PHYSICAL REVIEW D, VOLUME 61, 101101(R)

New limits on the gravitational Majorana screening from the Zürich *G* experiment

C. S. Unnikrishnan*

Gravitation Group, Tata Institute of Fundamental Research, Mumbai 400 005, India and NAPP Group, Indian Institute of Astrophysics, Koramangala, Bangalore - 34, India

G. T. Gillies†

School of Engineering and Applied Science, University of Virginia, Charlottesville, Virginia 22904-4746

(Received 2 November 1999; published 13 April 2000)

We analyze the results from an experiment being conducted at the Physik-Institut, Universität Zürich to measure the gravitational constant, and obtain a tight constraint on the Majorana gravitational shielding factor. The limit we obtain is two orders of magnitude lower than the positive results obtained by Majorana in the 1920s and a factor of about 5 better than the constraint obtained by Braginsky in a more modern laboratory experiment. The Zürich experiment is expected in the future to provide a better constraint which is another factor of 10 lower than our present estimate.

PACS number(s): 04.80.Cc

The possibility of shielding gravity is at present outside the scope of the standard theory of gravitation. This is mainly due to the fundamental feature of gravity that there is only one type (sign) of gravitational charge [1]. On the other hand, the question of whether gravity could be shielded or absorbed by an intervening medium has been important from an empirical point of view. The issue has been addressed experimentally during the last century in several laboratory experiments and astronomical tests, and theoretically by several physicists [2].

The experimental activity was brought to focus by Majorana who conducted several high precision experiments with a weighing balance. He was motivated by theoretical considerations which he himself developed that questioned if standard Newtonian gravity was unaffected by intervening matter. Majorana's formulation of the problem [3] introduced an absorption coefficient h which modified the gravitational acceleration g of a testbody when it is geometrically screened by an intervening body of density $\rho(r)$ as

$$g' = g \exp\left(-h \int \rho(r) dr\right).$$

$\int \rho(r)\, dr$ represents the gravitational opacity by some intervening matter. Majorana's estimate for h, from phenomenological considerations, was between 10^{-11} and 10^{-12} cm^2/g. Clearly, experimental determination of such a small shielding factor would be very difficult, requiring very sophisticated measurements capable of resolving fractional changes in force of the order of 10^{-10}. In experiments, normally a medium of uniform density ρ and spatial extent l is interposed between an attracting mass (such as Earth) and a test mass. Then the exponential factor is well approximated as $(1-h\rho l)$, where l is the effective screening length provided by the medium. The typical value for the quantity $h\rho l$, accessible for such experiments, is in the range of 10^{-9} to 10^{-10} and the corresponding values for h are in the range of $10^{-12}-10^{-13}$ cm^2/g.

Majorana conducted a series of experiments between 1920 and 1930 to see whether there was absorption of gravitational interaction by intervening matter. In a carefully conducted experiment, he compared the weights of two lead balls, one used as a tare mass and another as the test mass which could be ''shielded'' by about 100 kg of mercury from the Earth's gravitational field [3]. He observed an effect amounting to a weight correction of about 8×10^{-10}. The resolution of the modified weighing balance he used was about 6×10^{-10} in a single measurement and close to 10^{-10} from repeated observations. In another experiment conducted several years later with the same kind of balance and different arrangements with much larger masses (the shielding matter was about 10000 kg of lead), a slightly smaller shielding factor was measured [4]. The absorption coefficient h deduced from these experiments was 6.7×10^{-12} cm^2/g and 2.8×10^{-12} cm^2/g, respectively, consistent with Majorana's phenomenological estimates.

Braginsky investigated gravitational screening in a resonance experiment [5] and later in a torsion balance experiment [6]. The sensitivity of the first experiment was similar to that of Majorana and it was concluded that the probability of seeing an effect of magnitude comparable to that seen by Majorana was smaller than about 4%. The torsion balance experiment was an order of magnitude more sensitive, and gave a limit of 2.8×10^{-13} cm^2/g for h. An overview of other contemporary shielding experiments is available elsewhere [7].

Constraints on h obtained from planetary orbital data and gravimetric observations during eclipses are in general more stringent than those from direct laboratory experiments. Based on an early observation by Russel [8] that Majorana shielding could affect the universality of free fall, the most stringent limit of $h\leq1.0\times10^{-21}$ cm^2/g has been obtained by Eckhardt from an analysis of the laser ranging data on the Moon's orbit [9]. Reviews of these constraints are available in [2,7]. In this paper, though, we focus only on the results from laboratory experiments.

