ONSET OF HYDRODYNAMIC NON-EQUILIBRIUM IN HELMET STREAMERS

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Abstract. . The steady-state solar wind solution is examined for different geometries of the flow tube that mimics a helmet streamer. Onset of non-equilibrium is seen whenever the spatial variation of the flow geometry crosses critical values. It is suggested that the dynamical response of the flow to the onset of non-equilibrium can manifest as a coronal mass ejection.

1. Introduction

Early pictures of the solar corona were obtained as snapshots during the rare occurrences of total solar eclipses. A striking feature of these coronal images was the helmet streamer. These pictures of the helmet streamers conveyed the impression of static structures, existing without major changes for durations long compared with the dynamical time scales appropriate for their dimensions. The advent of space based coronagraphs showed that the corona was not steady, but frequently changed its structure. Most of these episodes of structural change were also associated with the ejection of matter, over and above the regular outflow in the solar wind. For a long time, these coronal mass ejections (CMEs) were considered to be the manifestation of a blast wave in the corona, initiated in response to the impulsive energy release in a solar flare (Dryer *et al.*, 1979). Careful analyses of the projected onset times of CMEs revealed that these often appeared to precede the impulsive phase of the associated flare (Harrison, 1986). Many of these ejections were seen to originate from the dynamical breakup of helmet streamers even without evidence for a flare being associated with the ejections (Munro *et al.*, 1979). Such inconsistencies with the blast wave model culminated in the realization that CMEs were manifestations of coronal activity in their own right and not necessarily caused by flares.

The helmet streamer was thus identified as the structure in which the CMEs are initiated. The next quest was for the physical mechanism that is responsible for initiating a CME in the helmet structure. There have been basically two approaches to this problem. One approach considered the evolution of the helmet structure subject to appropriate initial and boundary conditions (Dryer *et al.*, 1979; Nakagawa, Wu, and Han, 1981; Steinolfson, Suess, and Wu, 1982; Wu *et al.*, 1983; Steinolfson and Hundhausen, 1988). These models are essentially numerical and do not identify any generic physical process for initiating the ejection.

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Recently, Wu, Guo, and Wang (1995) constructed a numerical solution for a helmet streamer including a detached magnetic structure (bubble) within the helmet dome. Using this configuration, they investigated the conditions of dynamical equilibrium and non-equilibrium of the streamer due to the strength of the bubble in order to understand the initiation of CMEs. The other approach searches for the onset of non-equilibrium in a sequence of quasi-static field configurations that are essentially closed (Low, 1990). Although the latter approach holds better promise for identifying the physical process that initiates the ejection, it suffers from one drawback, viz., the neglect of solar wind flow. The hybrid nature of the streamer structure admitting the coexistence of open and closed field topologies, as well as the large scale of the helmet streamer together do not justify the neglect of the solar wind flow in the problem.

In what follows, I will attempt to remedy this defect by considering a simple model of equilibrium flow within a magnetic tube that mimics the spatial variation of the flow geometry within a helmet streamer. This approach assumes that the magnetic field is strong enough to be specified externally and is valid as long as the flow remains weaker than the field that confines the flow. In Section 2, I will describe the geometry and resulting equations for the flow. In Section 3, this equation will be analysed to show the possibility of non-equilibrium. In Section 4, the implications of this analysis for CME initiation will be discussed.

2. Basic Equations

The flow within a helmet streamer will be treated as a problem of flow along a magnetic tube of specified geometry. This approach is the same as was followed by Parker (1963) as well as Kopp and Holzer (1976) for steady flow, and by Hasan and Venkatakrishnan (1982) for unsteady flow. Figure 1 shows the geometry that will be assumed for the problem. The basic premise will be that there will be a portion of the tube where the area of cross-section will decrease with height, forming a neck that is seen in most pictures of streamers as well as in results of numerical simulation (Steinolfson and Hundhausen, 1988; Wu, Guo, and Wang, 1995). The neck begins at the height of the outermost closed field line. The area of cross-section of the tube will be assumed to increase with height until the base of the neck. The area of cross-section is assumed to increase once again beyond the top of the neck. For mathematical simplicity, the functional form for the variation of the cross-section is assumed as r^{α}. Regions of increasing area of cross-section will have positive values of α while the neck will have negative value of α. Reducing Equation (2) of Hasan and Venkatakrishnan (1982), for the equations for flow along a tube of specified geometry in a helio-centred coordinate system to that for isothermal steady flow, we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(\rho v A) = 0 \,, \tag{1}$$

