GENERAL RELATIVISTIC SPECTRA OF ACCRETION DISKS AROUND ROTATING NEUTRON STARS

Sudip Bhattacharyya,[1,2] Ranjeev Misra,[3] and Arun V. Thampan[4]

ABSTRACT

General relativistic spectra from accretion disks around rotating neutron stars in the appropriate spacetime geometry for several different equations of state, spin rates, and masses of the compact object have been computed. The analysis involves the computation of the relativistically corrected radial temperature profiles and the effect of Doppler and gravitational redshifts on the spectra. Light-bending effects have been omitted for simplicity. The relativistic spectrum is compared with the Newtonian one, and it is shown that the difference between the two is primarily a result of the different radial temperature profiles for the relativistic and Newtonian disk solutions. To facilitate direct comparison with observations, a simple empirical function has been presented which describes the numerically computed relativistic spectra well. This empirical function (which has three parameters including normalization) also describes the Newtonian spectrum adequately. Thus, the function can in principle be used to distinguish between the two. In particular, the best-fit value of one of the parameters (β-parameter) ≈ 0.4 for the Newtonian case, while it ranges from 0.1 to 0.35 for the relativistic case depending upon the inclination angle, equation of state (EOS), spin rate, and mass of the neutron star. Constraining this parameter by fits to future observational data of X-ray binaries will indicate the effect of strong gravity in the observed spectrum.

Subject headings: relativity — stars: neutron — stars: rotation — X-rays: binaries

1. INTRODUCTION

X-ray binaries are believed to harbor black holes or weakly magnetized neutron stars with an accretion disk. The X-ray emission arises from the hot ($\approx 10^7$ K) innermost region of the disk. In the case of a neutron star there will be emission, in addition, from a boundary layer between the accretion disk and neutron star surface. Since the observed emission arises from regions close to a compact object, these sources are possible candidates for studying strong field gravity.

In the standard theory (Shakura & Sunyaev 1973), the accretion disk is assumed to be an optically thick Newtonian one. In this model, the local emergent flux (assumed to be a blackbody) is equated to the energy dissipation at a particular radial point in the disk. The observed spectrum is then a sum of blackbody components arising from different radial positions in the disk. General relativistic effects modify this Newtonian spectrum in two separate ways. First, the local energy dissipation at a radial point is different from the Newtonian disk, giving rise to a modified temperature profile. Second, the observed spectrum is no longer a sum of local spectra because of effects like Doppler broadening, gravitational redshifts, and light bending. Modified spectra, incorporating these effects, but with different approximations, have been computed by several authors (e.g., Novikov & Thorne 1973; Asaoka 1989) for accretion disks around rotating (Kerr) black holes. These computations confirm the expected result, that the relativistic spectral shape differs from the Newtonian one by

around 10%. Thus, for comparison with observed data with systematic and statistical errors larger than 10%, the Newtonian approximation is adequate. Ebisawa, Mitsuda, & Hanawa (1991) showed that for typical data from *Ginga*, the relativistic spectrum cannot be differentiated from the Newtonian disk spectrum. They also found that the relativistic spectrum is similar in shape (at the sensitivity level of *Ginga*) to the Comptonized model spectrum. Although, *Ginga* was not sensitive enough to distinguish between the different spectra, better estimates of fit parameters like accretion rate and mass of the compact object were obtained when the data was compared to relativistic spectra rather than the standard Newtonian one.

The present and next generation of satellites (e.g., *ASCA*, *RXTE*, *Chandra*, *XMM*, *Constellation-X*), with their higher sensitivity and/or larger effective area than *Ginga*, are expected to differentiate between relativistic and Newtonian spectra from low-mass X-ray binaries (LMXBs) and black hole systems. However, as pointed out by Ebisawa et al. (1991), the presence of additional components (e.g., boundary layer emission from the neutron star surface) and smearing effects due to Comptonization may make the detection ambiguous. Nevertheless, the detection of strong gravity effects on the spectra from these sources will be limited by the accuracy of theoretical modeling of accretion disk spectra rather than by limitations on the quality of the observed data. Thus, it is timely to develop accurate relativistically corrected spectra for comparison with present and future observations. Apart from the importance of detecting strong gravity effects in the spectra of these sources, such an analysis may also shed light on the geometry and dynamics of innermost regions of accretion disks.

