THE ASTROPHYSICAL JOURNAL, 416:267–275, 1993 October 10

HIGH-DENSITY MATTER IN THE CHIRAL SIGMA MODEL

PRADIP K. SAHU
Institute of Physics, Bhubaneswar 751005, India

RAHUL BASU
Institute of Mathematical Sciences, Madras 600113, India

AND

BHASKAR DATTA
Indian Institute of Astrophysics, Bangalore 560034, India
Received 1992 March 5; accepted 1993 April 21

ABSTRACT

We present a field theoretical equation of state for nuclear matter and neutron-rich matter in β equilibrium using the chiral sigma model. The model includes an isoscalar vector field generated dynamically and reproduces the empirical values of the nuclear matter saturation density and binding energy and the isospin symmetry coefficient for asymmetric nuclear matter. The energy per nucleon of nuclear matter as predicted by our calculation is in very good agreement, up to about a density of $4n_s$ (n_s = nucleon number density for saturating nuclear matter), with the estimates inferred from heavy-ion collision data. An astrophysical application, relating to neutron star structure, is presented. An extension of the nuclear matter equation of state for finite temperatures (up to 15 MeV) is given, and the phase transition to (u, d, s) quark matter is discussed.

Subject headings: dense matter — equation of state — stars: neutron

1. INTRODUCTION

The equation of state (EOS) of high-density matter remains a focus of interest, despite two decades of work, due to persistent unresolved aspects relating to the nucleon-nucleon interactions at short separations (≤ 0.5 fm) and the appropriate many-body theory to be used and also due to the impetus coming from heavy-ion collision experiments (Stock 1986, 1989). The main theoretical approaches to derive the EOS are (1) a nonrelativistic method based on the use of a nucleon-nucleon potential together with a variational method for the many-body correlations and (2) a relativistic field-theoretical method based on a chosen form of the Lagrangian to describe the many-nucleon system. These approaches possess varying degrees of merit, and it is not firmly established at the present time which one is preferable to the other. However, what has become clear in recent years is the importance of the three-body forces in any calculation of the EOS at high densities, i.e., at densities exceeding n_s, the equilibrium nuclear matter density.

A chiral Lagrangian using the scalar field (the so-called sigma model) was originally introduced by Gell-Mann & Levy (1960) as an example to illustrate chiral symmetry and partial conservation of axial current. The importance of chiral symmetry in the study of nuclear matter properties was emphasized by Lee & Wick (1974). The nonlinear terms of the chiral Lagrangian can provide the three-body forces, important at high densities (Jackson, Rho, & Krotscheck 1985; Ainsworth et al. 1987) and can be relevant in applications to neutron star structure and supernova collapse dynamics calculations.

The usual theory of pions leads to a theory of nuclear matter that does not possess the empirically desirable saturation property for the energy per nucleon. For this reason, the isoscalar vector field is introduced in the theory via the Higgs mechanism. This way it becomes possible to have a saturating nuclear matter EOS (Boguta 1983). With the availability of experimental estimates for the incompressibility parameter of nuclear matter (denoted by K), there have been attempts to reproduce the desirable value of K (about 200 MeV) using the sigma model. In the "standard" sigma model, the value of K turns out to be quite large, several times the above-mentioned value, for plausible values of the coupling constants involved, and can be reduced only by introducing in the theory terms due to the scalar field self-interactions and/or vacuum fluctuations with adjustable coefficients.

In this work, we use the SU(2) × SU(2) chiral sigma model to describe nuclear matter and neutron star matter. We take the approach that the isoscalar vector field is generated dynamically. Inclusion of such a field is necessary to ensure the saturation property of nuclear matter. The effective mass of the nucleon then acquires a density dependence on both the scalar and the vector fields, and must be obtained self-consistently. We do this using mean-field theory wherein all the meson fields are replaced by their uniform, expectation values. To describe nuclear matter we have two parameters in the theory: the ratio of the coupling constant to the mass for the scalar, and the isoscalar vector fields. This procedure also gives a relatively high value for K at the saturation density. Although this is an undesirable feature as far as nuclear matter at saturation density is concerned, it need not be viewed as a crucial shortcoming for our purpose here, in view of the fact that a fit to K at saturation does not dictate the slope of the EOS at densities $\geq 4n_s$ (Prakash & Ainsworth 1987; Horowitz & Serot 1987; Stock 1989; Baym 1991; Ellis, Kapusta, & Olive 1991), which is the density regime of greatest importance insofar as astrophysical applications such as neutron star structure and supernova modeling calculations are concerned. To describe neutron star matter, we include the coupling to the isovector ρ meson, the coupling strength being determined by requiring a fit to the empirical value of the symmetry energy.

