CORRELATION BETWEEN MAGNETIC SHEAR AND MAGNETIC TENSION IN A SOLAR ACTIVE REGION

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Abstract. The difference between the magnetic tension and magnetic shear was calculated for four vector magnetograms of NOAA AR 4474. It was seen that this difference between the two independent angular measures of magnetic stress is less than 18° for more than 50% of the pixels. Magnetic tension is thus found to be fairly well correlated with magnetic shear for AR 4474.

1. Introduction

Flares are believed to be produced when magnetic fields in the solar corona relax from a highly stressed configuration to one with smaller stress. The amount of energy available for flaring is generally considered to be the excess of magnetic energy over that in a potential field having the same distribution of magnetic flux as the observed field. For force-free fields, the vector magnetic field measured at any given height level in the atmosphere is sufficient to calculate the excess energy in the region above the layer by means of the virial theorem (Molodensky, 1974; Low, 1985). For more general field configurations, one has to resort to indirect methods. One such way is to measure the so-called 'magnetic shear' of the field, defined as the angle between the observed transverse field and the transverse component of the potential field (Hagyard *et al.*, 1984). This parameter was found to have large values at the sites of flares on the polarity inversion lines of bipolar active regions (Hagyard *et al.*, 1984; Hagyard and Rabin, 1986; Hagyard, Venkatakrishnan, and Smith, 1990).

A physical basis for the correlation between shear and flares was suggested in terms of the loss in magnetic tension caused by the 'shearing' of the field line (Venkatakrishnan, 1990a, b). A basic assumption made in that suggestion was that large magnetic shear corresponded to low magnetic tension. This assumption cannot be theoretically justified in general. An attempt is made therefore to check the validity of the assumption in practice using data from four vector magnetograms of NOAA AR 4474 obtained with the Marshall Space Flight Center vector magnetograph (Hagyard *et al.*, 1982) in April 1984. In Section 2, we develop a definition for an angular measure of magnetic tension to enable direct comparison with magnetic shear. In Section 3 we present the results of such a comparison. In Section 4 we discuss the implications of these results.

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2. Angular Measure of Magnetic Tension

The Lorentz force is given by

$$\mathbf{F} = \frac{(\nabla \times \mathbf{B})}{4\pi} \times \mathbf{B}, \tag{1}$$

where **F** is the force and **B** is the field. The right-hand side of Equation (1) can be split as

$$\mathbf{F} = \frac{(\mathbf{B} \cdot \nabla)\mathbf{B}}{4\pi} - \frac{\nabla(\mathbf{B} \cdot \mathbf{B})}{8\pi} . \tag{2}$$

The first term in the right-hand side of Equation (2) is the tension and the second term is the magnetic pressure. The z-component (vertical component) of Equation (2) reduces to

$$F_z = \frac{(\mathbf{B}_T \cdot \nabla_T) B_z}{4\pi} - \frac{\partial}{\partial_z} \frac{(B_T^2)}{8\pi} , \tag{3}$$

where

$$\mathbf{B}_T = (B_x, B_y) \quad \text{and} \quad \nabla_T = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right).$$

At the polarity inversion line, $B_z = 0$ and therefore the first term on the right-hand side of z-component of Equation (2) is identical with the first term in the right-hand side of Equation (3). Thus, at the polarity inversion line, the vertical component of tension T_z can be written as