*Email address: unni@tifr.res.in

†Email address: gtg@virginia.edu

We have observed that a new beam balance experiment at the Physik-Institut, Universität Zürich, aimed at measuring the gravitational constant at 10 ppm level, can also give very useful information on Majorana gravitational screening. In this paper we describe an analysis of their results, which provided the value of *G* at the 200 ppm level, in terms of the gravitational screening hypothesis. We estimate that any gravitational screening in the experiment is smaller than a value corresponding to $h \simeq 5 \times 10^{-14}$ cm^2/g. Moreover, as the precision improves in the *G* measurement, the same experiment will be able to probe gravitational screening at the level of $h \leqslant 1 \times 10^{-14}$ cm^2/g. This enormous sensitivity comes about due to the fact that unlike torsion balances employed for the measurement of *G*, the beam balance measures the weight changes in the Earth's large attractive field and any small shielding of this field will result in a change in the weight which is a significant fraction of the gravitational signal from the source masses. In other words, the modification of the Earth's gravity due to the hypothetical screening and resulting change in weight is comparable or more than the gravitational signal sought for in the *G* experiment.

The principle and details of the Zürich experiment are described in several publications [10–12]. The heart of the experiment is an ultrahigh precision beam balance which has a precision of 100 ng in a single weighing, for weighing a 1 kg mass. Statistically averaged measurements can reach a precision of 10 ng. Two test masses are suspended on separate wires such that they hang on the same axis at different vertical positions, and the balance can be used for comparing the masses by alternatively connecting the test masses to the balance, keeping the load on the balance arm constant to about 1 gm. The weight difference is modulated by two cylindrical masses with a central bore. The positions of the test masses with respect to the attracting cylindrical masses are chosen such that they are at an extremum of the field, and this way there is no need to determine the positions of various masses to very high precision to obtain a high precision value of the gravitational constant. The cylindrical masses are tanks in which a liquid such as water or mercury can be filled.

Referring to Fig. 1, the gravitational signal (the weight difference between the two test masses for a fixed position of the attracting cylindrical masses) changes when the cylindrical masses are moved from position 1 to position 2. Two series of experiments have been performed so far, one with water filled in the tanks and a second series with mercury filled in the tanks. The main result is that the gravitational constant *G* is determined with a systematic uncertainty of about 200 ppm (the statistical errors are comparatively negligible). The aimed accuracy is about 10 ppm and efforts are underway to achieve this. An important result is that the two determinations of *G* with water filled and mercury filled tanks agree with each other within about 50 ppm. It is this observation we will use to make our estimate of the Majorana shielding parameter *h*.

From the relative positions of the masses it is clear that in position 1, the upper test mass is more or less completely shielded geometrically from the Earth's field (except for the opening of the bore, but this correction is small) by the *two* cylindrical masses whereas the lower test mass is completely unshielded. In position 2, the lower test mass is completely shielded (except for the bore) by the lower cylindrical mass and the upper test mass is partially shielded, being farther away, by the same cylindrical mass. Since the test masses are always at an extremum of the gravitational field, the total measured weight of each of the test masses can be written as $w_i = m_i g + \vec{F}_o + \vec{F}_d$ when not shielded and $w_i = m_i g(1 - h\rho_m l) + \vec{F}_o + \vec{F}_d$ when shielded. The subscript *i* denotes test mass 1 or 2, and $\vec{F}_o$ is the force due to the nearer mass on the test mass at the extremum position. $\vec{F}_o$ has the same magnitude for both test masses to a great accuracy (the difference between the two test masses is only of the order of a few milligrams at most). $\vec{F}_d$ is the force due to the farther attracting mass at an effective distance *d* from the test mass. Typically the values for *d* are 175 cm and 105 cm from the center of mass of the farther test mass in the two positions.

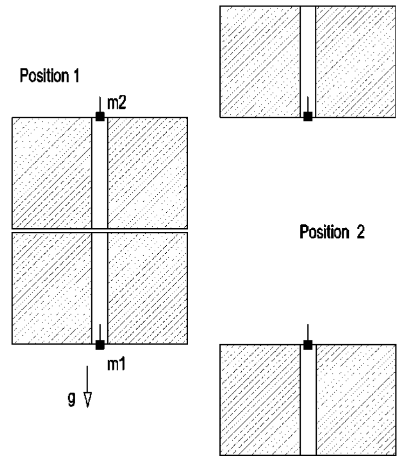

FIG. 1. Schematic diagram showing the two positions in which the cylindrical source masses (shaded) and the test masses (m1 and m2) are placed in the Zürich *G* experiment.

The basic sensitivity of the experiment to address the question of shielding is decided by the approximate expression $\Delta w \simeq mghl(\rho_{Hg} - \rho_{wat})$. Δw is the minimum weight change that can be measured and *l* is the effective screening length from one cylindrical mass. Since Δw is determined to about 10 ng, the sensitivity of the experiment for measuring *h*, limited by statistical errors, can be estimated to be 10 ng/$ml(\rho_{Hg} - \rho_{wat})$. With $l \simeq 50$ cm, $\rho_{Hg} - \rho_{wat} = 12.6$ g/cm^3, and $m_2 = 1000$ gm, the smallest *h* that can be probed is about 10^{-14} cm^2/g. In the experiment, the systematic errors are an order of magnitude larger and the actual constraint we arrive at is, therefore, larger.