There have been several earlier papers that have employed the chiral sigma model to obtain the EOS of high-density matter. Prakash & Ainsworth (1987) examined the role of the many-body effects provided by the chiral sigma model in the EOS of symmetric nuclear matter and neutron-rich matter. An EOS based on the chiral sigma model was also considered by Glendenning (1988). In these calculations the ω meson does not have a dynamically generated mass. In an earlier paper, Glendenning (1986) considered a chiral sigma model with dynamical masses, finite-temperature solutions, and application to neutron stars, but not with the ρ meson and its isospin symmetry influence.

Our paper is organized as follows. In § 2 we describe nuclear matter in the chiral sigma model. The EOS for neutron-rich matter in β equilibrium (representative of neutron star interiors), and its application with regard to neutron star structure, are considered in § 3. Extension of the nuclear matter EOS for finite temperatures ($k_B T \leq 15$ MeV), and speculations of a phase transition to (u, d, s) quark matter at high densities are discussed in § 5. Section 6 gives the summary and concluding remarks.

2. NUCLEAR MATTER IN THE CHIRAL SIGMA MODEL

2.1. *The Model*

The Lagrangian for an SU(2) × SU(2) chiral sigma model that includes (dynamically) an isoscalar vector field (ω_μ) is (we choose $\hbar = 1 = c$)

$$L = \frac{1}{2}(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4}(\boldsymbol{\pi} \cdot \boldsymbol{\pi} + \sigma^2 - x_0^2)^2 - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} g_\omega (\sigma^2 + \boldsymbol{\pi}^2)\omega_\mu \omega^\mu + g_\sigma \bar{\psi}(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})\psi + \bar{\psi}(i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu)\psi , \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, ψ is the nucleon isospin doublet, $\boldsymbol{\pi}$ is the pseudoscalar pion field, and σ is the scalar field. The vector field ω_μ couples to the conserved baryonic current $j_\mu = \bar{\psi}\gamma_\mu \psi$. The expectation value $\langle j_0 \rangle$ is identifiable as the nucleon number density, which we denote by n_B.

The interactions of the scalar and the pseudoscalar mesons with the vector boson generates a mass for the latter spontaneously by the Higgs mechanism. The masses for the nucleon, the scalar meson, and the vector meson are respectively given by

$$M = g_\sigma x_0, \quad m_\sigma = \lambda^{1/2} x_0 , \quad m_\omega = g_\omega x_0 . \quad (2)$$

To derive the thermodynamical quantities of the system of degenerate nucleons, characterized by the nucleon number density (n_B) or, equivalently, the Fermi momentum $k_F = (6\pi^2 n_B/\gamma)^{1/3}$ (where γ is the nucleon spin degeneracy factor), we need to know the dependence of the meson fields on n_B. For this, we resort to the mean-field approximation. This approach has been extensively used to obtain field-theoretical EOS models for high-density matter. In this approximation, expected to be valid for degenerate matter at high densities, the mesonic fields are assumed to be uniform (i.e., spacetime-independent with no quantum fluctuations). For the isoscalar vector field, then

$$\omega_\mu = \omega_0 \delta_\mu^0 , \quad (3)$$

where ω_0 is spacetime-independent but depends on n_B, and δ_μ^0 is the Kronecker delta. The equation of motion for the vector field specifies ω_0:

$$\omega_0 = \frac{n_B}{g_\omega x^2} , \quad (4)$$

where

$$x = (\langle \sigma^2 + \boldsymbol{\pi}^2 \rangle)^{1/2} . \quad (5)$$

The equation of motion for σ is written for convenience in terms of $y \equiv x/x_0$, and is of the form

$$y(1 - y^2) + \frac{c_\sigma c_\omega \gamma^2 k_F^6}{18\pi^4 M^2 y^3} - \frac{c_\sigma y}{\pi^2} \int_0^{k_F} \frac{dk\, k^2}{(\boldsymbol{k}^2 + M^{*2})^{1/2}} = 0 , \quad (6)$$

where $M^* \equiv yM$ is the effective mass of the nucleon and

$$c_\sigma \equiv g_\sigma^2/m_\sigma^2 , \quad c_\omega \equiv g_\omega^2/m_\omega^2 . \quad (7)$$

We consider here the normal state of high-density matter in which there is no pion condensation.