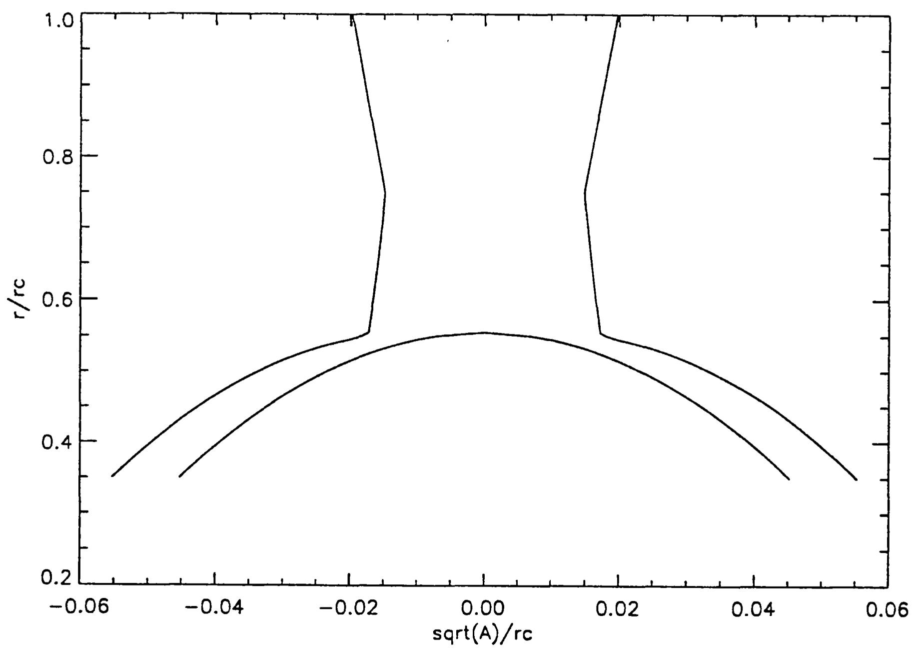

Fig. 1. Schematic outline of the tube geometry is shown. The abcissae denote the positions along a direction normal to the radial direction. The ordinates denote positions along the radial direction. All distances are measured in units of r_c, the distance to the Parker critical point. The solar surface is at 0.35 in these units. The outermost field line has its apex at 0.55. The area lying between the open line and the closed line varies as $x^{0.1}$ till the base of the neck at 0.55. From 0.55 to 0.75, the area goes as x^{-1}, while beyond 0.75, it increases as x^2.

$$\rho v \frac{\mathrm{d}}{\mathrm{d}r} v = -\rho \frac{GH}{r^2} - \frac{\mathrm{d}}{\mathrm{d}r} p \,, \tag{2}$$

where ρ is the plasma density, p is the pressure, A is the area of cross-section of the flow tube, v is the flow speed, G is the gravitational constant, and H is the solar mass.