Novikov & Thorne (1973) and Page & Thorne (1974) computed the spectra of accretion disks around rotating (Kerr) black holes. This formalism, when directly applied to rotating neutron stars, provides only a first-order estimate: the absence of an internal solution in the case of Kerr

[1] Joint Astronomy Program, Indian Institute of Science, Bangalore 560 012, India; sudip@physics.iisc.ernet.in.
[2] Indian Institute of Astrophysics, Bangalore 560 034, India; sudip@iiap.ernet.in.
[3] Department of Physics and Astronomy, Northwestern University, 2131 Sheridan Road, Evanston, IL 60208; ranjeev@finesse.astro.nwu.edu.
[4] Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune 411 007, India; arun@iucaa.ernet.in.

geometry makes it difficult to obtain, in a straightforward manner, the coupling between the mass and the angular momentum of the central accretor. On one hand, this coupling depends on the equation of state of neutron star matter, and on the other hand, it depends on the proper treatment of rotation within general relativity. Equilibrium configurations of rapidly rotating neutron stars for realistic equations of state have been computed by several authors (Bonazzola & Schneider 1974; Friedman, Ipser, & Parker 1986; Cook, Shapiro, & Teukolsky 1994; Stergioulas & Friedman 1994; Salgado et al. 1994a, 1994b; Datta, Thampan, & Bombaci 1998). One crucial feature in all these calculations is that the spacetime geometry is obtained by numerically and self-consistently solving the Einstein equations and the equations for hydrodynamic equilibrium for a general axisymmetric metric. With the aim of modeling spectra of LMXBs, we attempt, in this paper, to compute the spectrum of accretion disks around rotating neutron stars within such a spacetime geometry. This is particularly important since LMXBs are old (Population I) systems and the central accretor in these systems are expected to have large rotation rates (Bhattacharya & van den Heuvel 1993 and references therein).

Computation of the spectra is numerically time consuming and, hence, direct fitting to the observed data is impractical. For the sake of ease in modeling, we also present in this paper a simple empirical analytical expression that describes the numerically computed spectra. As shown later, the same expression (which has three parameters, including normalization) can also describe the Newtonian spectra. In particular, the value of one of the parameters (called the β-parameter here) determines whether the spectrum is relativistically corrected or not. This will facilitate comparison with observational data since only this β-parameter has to be constrained to indicate the effect of strong gravity in the observed spectrum.

The next section describes the method used to compute the spectra. In § 3 the results of the computation and the empirical fits are shown. Section 4 is devoted to discussion and summary.

2. SPECTRAL COMPUTATION

The disk spectrum is expressed as

$$F(E_{ob}) = (1/E_{ob}) \int I_{ob}(E_{ob}) d\Omega_{ob} , \qquad (1)$$

where the subscript "ob" denotes the quantity in the observer's frame, the flux F is expressed in photons s^{-1} cm^{-2} keV^{-1}, E is photon energy in keV, I is specific intensity, and Ω is the solid angle subtended by the source at the observer.

As I/E^3 remains unchanged along the path of a photon (see, e.g., Misner et al. 1973), one can calculate I_{ob}, if I_{em} is known (hereafter the subscript "em" denotes the quantity in the emitter's frame). We assume the disk to emit like a diluted blackbody, so I_{em} is given by

$$I_{em} = (1/f^4)B(E_{em}, T_c) , \qquad (2)$$

where f is the color factor of the disk assumed to be independent of radius (e.g., Shimura & Takahara 1995), B is the Planck function, and T_c (the temperature in the central plane of the disk) is related to the effective temperature T_{eff} through the relation $T_c = f T_{eff}$. The effective temperature, T_{eff}, is a function of the radial coordinate r and for a rotat-

ing accretor and is given by (Page & Thorne 1974)

$$T_{eff} = (F/\sigma)^{1/4} , \qquad (3)$$

where

$$F(r) = -\frac{\dot{M}}{4\pi r} \Omega_{K,r}(\tilde{E} - \Omega_K \tilde{l})^{-2} \int_{r_{in}}^{r} (\tilde{E} - \Omega_K \tilde{l})\tilde{l}_{,r} \, dr , \qquad (4)$$

is the flux of energy from the disk in an orbiting particle's frame. The factor σ is the Stephan-Boltzmann constant, r_{in} is the disk inner edge radius, $\tilde{E}$, $\tilde{l}$ are the specific energy and specific angular momentum of a test particle in a Keplerian orbit, and Ω_K is the Keplerian angular velocity at radial distance r. In our notation, a comma followed by a variable as subscript to a quantity represents a derivative of the quantity with respect to the variable. Also, in this paper, we use the geometric units $c = G = 1$.