The diagonal components of the conserved total stress tensor corresponding to the Lagrangian in equation (1) together with the equation of motion for the Fermion field (and a mean-field approximation for the meson fields) provide the following identification for the total energy density (ε) and pressure (P) of the many-nucleon systems (assumed to be a perfect fluid):

$$\varepsilon = \frac{M^2(1 - y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega k_F^6}{72\pi^4 y^2} + \frac{\gamma}{2\pi^2} \int_0^{k_F} dk\, k^2(\boldsymbol{k}^2 + M^{*2})^{1/2} , \quad (8)$$

$$P = -\frac{M^2(1 - y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega k_F^6}{72\pi^4 y^2} + \frac{\gamma}{6\pi^2} \int_0^{k_F} \frac{dk\, k^4}{(\boldsymbol{k}^2 + M^{*2})^{1/2}} . \quad (9)$$

The energy per nucleon is

$$E = \frac{3\pi^2 M^2(1 - y^2)^2}{4\gamma c_\sigma k_F^3} + \frac{\gamma c_\omega k_F^3}{12\pi^2 y^2} + \frac{3}{k_F^3} \int_0^{k_F} dk\, k^2(\boldsymbol{k}^2 + M^{*2})^{1/2} . \quad (10)$$

For pure neutron matter $\gamma = 2$, and for nuclear matter $\gamma = 4$. A specification of the coupling constants c_σ, c_ω now specifies the EOS.

2.2. *Nuclear Matter Equation of State*

For nuclear matter we fix c_σ and c_ω by fits to two nuclear matter properties: the saturation density (n_s) and the binding energy per particle at $n_B = n_s$. For these we choose the values 0.153 fm^{-3} and -16.3 MeV, respectively, as suggested from analysis of experimental data (Möller et al. 1988). This gives

$$c_\sigma = 6.2033 \text{ fm}^2 , \quad c_\omega = 2.9378 \text{ fm}^2 . \quad (11)$$

This leads to a value of $0.78M$ for the effective mass of the nucleon in saturating nuclear matter. The value of K at saturation density that we get is ~ 700 MeV.

Values of the various thermodynamical quantities for nuclear matter for various densities are presented in Table 1. The quantity μ appearing in this table is the chemical potential, given by the relationship $\mu = (P + \varepsilon)/n_B$, and ρ is the total mass-energy density, ε/c^2.

TABLE 1

EQUATION OF STATE OF DEGENERATE NUCLEAR MATTER AS GIVEN BY PRESENT MODEL

k_F (fm^{-1}) (1)	n_B (fm^{-3}) (2)	y (3)	ρ (g cm^{-3}) (4)	P (dyn cm^{-2}) (5)	E (MeV) (6)	μ (MeV) (7)
1.0..........	0.068	0.91	1.12E14	−1.14E33	930.98	920.42
1.1..........	0.090	0.87	1.49E14	−1.81E33	927.64	915.07
1.2..........	0.117	0.83	1.92E14	−2.41E33	924.39	912.95
1.3..........	0.148	0.79	2.44E14	−4.82E33	922.52	920.49
1.4..........	0.185	0.75	3.05E14	6.57E33	924.41	946.56
1.5..........	0.228	0.72	3.79E14	2.23E34	932.77	993.76
1.6..........	0.277	0.72	4.68E14	4.73E34	948.91	1055.60
1.7..........	0.332	0.72	5.75E14	8.11E34	972.47	1125.00
1.8..........	0.394	0.74	7.04E14	1.23E35	1002.30	1197.90
1.9..........	0.463	0.76	8.57E14	1.75E35	1037.30	1272.40
2.0..........	0.540	0.78	1.04E15	2.35E35	1076.30	1347.60
2.1..........	0.625	0.80	1.25E15	3.05E35	1118.40	1423.00
2.2..........	0.719	0.83	1.49E15	3.86E35	1163.10	1498.40
2.3..........	0.822	0.86	1.77E15	4.79E35	1209.70	1573.80
2.4..........	0.934	0.88	2.09E15	5.85E35	1257.90	1649.00
2.5..........	1.055	0.91	2.46E15	7.05E35	1307.40	1724.00

NOTE.—Columns (1)–(7), respectively, give the Fermi momentum, the nucleon number density, the nucleon effective mass factor, the total mass-energy density, the pressure, the energy per nucleon, and the nucleon chemical potential. The numbers following the letter E represent powers of 10 in all the tables.