$$T_z = (\mathbf{B}_T \cdot \nabla_T) B_z / 4\pi . \tag{4}$$

We can thus define an angular measure of tension as

$$\Theta = \cos^{-1}(4\pi \, |T_z| / |B_T| \, |\nabla_T B_z|) . \tag{5}$$

Note that by taking the absolute value of T_z we avoid the 180° ambiguity in the measured transverse field azimuth. Likewise, we will always choose the acute angle solution of Equation (5) for the definition of the tension angle. For force-free fields, $F_z = 0$ in Equation (3). Thus the two terms in the right-hand side of Equation (3) must balance each other. Therefore any decrease in the first term in the right-hand side necessitates a decrease in the second term. This means that B_T^2 becomes distributed less steeply with height and therefore implies a vertical extension of the field. Thus, even for $B_z \neq 0$, we will continue to call Θ (as defined by Equation (5)) as the tension angle in the sense that $\Theta \to 90°$ indicates a large vertical extension of the transverse field. In any case, large values of the 'tension angle' off the polarity inversion line serve to denote a lower vertical gradient of the transverse field (in the case of force-free field) or alternatively, a predominance of an upward force (for a non-force-free field), if B_T^2 decreases with z. Either situation is potentially vulnerable for flaring (Venkatakrishnan, 1990a, b). Thus, the utility of this parameter (the tension angle) does not diminish even for regions away from the polarity inversion line.

3. Results

The computation of magnetic shear involves two steps. First, the potential field B_p must be calculated from the distribution of B_z, the vertical component of the magnetic field. Next, the azimuth of this potential field must be subtracted from the observed azimuth. There could be other complications when the observed region is far away from disc centre (Venkatakrishnan, Hagyard, and Hathaway, 1989). In the present case, the magnetograms were obtained when the active region was near disc centre and hence the transformation to heliographic coordinates was not attempted. The magnetic shear was computed for the four magnetograms labelled A, B, C, and D (Figures 1(a–d)) which were obtained by the Marshall Space Flight Center vector magnetograph at respectively 19^h53^m, 20^h31^m, 21^h17^m UT of April 28, 1984 and at 13^h54^m UT of April 30, 1984.

For determining the tension angle Θ_T, a 3-point Lagrange interpolation scheme was adopted to obtain $\partial/\partial x$ and $\partial/\partial y$ of B_z. As mentioned earlier, only acute angle solutions of Equation (5) were accepted. This assumes that the magnetic tension is always a downward force, which is generally true for 'closed' field line configurations that are concave towards the photosphere.

The pixels on the polarity inversion lines were then picked out. The number of pixels thus selected were 75, 72, 52, and 72, respectively, for magnetograms A, B, C, and D. The distribution of the percentage of the total number of these pixels was then plotted as a function of the absolute value of the difference between shear and tension (Figure 2(a)). The solid curve in the figure represents the distribution for magnetogram A while the large dashes depict the behaviour for magnetogram B. Magnetogram C is represented by the line with small dashes while dots make up the line for magnetogram D. We essentially see no difference in the distribution for the four magnetograms, although magnetogram D has a somewhat broader distribution. For comparison we have also plotted in Figure 2(b), the distribution followed by all pixels in the magnetograms irrespective of the position with respect to a polarity inversion line. We discarded pixels where the transverse field strength was less than 100 G and thus were left with 13443, 13195, 13454, and 10581 pixels for magnetograms A, B, C, and D, respectively. Here, the difference in the behaviour amongst the magnetograms is even less, but the distribution in Figure 2(b) resembles very closely the distribution for magnetogram D in Figure 2(a).

4. Discussion

Figures 2(a) and 2(b) show that at more than 50% of the pixels, shear and tension do not differ beyond 18°. This result is consistent with the expectation that the transverse