For isothermal stratification, pressure p is related to density ρ as $p = 2\rho S^2$, where S is the sound speed. The factor 2 takes care of the fact that the plasma is fully ionized. Using the mass continuity equation to relate ρ with the flow speed v, we can write the equation of motion as

$$\frac{(1 - M^2)}{M^2} \frac{\mathrm{d}}{\mathrm{d}r} M^2 = \frac{(2r_c - \alpha r)}{r^2} \,, \tag{3}$$

where $r_c = GH/S^2$ is the Parker critical point, $\alpha = \mathrm{d} \ln A / \mathrm{d} \ln r$, and $M = v/S$. Rewriting Equation (3) in terms of $x = r/r_c$, we have

$$\frac{(1 - M^2)}{M^2} \frac{\mathrm{d}}{\mathrm{d}r} M^2 = \frac{(2 - \alpha x)}{x^2} \,. \tag{4}$$

Equation (4) can be analysed to look at existence of continuous solutions connecting the solar surface to regions far away from the surface.

TABLE I

Continuous solution

x range	x_s, x_1	x_1, x_2	x_2, $2/\alpha_2$	$2/\alpha_2$, ∞
α	< 2	< 0	< 2	< 2
$2 - \alpha x$	$+$	$+$	$+$	$-$
$1 - M^2$	$+$	$+$	$+$	$-$

TABLE II

No continuous solution

x range	x_s, x_1	x_1, x_2	x_2, ∞
α	< 2	< 0	$> 2/x_2$
$2 - \alpha x$	$+$	$+$	$-$
$1 - M^2$	$+$	$+$	

3. Analysis of the Equation and Results

Both sides of Equation (4) have factors that can change sign after crossing zero. The left-hand side will vanish when $M^2 = 1$. Physically, it means a transition from subsonic to supersonic flow or *vice versa*. The right-hand side will vanish for $\alpha x = 2$. This possibility does not exist for $\alpha < 0$. Thus, there is no chance for a transition in the neck of the structure where α is assumed to be < 0. In the regions where $\alpha = 2$, there is only one turning point at $x = 1$. If the regions where $\alpha = 2$ do not contain $x = 1$, then there is no possibility of a turning point. For $\alpha \neq 2$, a turning point exists at $x = 2/\alpha$. This can occur in $x < 1$ for $\alpha > 2$, and in $x > 1$ for $\alpha < 2$. If there is no transonic transition at a turning point, then the flow solution goes through a maximum or a minimum for M^2. A transonic transition is not allowed when $\alpha x \neq 2$. A transition across $M^2 = 1$ at $\alpha x = 2$ is allowed and is called the critical solution for solar wind flow (Parker, 1958).

A better appreciation of the various possibilities can be achieved by drawing up a table showing the regions and signs for the factors $1 - M^2$ and $2 - \alpha x$. Let $\alpha = \alpha_1$ for $x < x_1$ (below the neck) and let $\alpha = \alpha_2$ for $x > x_2$ (above the neck) with $x_2 < 1$. For $\alpha_1 < 2$ and $\alpha_2 < 2$, the right-hand side will be positive until $x = x_2$. A transition from subsonic to supersonic flow is possible at $x = 2/\alpha_2$ (Table I).

Suppose now that there is a gradual evolution in the geometry so that α_2 increases. There will be no turning point for $\alpha_2 > 2/x_2$ and the continuous solution will cease to exist (Table II).

TABLE III

No continuous solution

x range	x_s, x_1	x_1, x_2	x_2, x_3	x_3, ∞
α	< 2	< 0	$< 2/x_3$	2
$2 - \alpha x$	$+$	$+$	$+$	$-$
$1 - M^2$	$+$	$+$	$+$	

There are several other ways for the cessation of the solution. For example, α_2 can be $< 2/x_3$ until $x = x_3 > 1$, and thereafter the tube can expand radially with $\alpha = 2$. Even in this case, there is no continuous solution (Table III).

The value of α_1 does not play a major role except when $\alpha_1 > 2/x_1$. In this case, a lower transition is possible at $x = 2/\alpha_1$. We do not expect such a transition normally since we require a steady acceleration, strong enough to produce a supersonic flow below the neck of the structure. However, the flow could switch to the lower transition during epochs of cessation of the outer transition above the neck.

The main result of the analysis is then that the existence of a continuous solution in the streamer depends very much on the spatial variation in the area of cross-section of the flow tube. We have also seen some possibilities for the cessation of continuous flow solutions during gradual evolution of the tube geometry.