The quantities E_{ob} and E_{em} are related through the expression $E_{em} = E_{ob}(1 + z)$, where $(1 + z)$ contains the effects of both gravitational redshift and Doppler shift. For a general axisymmetric metric (representing the spacetime geometry around a rotating neutron star), the factor $(1 + z)$ is expressed as (see, e.g., Luminet 1979)

$$1 + z = (1 + \Omega_K b \sin \alpha \sin i)$$
$$\times (-g_{tt} - 2\Omega_K g_{t\phi} - \Omega_K^2 g_{\phi\phi})^{-1/2} , \qquad (5)$$

where the $g_{\mu\nu}$ terms are the metric coefficients, and t and ϕ are the time and azimuthal coordinates. In the above expression (which includes light-bending effects), i is the inclination angle of the source, b the impact parameter of the photon relative to the line joining the source and the observer, and α the polar angle of the position of the photon on the observer's photographic plate. For the sake of illustration and simplicity in calculations, we neglect light bending. We thus write $b \sin \alpha = r \sin \phi$ and

$$d\Omega_{ob} = \frac{r dr d\phi \cos i}{D^2} , \qquad (6)$$

where D is the distance of the source from the observer.

The spacetime geometry around a rotating neutron star can be described by a general axisymmetric, stationary metric (see, e.g., Komatsu, Eriguchi, & Hachisu 1989). Assuming the matter to be a perfect fluid and the metric to be asymptotically flat, the Einstein field equations reduce to three nonhomogeneous, second-order, coupled differential equations and one ordinary differential equation in terms of the energy density and the pressure of the high-density neutron star matter (Cook, Shapiro, & Teukolsky 1994). An important input in solving these equations is the equation of state (EOS) of high-density matter comprising the neutron star. Assuming rigid rotation, we solve these equations numerically and self-consistently for four representative EOS models: (1) Pandharipande (1979) (hyperons), (2) Baldo, Bombaci, & Burgio (1997) (AV14 + 3bf), (3) Walecka (1974), and (4) Sahu, Basu, & Datta (1993). The values of stiffness parameters for each of these EOS models are widely different and increases from model 1 to model 4. We, therefore, expect the results of our computations to be of sufficient generality.

The solution of the field equations yield the metric coefficients, numerically, as functions of r and θ. Using these metric coefficients, it is straightforward to calculate the structure parameters of rapidly rotating neutron stars. The

details of these calculations are given in Datta, Thampan, & Bombaci (1998) and references therein. The quantities r_{in}, $\tilde{E}$, $\tilde{l}$, and Ω_K are obtained by solving the equation of motion of material particles within the spacetime geometry given by the above metric (Thampan & Datta 1998; Bhattacharyya et al. 2000). For our purpose here we compute constant gravitational mass sequences whose rotation rates vary from zero to the centrifugal mass shed limit (where gravitational forces balance centrifugal forces). For realistic neutron stars, the inner radius r_{in} may be located at either the marginally stable orbit or the surface of the neutron star depending on its central density and rotation rate (Thampan & Datta 1998; Bhattacharyya et al. 2000), having important implications for the gravitational energy release as well as the temperature profiles of accretion disks. For the procedure of calculating T_{eff}, for rapidly rotating neutron stars, considering the full effect of general relativity, we refer to Bhattacharyya et al. (2000). These authors have also shown (in their Fig. 2) that the difference between Newtonian temperature profile and general relativistic temperature profile is substantial at the inner portion of the disk. As will be shown herein, it turns out that this is the major reason for the difference between Newtonian and general relativistic spectra at high energies.

To summarize this section, we calculate the accretion disk spectrum using equation (1), taking the radial integration limits as r_{in} and r_{out} and the azimuthal integration limits as 0 and 2π. We choose a very large value ($\approx 10^5$ Schwarzschild radius) for r_{out}.