In heavy-ion collision experiments, hot hadronic matter is produced at temperatures up to 100 MeV, which contain up to 25% of their energy in nuclear resonances and mesonic degrees of freedom. After making allowance for (model-dependent) corrections for the thermal part of the energy, first estimates of the energy per nucleon of nuclear matter (for $k_B T = 0$) have been made (see Stock 1989 for a discussion). In Figure 1 we present a comparison of available such estimates with the prediction of the EOS considered by us here. Up to a density of $4n_s$, there is satisfactory agreement between the two.

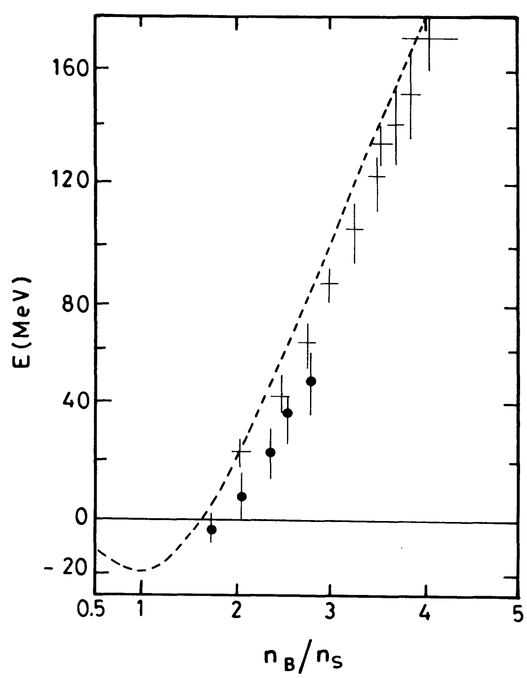

FIG. 1.—First generation of estimates, from heavy-ion collision data, of energy per nucleon (E) of nuclear matter plotted against n_B (in units of n_s). The crosses correspond to pion data and the circles to radial energies (see Stock 1989 for a detailed discussion). The dashed curve corresponds to the present model (nuclear matter).

3. NEUTRON STAR MATTER

3.1. *Equation of State*

At high densities typical of interiors of neutron stars, the neutron chemical potentials will exceed the combined masses of proton and electron. Asymmetric nuclear matter with an admixture of electrons (rather than pure neutron matter) is, therefore, a more likely composition of matter in neutron star interiors. The concentrations of protons and electrons (denoted by n_p and n_e, respectively) can be determined using conditions of β equilibrium and electrical charge neutrality:

$$\mu_n = \mu_p + \mu_e , \quad (12)$$

$$n_p = n_e \quad (13)$$

(μ_i is the chemical potential of particle species i).

Since nuclear force is known to favor isospin symmetry, and since the symmetry energy arising solely from the Fermi energy is known to be inadequate to account for the empirical values of the symmetry energy ($\simeq 32$ MeV), we include the interaction due to the isospin triplet ρ meson in equation (1) for the purpose of describing neutron-rich matter. That is, we add the following terms,

$$-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \tfrac{1}{2}m_\rho^2 \rho_\mu \cdot \rho^\mu - \tfrac{1}{2}g_\rho \bar{\psi}(\rho_\mu \cdot \tau\gamma^\mu)\psi ,$$

to the right-hand side of equation (1) in order to describe the asymmetric matter. Here ρ_μ stands for the ρ meson field with mass m_ρ, g_ρ is the coupling strength, and

$$G_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu . \quad (14)$$

Strictly speaking, the ρ meson should couple to the total conserved current (Glendenning, Banerjee, & Gyulassy 1983). In the above, we have coupled the ρ meson to the baryons, which are not the only possible source of isospin current.

However, for the ground-state EOS, in the mean-field approximation, only the baryon part of the isospin current will survive (Glendenning 1988).