Magnetogram B

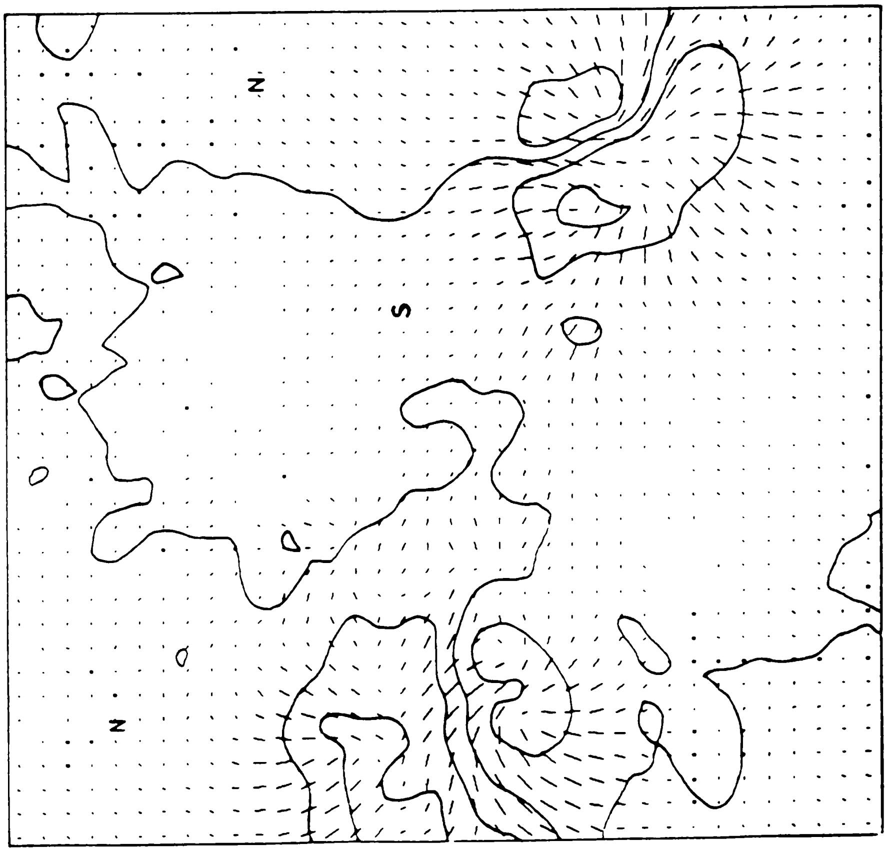

Fig. 1b.

Magnetogram A

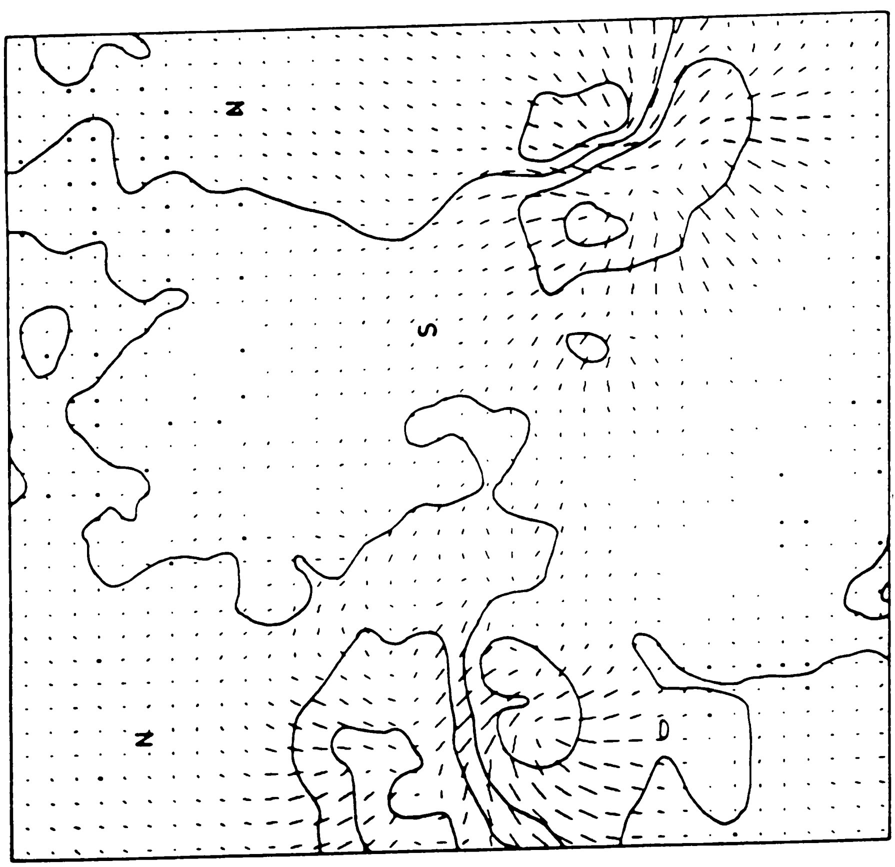

Fig. 1a.