3. RESULTS

To illustrate the differences between the relativistic and Newtonian spectra, we show in Figure 1 the computed relativistic spectrum (*solid line*) and the Newtonian spectrum (*dashed line*) for the same parameters. The Newtonian spectrum is the spectrum expected from a standard nonrelativistic disk (Shakura & Sunyaev 1973), but with the intensity and the effective temperature modified by the color factor (eqs. [2] and [3]). In order to isolate the different contribu-
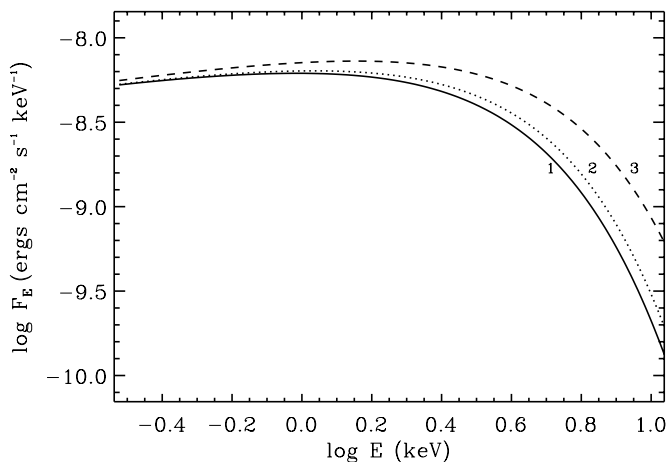
tions, we have also plotted in Figure 1 the theoretical spectrum arising from relativistic temperature profile, but without the effect of Doppler/gravitational redshift (*dotted line*). The relativistic spectrum is underluminous compared to the Newtonian one at high energies—this is primarily because of the the difference in the radial temperature profile (Bhattacharyya et al. 2000). The difference between the two spectra is nearly 50% at 2 keV. We emphasize here that such a high difference is true only when both the spectra are calculated for the same disk parameters. If the Newtonian spectra is calculated for slightly different values of disk parameters (e.g., accretion rate, inclination angle, distance to the source), the average discrepancy between the two spectra will be less (Ebisawa, Mitsuda, & Hanawa 1991).

In order to facilitate comparison with observations, we introduce a simple analytical expression which empirically describes the computed relativistic (and Newtonian) spectra,

$$S_f(E) = S_o E_a^{-2/3} \left(\frac{E}{E_a}\right)^\gamma \exp\left(-\frac{E}{E_a}\right), \qquad (7)$$

where $\gamma = -(2/3)(1 + E\beta/E_a)$, E_a, β, and S_o are parameters, and E is the energy of the photons in keV; $S_f(E)$ is in units of photons s^{-1} cm^{-2} keV^{-1}. To compare this empirical function with the computed spectra, we use a reduced χ^2 technique. In particular, we define a function

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{S_c(E_i) - S_f(E_i)}{0.1S_c(E_i)}\right]^2, \qquad (8)$$

where $S_c(E)$ is the computed spectra. The spectra are divided into N logarithmic energy bins. We have chosen the range of energy used in calculating χ^2 to be dependent on the location of the maximum of the energy spectrum ($ES_c[E]$), which is typically at 2 keV. The minimum energy is set to be one-hundredth of this value (typically 0.02 keV) while the maximum is set at 10 times (typically 20 keV). The value of χ^2 is fairly insensitive to the number of energy bins; we take $N = 200$. For each $S_c(E)$ the best-fit parameters (E_a, β, and S_o) are obtained by minimizing χ^2. Figure 2 shows the relativistic spectra for three different inclination angles

FIG. 1.—General relativistic spectrum (*solid line*) for a neutron star configuration with mass $M = 1.4\ M_\odot$, spin rate $\Omega_* = 0$, distance to the source $D = 5$ kpc, inclination angle $i = 30°$, accretion rate $\dot{M} = 10^{18}$ g s^{-1}, and color factor $f = 2$. *Dashed line*: The spectrum expected from a source with the same disk parameters but without the relativistic effects (Newtonian spectrum). *Dotted line*: The spectrum for the same disk parameters but without the effect of Doppler and gravitational redshifts (i.e., z is set to zero). The EOS model (2) is used here.
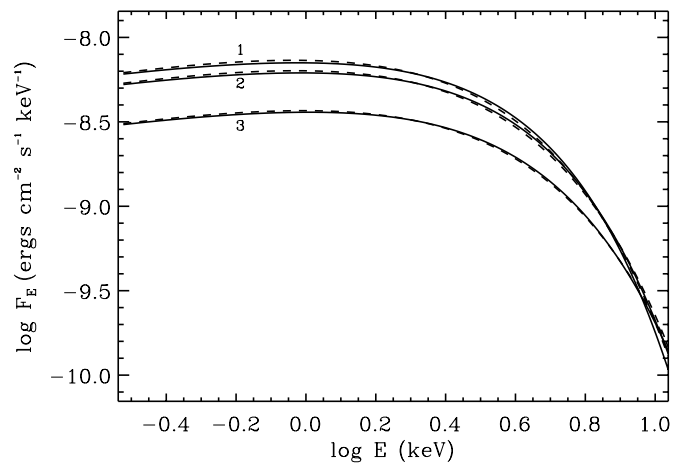