The equation of motion for ρ_μ, in the mean-field approximation where ρ_μ is replaced by its uniform value ρ_0^3 (here the superscript 3 stands for the third component in isospin space), gives

$$m_\rho^2 \rho_0^3 = \tfrac{1}{2} g_\rho \sum_{B=n,p} \langle \bar{\psi} \gamma_0 \tau^3 \psi \rangle_B , \tag{15}$$

where the sum is over neutrons and protons. This gives the following density dependence for the field variable ρ_0^3:

$$\rho_0^3 = \frac{g_\rho}{2m_\rho^2} (n_p - n_n) . \tag{16}$$

The symmetric energy coefficient that follows from the semi-empirical nuclear mass formula (that is, the coefficient of the term $(n_p - n_n)^2/(n_p + n_n)^2$ in the mass formula) is

$$a_{\rm sym} = \frac{c_\rho k_{\rm F}^3}{12\pi^2} + \frac{k_{\rm F}^2}{6(k_{\rm F}^2 + M^{*2})^{1/2}} , \tag{17}$$

where $c_\rho \equiv g_\rho^2/m_\rho^2$. We fix the coupling constant c_ρ by requiring that $a_{\rm sym}$ correspond to the empirical value 32 MeV. This gives

$$c_\rho = 4.6617 \ {\rm fm}^2 . \tag{18}$$

Note that the ρ meson will contribute a term $= m_\rho^2(\rho_0^3)^2/2$ to the energy density and pressure. Table 2 lists the pressure versus the total mass-energy density for the neutron-rich matter in β equilibrium.

3.2. *Neutron Star Structure*

The composite EOS for the entire neutron star density span was constructed by joining the EOS of neutron-rich matter as given by equations (8) and (9) to that given by (*a*) Negele & Vautherin (1973) for the density region (10^{14} to 5×10^{10}) g cm^{-3}, (*b*) Baym, Pethick, & Sutherland (1971) for the region (5×10^{10} to 10^3) g cm^{-3}, and (*c*) Feynman, Metropolis, & Teller (1949) for $\rho < 10^3$ g cm^{-3}.

TABLE 2

PRESSURE VERSUS DENSITY FOR NEUTRON-RICH MATTER

ρ (g cm^{-3})	P (dyn cm^{-2})
4.585E15..........	1.628E36
3.666E15..........	1.267E36
2.686E15..........	8.881E35
2.107E15..........	6.663E35
1.786E15..........	5.440E35
1.510E15..........	4.395E35
1.274E15..........	3.507E35
1.073E15..........	2.757E35
9.049E14..........	2.131E35
8.313E14..........	1.859E35
7.030E14..........	1.390E35
5.970E14..........	1.008E35
5.512E14..........	8.527E34
5.096E14..........	7.117E34
4.719E14..........	5.878E34
4.065E14..........	3.867E34
3.517E14..........	2.403E34
3.044E14..........	1.028E34

The gravitational mass (M_G) and radius (R) for nonrotating neutron stars are obtained by integrating the structure equations (Misner, Thorne, & Wheeler 1970):

$$\frac{dP}{dr} = -\frac{G(\rho + P/c^2)(m + 4\pi r^3 P/c^2)}{r^2(1 - 2Gm/rc^2)} , \tag{19}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho . \tag{20}$$

The potential function, $\nu(r)$, relating the element of proper time to the element of time at $r = \infty$ is given by

$$\frac{d\nu}{dr} = \frac{2G}{r^2} \left(\frac{m + 4\pi r^3 P/c^2}{1 - 2Gm/rc^2} \right) . \tag{21}$$

For a given EOS, $P(\rho)$, and a given central density $\rho(r = 0) = \rho_c$, equations (19)–(21) are integrated numerically with the boundary condition

$$m(r = 0) = 0 , \tag{22}$$

to give R and M_G. The radius R is defined by the point where $P \simeq 0$, or, equivalently, $\rho = \rho_s$, where ρ_s is the density expected at the neutron star surface (about 7.8 g cm^{-3}). The total gravitational mass is then given by $M_G = m(R)$.

The moment of inertia (I) of the rotating neutron star (Ω is the angular velocity of the star as seen by a distant observer) is given by

$$I = \frac{1}{\Omega} \frac{c^2 R^4}{6G} \left(\frac{d\bar{\omega}}{dr} \right)_{r=R} , \tag{23}$$

where $\bar{\omega}(r)$, the angular velocity of the star fluid relative to the local inertial frame, is given by

$$\frac{d}{dr} \left(r^4 j \frac{d\bar{\omega}}{dr} + 4r^3 \bar{\omega} \frac{dj}{dr} \right) = 0 , \tag{24}$$

where

$$j(r) = e^{-\nu/2}(1 - 2GM_G/rc^2)^{1/2} , \tag{25}$$

and satisfies the boundary condition

$$\left(\frac{d\bar{\omega}}{dr} \right)_{r=R} = 0; \quad \bar{\omega}(\infty) = \Omega . \tag{26}$$