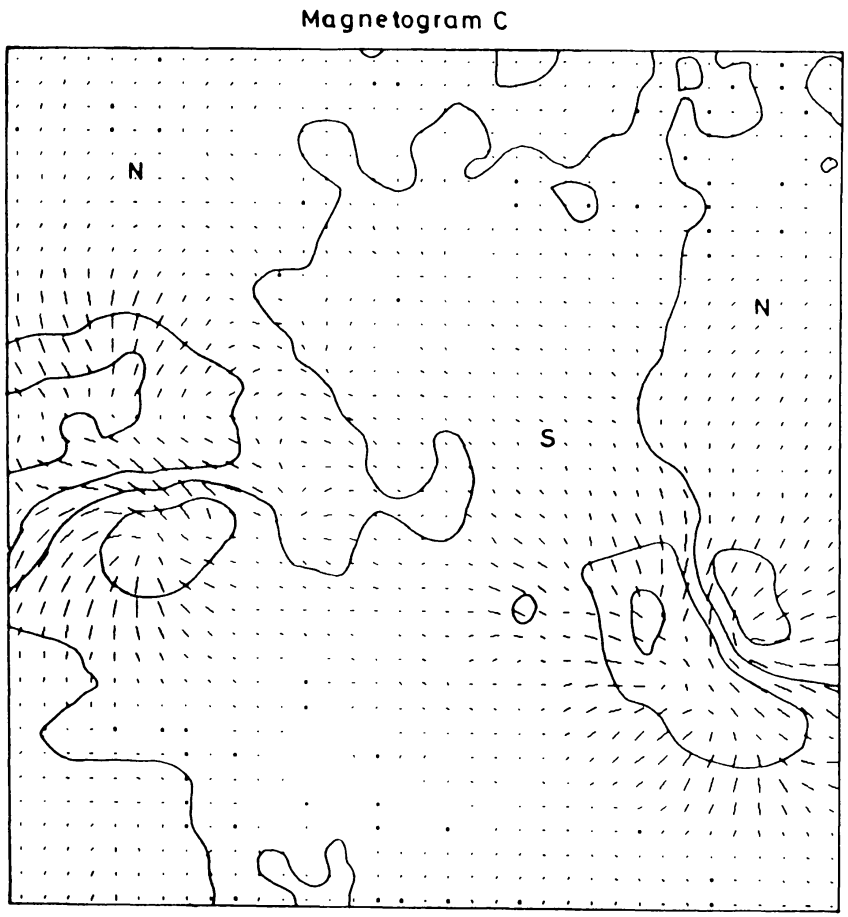

Fig. 1c.