FIG. 2.—Relativistic spectra for three different inclination angles ($i = 0°$, $30°$, $60°$) with rest of the parameters same as in Fig. 1 (*solid lines*). *Dashed lines*: Empirical fit to the relativistic spectra using eq. (7). The minimum $\chi^2 = 0.073, 0.049$, and 0.026 for $i = 0°$, $30°$, and $60°$, respectively.
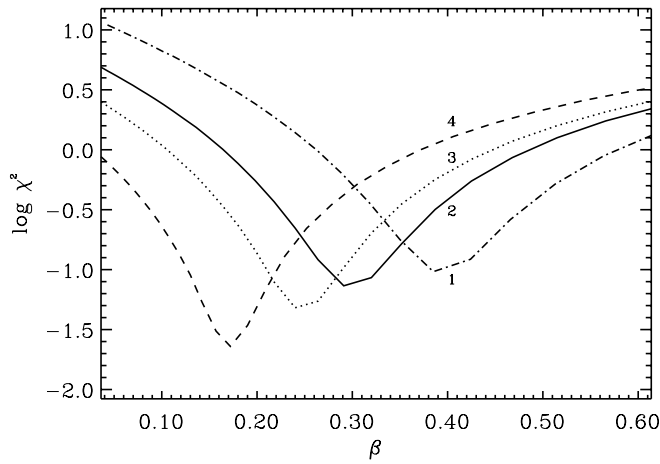
FIG. 3.—Variation of minimum χ^2 (i.e., minimized with respect to parameters E_a and S_o) with parameter β. Curves marked 2, 3, and 4 correspond to the spectra shown in Fig. 3 for $i = 0°$, $30°$, and $60°$, respectively. Curve marked 1 is for the Newtonian spectra shown in Fig. 1.
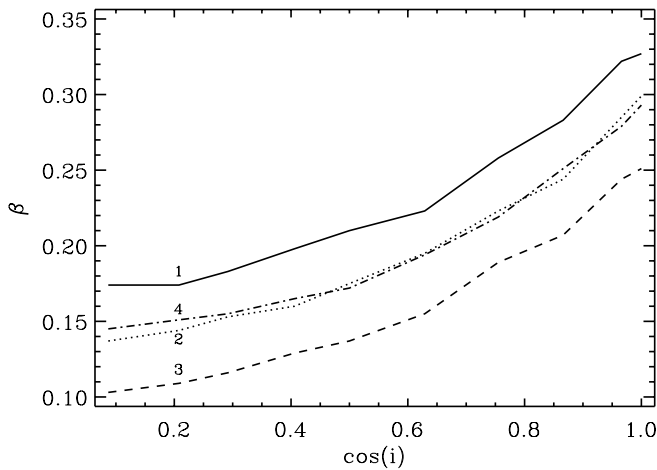

FIG. 4.—Variation of the best-fit β-parameter with inclination angle for different equation of states. Curve 1: Pandharipande(Y) (softest); curve 2: Bombaci; curve 3: Walecka; curve 4: SBD (stiffest). The values of the other parameters are as in Fig. 1.
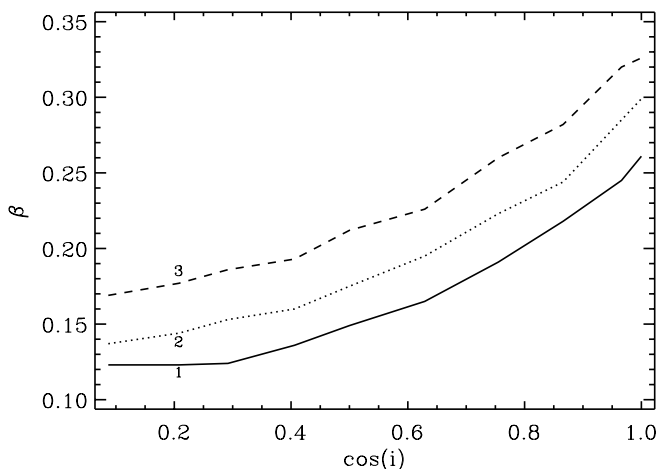