4. Discussion and Conclusions

Although we do show possibilities of cessation of continuous solutions within a streamer-like geometry, only a time-dependent calculation can demonstrate the dynamical effects on the flow during the evolution of the geometry across such critical epochs. One such case was considered by Hasan and Venkatakrishnan (1982) in the context of coronal hole flow. The flow did show large changes tending to shock-like discontinuities. We might therefore expect similar dynamical effects even for the critical geometries discussed in the present paper.

Shocks can lead to other physical effects like particle acceleration and energy release. Can a coronal mass ejection result from this process? The present steady-state analysis cannot answer the question and we must await the results of unsteady MHD calculations. However, the recent numerical simulations of Wu, Guo, and Wang (1995) do show two regimes of behaviour. In the first regime, involving a smaller initial current for their magnetic bubble structure, the flow tends to an asymptotic steady state. In the second regime with a larger initial current for the bubble, the flow did not attain a steady state. Rather, it grew nonlinearly to large values, simulating a CME type behaviour. The existence of such completely different responses to the different initial states supports the results of our present analysis showing critical geometries.

Physically, the transition of flow regimes seems to be due to the inability of gravitational forces to contain the outward magnetic pressure. Our geometrical study contains the pressure information in the spatial variation of the cross-section of the flow tube. The magnetic field approaches zero at the singularity above the outermost closed field lines (the base of the neck). Thus magnetic forces like pressure and tension will also be vanishingly small. We can therefore expect a significant outward pressure gradient at the outermost closed field line. A gradual increase of the height of this outermost field line relative to the Parker critical point can occur either by an increase in shear at the footpoints (Mikić and Linker, 1994), or by emerging currents (Wu, Guo, and Wang, 1995), or by an increase in the plasma temperature (which depresses the Parker critical point). When the flow tube crosses a critical geometrical shape as envisaged in this paper, we expect a drastic change in the flow topology, driven by an imbalance of pressure against gravity. The dynamical effects associated with this change may well be manifested as a CME.

References

Dryer, M., Wu, S. T., Steinolfson, R. S., and Wilson, R. M.:1979, *Astrophys. J.* **227**, 1059.
Harrison, R. A.: 1986, *Astron. Astrophys.* **162**, 283.
Hasan, S. S. and Venkatakrishnan, P.: 1982, *Solar Phys.* **80**, 385.
Kopp, R. A. and Holzer, T. E.: 1976, *Solar Phys.* **49**, 43.
Low, B. C.: 1990, *Ann. Res. Astron. Astrophys.* **28**, 491.
Mikić, Z. and Linker, J. A.: 1994, *Astrophys. J.* **430**, 898.
Munro, R. H., Gosling, J. J., Hildner, E., Macqueen, R. M., Poland, A. I., and Ross, C. L.: 1979, *Solar Phys.* **61**, 201.
Nakagawa, Y. and Steinolfson, R. S.: 1976, *Astrophys. J.* **207**, 276.
Nakagawa, Y., Wu, S. T., and Han, S. M.: 1981, *Astrophys. J.* **244**, 331.
Parker, E. N.: 1958, *Astrophys. J.* **128**, 664.
Parker, E. N.: 1963, *Interplanetary Dynamical Processes*, Interscience, New York.
Steinolfson, R. S. and Hundhausen, A. J.: 1988, *J. Geophys. Res.* **93**, 14269.
Steinolfson, R. S., Suess, S. T., and Wu, S. T.: 1982, *Astrophys. J.* **255**, 730.
Wu, S. T., Guo, W. P., and Wang, J. F.: 1995, *Solar Phys.* **157**, 325.
Wu, S. T., Wang, S., Dryer, M., Poland, A. I., Sime, D. G., Wolfson, C. J., Orwig, L. E., and Maxwell, A.: 1983, *Solar Phys.* **85**, 351.