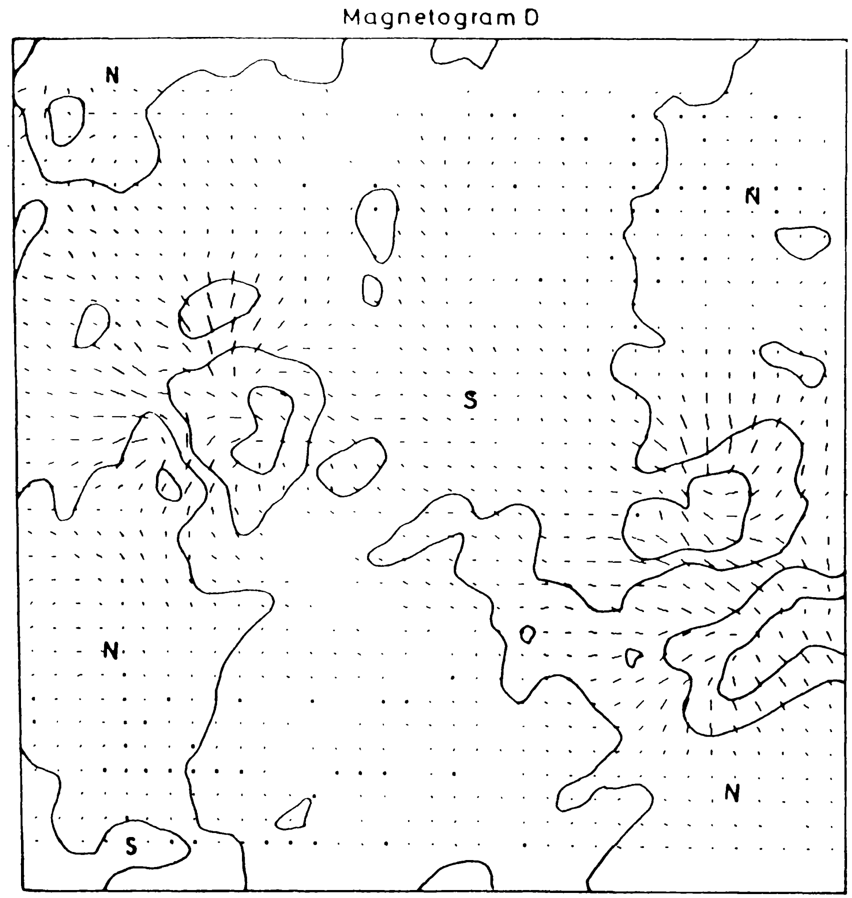

Fig. 1d.

Fig. 1. Vector magnetograms of NOAA AR 4474 obtained on April 28, 1984 at (a) 19:53 UT, (b) 20:31 UT, (c) 21:17 UT, and (d) on April 30, 1984 at 13:54 UT. The dashes are oriented along the azimuth direction of the transverse field, with the length of the dash proportional to the transverse field strength. A transverse field of 1000 G would thus be represented by a line segment of length equal to the pixel separation. Each figure is 75 pixels by 75 pixels in size, corresponding to a field of view of $3' \times 3'$. Heliographic north is at the top in (a), (b) and (c), and at the bottom in (d). Heliographic east is on the left in (a), (b) and (c), and is on the right in (d). The contours represent the strength of the longitudinal component, ranging from -2500 G to $+2500$ G except for (d) where the range is from -2000 G to $+2000$ G. The north polarity regions are labelled N while the south polarity regions are labelled S.