FIG. 5.—Variation of the best-fit β-parameter with inclination angle for different neutron star masses. Curve 1: $M = 1.0 M_{\odot}$, curve 2: $M = 1.4 M_{\odot}$, curve 3: $M = 1.788 M_{\odot}$. The values of the other parameters are as in Fig. 1.
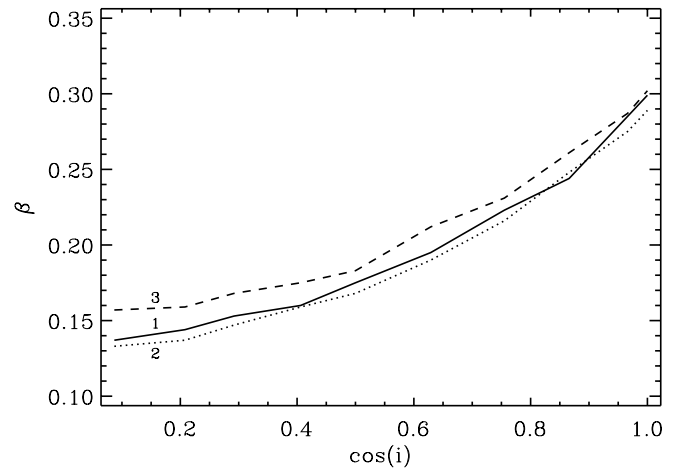

FIG. 6.—Variation of the best-fit β-parameter with inclination angle for different neutron star spin rates. Curve 1: $\Omega_* = 0$ radians s^{-1}; curve 2: $\Omega_* = 2044$ radians s^{-1}; curve 3: $\Omega_* = 7001$ radians s^{-1} (mass-shed limit). The values of the other parameters are as in Fig. 1.

(*solid lines*) and the corresponding empirical fits using equation (7) (*dotted lines*). The minimum χ^2 obtained while fitting these spectra was less than 0.1, which means that the average discrepancy is less than 3%. This is also true for other disk parameters and EOS considered in this work. Thus the empirical function (eq. [7]) is a reasonable approximation to the computed relativistic spectra. It also describes the Newtonian spectra to a similar degree of accuracy.

The S_o parameter in equation (7) is the normalization factor and is independent of the relativistic effects. It depends only on the mass of the star (M), accretion rate ($\dot{M}$), distance to the source (D), color factor (f), and inclination angle (i), i.e., $S_o \propto \dot{M}^{2/3} f^{-4/3} M^{1/2} D^{-2} \cos i$. The E_a parameter (which is in units of keV) describes the high-energy cutoff of the spectrum. Its dependence on the spacetime metric and inclination angle is complicated, but it scales as $E_a \propto \dot{M}^{1/4} f$. The β-parameter depends only on the spacetime metric and not on accretion rate, distance to the source, or color factor. This makes the β-parameter useful as a probe into the underlying spacetime metric. We show
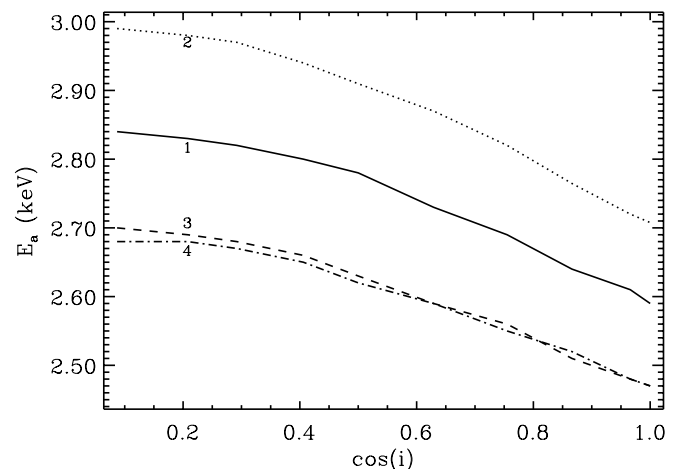