For the numerical integration to obtain the structure parameters, it is sufficient to start with an arbitrary value of $\nu(0)$, which is then rescaled to satisfy the surface condition

$$\nu(R) = \ln\left(1 - \frac{2GM_G}{Rc^2}\right) , \tag{27}$$

so that $\nu(\infty) = 0$. Likewise, $\bar{\omega}(0)$ is initially chosen to be an arbitrary constant, and a value of Ω given by

$$\Omega = \bar{\omega}(R) + \frac{1}{3} R \left(\frac{d\bar{\omega}}{dr} \right)_{r=R} \tag{28}$$

is obtained. A new starting value $\bar{\omega}_{\rm new}(0)$ corresponding to any specified $\Omega_{\rm new}$ is given by

$$\bar{\omega}_{\rm new}(0) = \Omega_{\rm new}\, \bar{\omega}(0)/\Omega . \tag{29}$$

4. NUCLEAR MATTER AT FINITE TEMPERATURES

The extension of the EOS for nuclear matter to the case of finite temperatures is straightforward, and is done by considering the usual thermodynamical potential function. For a first estimate of the effect of finite temperatures, we can restrict our consideration to the nucleons only as far as the contributions to the thermodynamical potential are concerned, and neglect the free thermodynamical potentials due to the mesons.

For the vector meson field ω_0, the thermodynamic average of the equation of motion gives

$$\omega_0 = \frac{c_\omega n_B}{m_\omega^2 y^2}, \tag{30}$$

where n_B is the baryonic number density, given by

$$n_B = \frac{\gamma}{(2\pi)^3} \int_0^\infty d^3k[n(T) - \bar{n}(T)], \tag{31}$$

with

$$n(T) = [\![\exp\{[(k^2 + y^2M^2)^{1/2} - \nu]\beta\} + 1]\!]^{-1}, \tag{32}$$

$$\bar{n}(T) = [\![\exp\{[(k^2 + y^2M^2)^{1/2} + \nu]\beta\} + 1]\!]^{-1}, \tag{33}$$

$$\beta = 1/(k_B T), \tag{34}$$

$$\nu \equiv \mu - c_\omega n_B/y^2. \tag{35}$$

The effective mass factor y is now different from the $T = 0$ case, and must be determined self-consistently from the equation for the sigma field that minimizes the thermodynamic potential. It is given by

$$y(1 - y^2) + \frac{2c_\sigma c_\omega}{m_\omega^2}\frac{n_B^2}{y^3} - \frac{\gamma c_\sigma}{\pi^2}\int_0^\infty \frac{dk\, k^2(n + \bar{n})}{(k^2 + y^2M^2)^{1/2}} = 0. \tag{36}$$

The energy density and pressure at finite temperatures are given by

$$\varepsilon = \frac{M^2}{8c_\sigma^2}(1 - y^2)^2 + \frac{n_B^2}{2y^2} + \frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2(k^2 + y^2M^2)^{1/2}(n + \bar{n}), \tag{37}$$

$$P = -\frac{M^2}{8c_\sigma^2}(1 - y^2)^2 + \frac{n_B^2}{2y^2} + \frac{\gamma}{6\pi^2}\int_0^\infty \frac{dk\, k^4(n + \bar{n})}{(k^2 + y^2M^2)^{1/2}}. \tag{38}$$

The EOS is evaluated by first solving numerically equations (31) and (36) for n_B and y for fixed chosen values of the inverse temperature β and the chemical potential μ, and making the substitutions in equations (37) and (38). We assume that the coupling constants c_σ and c_ω are temperature-independent, and have the same values as in the degenerate nuclear matter case. The EOS for nuclear matter ($\gamma = 4$) for $k_B T = 5$, 10, and 15 MeV are given in Tables 3–5. Note that the upper limit of all the integrals in the finite-temperature case is ∞, and not k_F. The entropy per particle, S/A, at temperature T is calculated using the relationship (Barranco & Treiner 1981)

$$\frac{S}{A} = \frac{\pi^2}{\hbar^2 k_F^2} M^* T. \tag{39}$$

5. PHASE TRANSITION TO QUARK MATTER

The success of the quark model in explaining hadronic spectra and the jet phenomena observed in hadron collider experiments strongly suggests that a more fundamental description of matter at very high densities (internucleon separations $\sim$1 fm) should be in terms of a quark-gluon plasma. This raises the possibility of the existence of quark matter cores in heavy neutron stars and/or degenerate stars made up entirely of (u, d, s) quark matter (Witten 1984; Farhi & Jaffe 1984; Haensel, Zdunick, & Schaeffer 1986; Alcock, Farhi, & Olinto (1986). The presence of quarks in neutron star interiors will substantially enhance the neutrino emissivity (Datta, Raha, & Sinha 1988) and may be a contributing factor for anomalously low surface temperature of some pulsars.