In positions 1 and 2, respectively, the weight (force) differences are of the form

$$\Delta w_1 = (m_1 - m_2)g + 2m_2 gh\rho l - 2F_o - 2F_{105},$$

$$\Delta w_2 = (m_1 - m_2)g - m_1 gh\rho l + 2F_o - 2F_{175} + m_2 g' h\rho l'.$$

The second equation needs some explanation. The last term comes from the shielding of the upper test mass by the lower attracting mass. Since the shielding is geometrically partial an effective value for the gravitational attraction of the Earth needs to be used and the spatial extent of the intervening matter is different from the value denoted by l. An estimate of the effective gravity could be made from the geometrical factors, and the volume which is geometrically shielded is 18% of the total volume of Earth. This has to be corrected for the increased density at the central regions to get an effective value for gravity. But the general conclusion is that the last term contributes only about 10% of the total contribution from shielding when we take the measured gravitational signal $\Delta w_1 - \Delta w_2$, and therefore we neglect it in this estimate. Since $m_1 \simeq m_2 \equiv m$, the measured difference signal is

$$S_g \equiv \Delta w_1 - \Delta w_2 \simeq 3mgh\rho l - 4F_o - 2F_{105} + 2F_{175}.$$

The quantity on the left-hand side is measured with a statistical accuracy of 10 ng and a systematic uncertainty of 160 ng (200 ppm of the gravitational signal from mercury filled tanks). The last three terms on the right-hand side can be estimated to an accuracy decided by the accuracy with which the gravitational constant is known. Since the experiment was designed to measure the gravitational constant we cannot use the best value of G for this estimate. Instead we rely on the important fact that the values of G measured with water and mercury as the source masses are identical within about 50 ppm. This means that any contribution of the first term in the gravitational signal in the experiment is limited to about 50 ppm. (It may be argued that such a contribution could be at the level of the quoted systematic errors. While this does not alter the constraint on h very much, we believe that the agreement is better than the quoted systematic errors since the main contribution to the systematic error arises from a nonlinearity of the balance and the quoted number is an upper limit.) For estimating an upper limit on h, we take a conservative value for the difference in G values to be 100 ppm. This residual Δw_{res} amounts to about 80 ng in the mercury experiment. The absorption length varies from 50 to 70 cm, and it is larger than 50 cm for most of the volume of the cylindrical masses. We take l conservatively at 50 cm. We now estimate the admissible value of h from

$$\Delta w_{res} \simeq 3mhl(\rho_{Hg} - \rho_{wat}).$$

This gives $h \leqslant \Delta w_{res}/3ml(\rho_{Hg} - \rho_{wat}) \simeq 4.3 \times 10^{-14}$ cm^2/g.

These estimates represent the best limits on the Majorana shielding factor from a laboratory experiment. They not only rule out the Majorana hypothesis of shielding with a sensitivity approaching a factor 100 compared to Majorana's positive results, but also provide a new constraint which is a factor of about 5 better than the best direct laboratory experiment done so far.

The Zürich experiment can provide a factor of 5 to 10 better limits on the Majorana shielding factor in the future, as the systematic error is studied and eliminated and as the measurements are improved. Also, it may be possible to arrange the positions of the attracting masses to get a direct estimate of shielding without making a comparison between measurements with water filled and mercury filled tanks.

In summary, we have obtained a tight constraint on the Majorana gravitational shielding factor from the analysis of the results from an experiment that is being conducted at the Physik-Institut, Universität Zürich to measure the gravitational constant. This new constraint is about a factor of 100 lower than the positive results obtained by Majorana in the 1920s and a factor of about 5 better than the constraint obtained by Braginsky in a more modern laboratory experiment.

[1] C. S. Unnikrishnan, Physica C **266**, 133 (1996); see the introductory section.
[2] C. S. Unnikrishnan and G. T. Gillies, ''Gravitational Screening—A Review'' (in preparation).
[3] Q. Majorana, Philos. Mag. **39**, 488 (1920).
[4] Q. Majorana, J. Phys. Radium **1**, 314 (1930).
[5] V. B. Braginsky, V. N. Rudenko, and G. I. Rukman, Zh. Éksp. Teor. Fiz. **43**, 51 (1962) [Sov. Phys. JETP **16**, 36 (1963)].
[6] V. B. Braginsky and V. K. Martynov, Moscow Univ. Phys. Bull. **21**, 35 (1968).
[7] G. T. Gillies, Rep. Prog. Phys. **60**, 151 (1997).
[8] H. N. Russel, Astrophys. J. **54**, 334 (1921).
[9] D. H. Eckhardt, Phys. Rev. D **42**, 2144 (1990).
[10] J. Schurr, F. Nolting, and W. Kundig, Phys. Lett. A **248**, 295 (1998).
[11] J. Schurr, F. Nolting, and W. Kundig, Phys. Rev. Lett. **80**, 1142 (1998).
[12] F. Nolting, J. Schurr, St. Schlamminger, and W. Kundig, Meas. Sci. Technol. **10**, 487 (1999).