FIG. 7.—Variation of the best-fit E_a-parameter with inclination angle for different equation of states. The curve numbers correspond to the same equations of state as listed in Fig. 4. The values of the other parameters are as in Fig. 1.

in Figure 3 the variation of minimum χ^2 (i.e., minimized with respect to parameters E_a and S_o only) as a function of the β-parameter for the three spectra shown in Figure 2 and for the Newtonian one. For the Newtonian case the minimum χ^2 occurs for $\beta \approx 0.4$ while it is lower for the relativistic cases. For example, consider the relativistic spectrum for parameters listed in Figure 1 and for $i = 30°$ (line marked as 3 in Fig. 3). If this spectrum is fitted with the empirical function the minimum $\chi^2 = 0.05$ (corresponding to an average discrepancy of 2%) and the best-fit β-parameter is $\beta \approx 0.25$. For a Newtonian β-parameter value of ≈ 0.4, the minimum χ^2 increases to 0.1, corresponding to an average discrepancy of more than 3%. Thus, the empirical function can resolve the difference between the Newtonian and the relativistic one at the 10% level. For an observed spectrum fitted using the empirical function, if the best-fit range of β-parameter excludes the Newtonian value of 0.4, that would strongly indicate that the spectrum has been modified by strong gravitational effects. Since the β-parameter deviation from 0.4 increases with inclination angle, nearly edge-on disks are more promising candidates for detecting the presence of strong gravity. To show the robustness of this result, we show in Figures 4, 5, and 6 the variation of the best-fit β-parameter with i for different EOSs, masses of the central object, and spin rates, respectively. For all these cases the best-fit β-parameter is less than 0.4. Parameter E_a is useful to determine the accretion rate. However, it also depends on the metric and inclination angle. We show this dependence in Figure 7.

4. SUMMARY AND DISCUSSION

In this paper, we have computed relativistic spectra from accretion disks around rotating neutron stars for the appropriate spacetime geometry. Several different EOS, spin rates, and masses of the compact object have been considered. The Doppler and gravitational effects on the spectra have been taken into account, while light-bending effects have been omitted for simplicity. The spectrum differs from the Newtonian one, with the main difference being due to the different radial temperature profile for the relativistic and Newtonian disk solutions.

A simple empirical function has been presented which describes the numerically computed relativistic spectra well. This will facilitate direct comparison with observations. The empirical function (eq. [7]) has three parameters, including normalization. Another important advantage of this func-

tion is that it also describes the Newtonian spectrum adequately, and the value of one of the parameters (β-parameter) distinguishes between the two. In particular, the best-fit β-parameter ≈ 0.4 for the Newtonian case, while it ranges from 0.1 to 0.35 for the relativistic case depending upon the inclination angle, EOS, spin rate, and mass of the neutron stars.

In principle, for sufficiently high-quality data, the effects of strong gravity on the disk spectrum can be detected using this empirical function as a fitting routine and constraining the b-parameter. However, it must be emphasized that there are several reasons why this may not be possible. There could be systems which have additional components in the X-ray spectra, for example, boundary layer emission from the neutron star surface. Uncertainties in modeling these additional components may lead to a wider range in the best-fit β-parameter. Thus, accurate spectra of the boundary layer (with relativistic corrections) is also needed for modeling these systems. Moreover, X-rays could be emitted from hotter regions (e.g., an innermost hot disk or a corona) giving rise to a Comptonized spectra instead of the sum of local emission assumed here. In this case, the empirical fit will probably not describe the observed data well. It has been assumed here that the color factor is independent of radius. Shimura & Takahara (1995) have shown from numerical computation that this could be the case for an accretion disk in a Schwarzschild metric. Apart from the fact that this was done for a Schwarzschild metric, their numerical computation depends on the vertical structure of the disk, which in turn depends on the unknown viscosity mechanism in the disk. If the color factor has a radial dependence, the spectral shape might change, which may be confused with a relativistic effect.

Despite these caveats the method described in this paper will be a step forward in the detection of strong gravity effects in the spectra of X-ray binaries. Future comparison with high-quality observational data will highlight the theoretical requirements that have to be met before concrete evidence for strong gravity are detected in these systems and the enigmatic region around compact objects is probed.

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