The status of phase transition to quark matter at high densities is far from clear at the present time. Earlier theoretical works (Baym & Chin 1976; Chapline & Nauenberg 1977) suggested that a first-order phase transition to (u, d) quark matter will occur at densities too high to be of interest for neutron star physics. Later, Farhi & Jaffe (1984) and Haensel et al. (1986) suggested, following Witten (1984), that such a transition to (u, d, s) quark matter may take place inside neutron stars at densities not far above n_s. Similar conclusions are also reported in the calculations of Glendenning, Weber, & Moszkowski (1991) and Rosenhauer et al. (1992). A completely opposite viewpoint was reported by Bethe, Brown, & Cooperstein (1987), Brown et al. (1990), and Brown, Bethe, & Pizzochero (1991). On the

TABLE 3

EQUATION OF STATE OF NUCLEAR MATTER: $k_B T = 5$ MeV

μ (MeV) (1)	n_B (fm^{-3}) (2)	y (3)	ρ (g cm^{-3}) (4)	P (dyn cm^{-2}) (5)	E MeV (6)	S/A (7)
947.18..........	0.186	0.748	3.07E14	7.12E33	925.70	0.48
957.04..........	0.196	0.740	3.24E14	1.01E34	927.08	0.46
966.91..........	0.205	0.734	3.40E14	1.33E34	928.66	0.45
976.78..........	0.214	0.730	3.55E14	1.66E34	930.44	0.43
986.64..........	0.222	0.727	3.70E14	2.01E34	932.38	0.42
996.51..........	0.230	0.725	3.84E14	2.37E34	934.48	0.41
1006.38..........	0.238	0.723	3.98E14	2.74E34	936.73	0.40
1016.24..........	0.246	0.721	4.12E14	3.12E34	939.12	0.39
1026.11..........	0.254	0.720	4.26E14	3.52E34	941.64	0.39
1035.98..........	0.262	0.720	4.40E14	3.92E34	944.30	0.39
1045.84..........	0.269	0.719	4.55E14	4.34E34	947.07	0.37
1055.71..........	0.277	0.719	4.69E14	4.77E34	949.97	0.37

NOTE.—In Tables 3–5, columns (1)–(7), respectively, give the chemical potential, the nucleon number density, the nucleon effective mass factor, the total mass-energy density, the pressure, the energy per nucleon, and the entropy per nucleon.

TABLE 4

EQUATION OF STATE OF NUCLEAR MATTER: $k_B T = 10$ MeV

μ (MeV) (1)	n_B (fm^{-3}) (2)	y (3)	ρ (g cm^{-3}) (4)	P (dyn cm^{-2}) (5)	E (MeV) (6)	S/A (7)
937.31..........	0.175	0.761	2.89E14	5.30E33	927.97	0.96
947.18..........	0.187	0.748	3.10E14	8.19E33	929.20	0.93
957.04..........	0.197	0.741	3.27E14	1.12E34	930.48	0.90
966.91..........	0.206	0.736	3.43E14	1.44E34	931.99	0.87
976.78..........	0.215	0.732	3.58E14	1.77E34	933.71	0.85
986.64..........	0.223	0.728	3.72E14	2.12E34	935.60	0.83
996.51..........	0.231	0.726	3.87E14	2.48E34	937.66	0.81
1006.38..........	0.239	0.724	4.01E14	2.85E34	938.87	0.79
1016.24..........	0.247	0.722	4.15E14	3.24E34	942.23	0.77
1026.11..........	0.255	0.721	4.29E14	3.68E34	944.72	0.76
1035.98..........	0.263	0.721	4.43E14	4.04E34	947.35	0.75
1045.84..........	0.270	0.721	4.58E14	4.46E34	950.10	0.73
1055.71..........	0.278	0.721	4.72E14	4.90E34	952.97	0.72
1065.57..........	0.286	0.721	4.87E14	5.34E34	955.82	0.70

experimental side, efforts are presently being made to detect signals for quark-gluon plasma (QGP) in accelerator experiments involving heavy-ion collisions. However, a persistent question is: What constitutes an acceptable, clear signature for QGP in heavy-ion collisions? This question is still unresolved. In this section we briefly reexamine the problem, using the chiral sigma model description for high-density neutron star matter.

