{\sqrt{2}}[\chi_1(r)|1,0\rangle+\chi_2(r)|0,1\rangle]|g\rangle. \tag{2}$$

The ket $|1,0\rangle$ represent the situation in which there is a photon in the upper cavity and no photon in the lower one. The ket $|0,1\rangle$ indicates that there is a photon in the lower cavity and no photon in the upper one. These are the kets representing the correlations. Since $\langle 1,0|0,1\rangle=0$, there is no interference pattern. But, as mentioned earlier, such correlations with detectors are expected to take a finite time and it is desirable to get the result of loss of interference from the interfering amplitudes that are associated with only the atom.

We do the entire analysis employing the wave function of the atom, without using the detector states, before and after passing through the cavities. If a photon is registered in the upper detector, then the atom wave function after passing through the detector would pick up a phase of $\pm\pi/2$, since the transition from the excited state to the ground state of the two-level atom is equivalent to rotating a spinor by $\pm\pi$ (the sign of the rotation cannot be determined for the spontaneous emission and we have to write both signs). Similarly, if the photon is emitted in the lower detector there is a similar change in phase. We can write the wave function of the atom after passing through the cavities as

$$\psi(r)=\frac{1}{\sqrt{2}}[\chi_1(r)\exp(\pm i\pi/2)+\chi_2(r)]|g\rangle, \tag{3}$$

for the case in which the spontaneous emission takes place in the upper cavity, and

$$\psi(r)=\frac{1}{\sqrt{2}}[\chi_1(r)+\chi_2(r)\exp(\pm i\pi/2)]|g\rangle, \tag{4}$$

for the case in which the emission takes place in the lower cavity. The net result is the overlap of two interference patterns, both of which are shifted with respect to the interference pattern without the cavities by $\pm\pi/2$, and the bright fringes of one pattern will overlap with the dark fringes of the other, resulting in an *apparently washed out interference*. Similar analysis is applicable to the experiment by Durr, Nonn, and Rempe [6]. Of course, in the micromaser kind of experiments where the information on the nature of the spinor rotation can be retrieved, the two interference patterns can also be retrieved and this is the scheme of the quantum eraser [1]. Our analysis also *predicts that the interference pattern vanishes even if there is only one good cavity*. If the experiment is done with only the upper cavity containing no photons initially and the lower cavity containing a large number of photons, the upper cavity is a good which-path detector and the lower one is not [12]. This is because the addition of one photon cannot be distinguished when the photon number is uncertain by $\sqrt{N}$, in a cavity containing N photons with $N\gg 1$. There is still complete loss of interference since even with one cavity there are two overlapping interference patterns shifted with respect to each other by π. (This is not surprising. Since the probability of emission when passing through the cavity is 1, if there is no photon in the upper cavity then the atom has taken the other path. So, even with only one good cavity we have 100% which-path information.)

There is an important corollary to our analysis. *If quantum complementarity is taken as a fundamental principle, then it implies that two-level atoms, neutrons, etc. should behave like spinors under rotations*. They should pick up the phase factor $\pi/2$ under rotation through π; otherwise quantum complementarity will be violated in interference experiments.

The wave functions of Eqs. (3) and (4) can be written only if the initial state of the cavities is such that we can distinguish the cavity states in principle before and after the emission. If the occupation number in the cavity is already very large, the two cavities are indistinguishable even after the emission. Then the wave functions $\chi_1(r)$ and $\chi_2(r)$ are coherent and the normal interference pattern emerges (the visibility will be a function of the photon occupation inside the cavities) [12]. This could also be thought of as related to the uncertainty relation between the occupation number and the phase angle, $\Delta N\Delta\phi\geq 1$, and when ΔN is large due to photon statistical fluctuations, the phase fluctuation is small [12, 13]. But the uncertainty relation gives only a bound and not the right solution of two overlapping interference patterns. Also, this particular uncertainty relation is not on as rigorous a footing as the momentum-position uncertainty relation is. (An example in which the fluctuations in the geometric phase destroy interference in a two-slit which-path experiment with photons has been discussed by Bhandari [13], and the similarity between the optics experiment and the atom-interferometry experiments has been mentioned in terms of the uncertainty principle between number and phase.)

It is important to mention that there is quantum back action even in the case we are discussing; but the back action is on the internal angular state rather than on the linear momentum. The discrete and random back action is on the spin part of wave function. The disturbance on the spatial wave function is too small to be responsible for the loss of interference. This agrees with the assertion by Scully, Englert, and Walther [1,3] that the loss of interference in the micromaser which-path experiment cannot be explained by Heisenberg momentum back action and recoil.

The analysis presented in this paper has identified the physical mechanism for the loss of interference in which-path atom-interferometry experiments. This answers an issue that was debated considerably recently. Our approach provides a simple picture of the physical (or geometrical) origin of the complementarity in which-path atom-interferometry experiments in which the Heisenberg back action on momentum is clearly not the source of the phase shifts. The real cause is traced to the phase change resulting from the rotation of the spinorial wave function. The information in the rotation (emission) is also encoded in the emitted quantum that establishes a correlation that is transferred subsequently to the which-path detector.

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