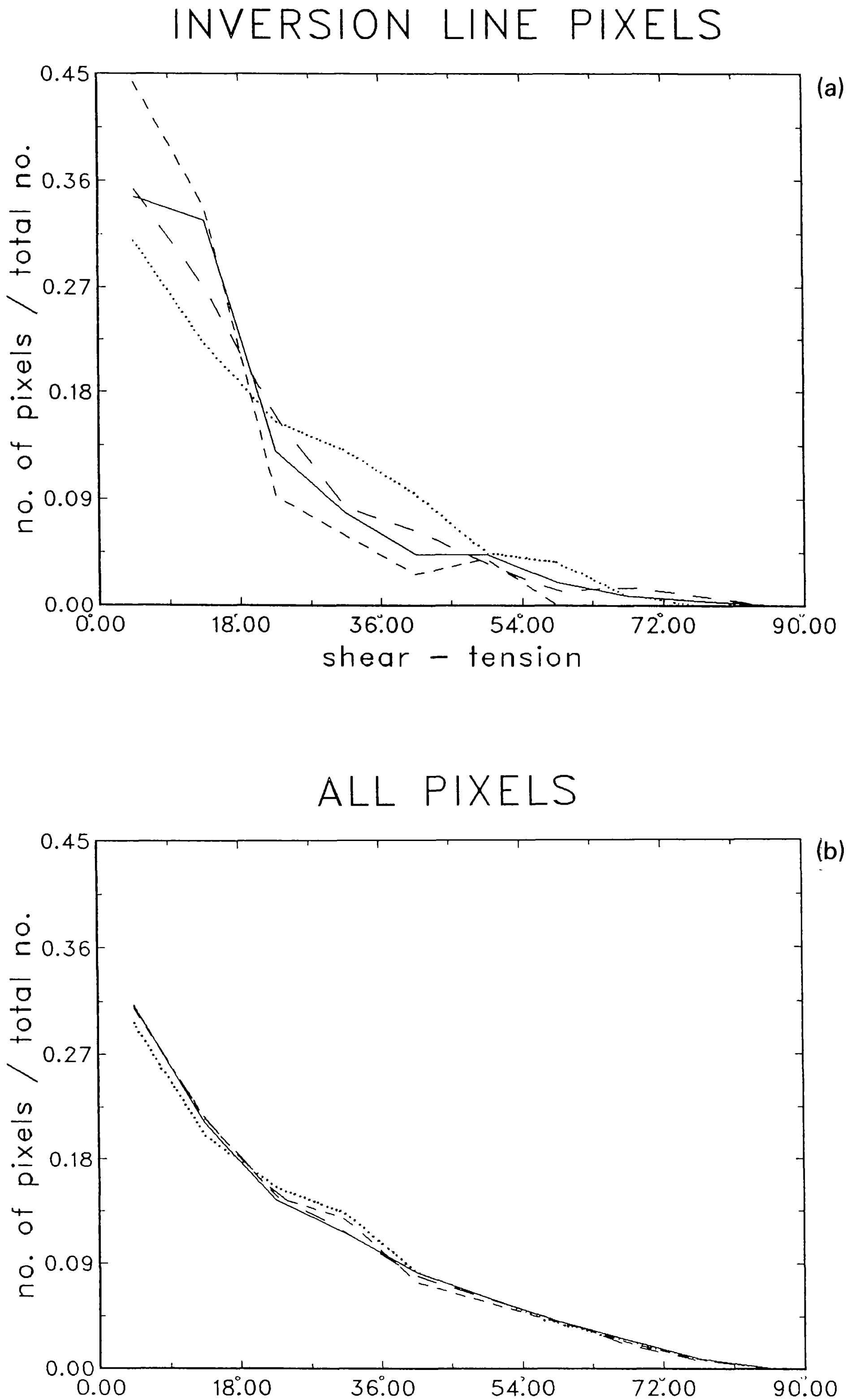

Fig. 2. Distribution of the fraction of pixels as a function of the difference between shear and tension for (a) points along the polarity inversion lines and (b) all points in the magnetogram where transverse field strength is greater than 100 G. The solid line is the distribution for magnetogram A, the long dashes for magnetogram B, the short dashes denote magnetogram C, while the dots comprise the data for magnetogram D.

potential field is perpendicular to the inversion line. It provides a practical confirmation of the assumption made in Venkatakrishnan (1990a, b) that strong shear corresponds to low magnetic tension on the polarity inversion line. Observers can therefore use either parameter with equal effect in their task of characterizing the magnetic non-potentiality of active regions. In the case of filter magnetographs, the time required for the computation of either parameter is not much different since FFTs can be used, both for calculating the potential field as well as the lateral gradient of the vertical field.

In the case of magnetographs that use the entire line profile for magnetic field evaluation, calculation of the potential field requires measurements at many different positions of the solar image on the entrance slit of the spectrograph to provide adequate 2-D coverage of the flux distribution. Here, the calculation of tension at any given position of the slit requires the measurement at only two neighbouring positions, flanking this particular position. Thus, tension is a more convenient parameter for such instruments especially if on-line results are intended.

Theoretically, the correlation of shear and tension implies the correlation of the direction of the lateral gradient of the vertical field with that of the transverse potential field. Now, the vertical gradient of the transverse potential field is by definition equal to the lateral gradient of the vertical field. Thus, the correlation of shear and tension angles implies that the transverse component of the potential field must point along the direction of its vertical gradient. This is not universally true, requiring, for example, that the potential be expressible as the product of a function independent of z with another independent of x and y. Examination of the general solutions of the Laplace equation with boundary values given on a plane of constant z reveal that such peculiar situations might obtain wherever the flux distribution is dominated by a few pockets of strong sunspot field. It would be interesting to see whether the distribution of the difference between shear and tension would become broader for regions less dominated by sunspot-like fields. The broader distribution for the last magnetogram (where there is significant reduction in magnetic field gradients at the polarity inversion line) is perhaps an indication of this effect. However, such an investigation is hardly an urgent requirement for flare research. The situation is best summarized by stating that for the purposes of characterizing the non-potential nature of the active region magnetic fields, both tension and shear would serve equally well. The fact that the tension angle directly reflects the loss of magnetic tension at the polarity inversion line should then provide sufficient justification for its greater utilisation as a parameter in future studies of the vector magnetic field configuration of active regions.

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