For the present discussion only u, d, and s quarks need be considered, the other quarks being too massive to be present in any significant number. Note that an equilibrium number density of s quarks will be present even though the neutron matter is made up of only u and d quarks (Witten 1984), because with increasing density of (u, d) quark matter, it is energetically more favorable for d quarks at the top of the Fermi sea to transform themselves into s quarks via the weak interaction process

$$u + d \rightarrow u + s\ .$$

For the (u, d, s) quark matter, we use an EOS that incorporates short-range quark-gluon interactions perturbatively, along the lines adopted by Farhi & Jaffe (1984). These authors retained terms to first order in the strong interaction coupling constant (α_c). Here we incorporate terms to second order in α_c, following Goyal & Anand (1990). The long-range interactions are taken into account phenomenologically using the MIT bag model. The treatment incorporates the density dependence of α_c which are solutions of the Gell-Mann–Low equation for the screened charge. The values of the parameters involved, namely, the strange quark mass (m_s), the bag pressure term (B), and the renormalization point (μ_0) correspond to the requirement that bulk strange matter be stable at zero temperature and pressure, with energy per baryon less than the lowest energy per baryon found in nuclear matter.

To second order in α_c, and assuming u and d quarks to be massless, the thermodynamical potential of the (u, d, s) quark matter is

$$\Omega = \sum_i \Omega_i + \Omega_{\rm int}\ , \tag{40}$$

with Ω_i ($i = u, d, s, e^-$) represent the contributions to the thermodynamical potential due to u, d, s quarks and electrons, and $\Omega_{\rm int}$ is the contribution due to interference between u and d quarks, and is of order α_c^2. Expressions for Ω_i and $\Omega_{\rm int}$ are given in Goyal & Anand (1990) and so are not reproduced here. The total energy density and the external pressure of the quark

TABLE 5

EQUATION OF STATE OF NUCLEAR MATTER: $k_B T = 15$ MeV

μ (MeV) (1)	n_B (fm^{-3}) (2)	y (3)	ρ (g cm^{-3}) (4)	P (dyn cm^{-2}) (5)	E (MeV) (6)	S/A (7)
937.31..........	0.178	0.756	2.97E14	6.98E33	933.41	1.37
947.18..........	0.189	0.749	3.15E14	9.89E33	934.32	1.32
957.04..........	0.199	0.742	3.32E14	1.30E34	935.54	1.28
966.91..........	0.208	0.737	3.47E14	1.62E34	937.00	1.24
976.78..........	0.217	0.733	3.62E14	1.95E34	938.66	1.21
986.64..........	0.225	0.730	3.77E14	2.30E34	940.51	1.19
996.51..........	0.233	0.727	3.91E14	2.66E34	942.53	1.16
1006.38..........	0.241	0.725	4.06E14	3.04E34	944.71	1.14
1016.24..........	0.249	0.724	4.20E14	3.42E34	947.03	1.12
1026.11..........	0.256	0.723	4.34E14	3.82E34	949.50	1.10
1035.98..........	0.264	0.723	4.48E14	4.24E34	952.09	1.08
1045.84..........	0.272	0.722	4.62E14	4.66E34	954.82	1.06
1055.71..........	0.279	0.722	4.77E14	5.09E94	957.67	1.05
1065.57..........	0.287	0.723	4.92E14	5.54E34	960.64	1.03

matter will be given respectively by

$$\varepsilon_q = \Omega + B + \sum_i \mu_i n_i \,, \tag{41}$$

$$P_q = -\Omega - B \,, \tag{42}$$

where n_i is the number density of the ith particle species. For specific choices of the parameters of the theory, the EOS is obtained by calculating ε_q and P_q for a given value of μ,

$$\mu \equiv \mu_d = \mu_s = \mu_\mu + \mu_e \,, \tag{43}$$

by solving for μ_e from the condition that the total electric charge of the system be zero.

There is an unphysical dependence of the EOS on the renormalization point μ_0, which, in principle, should not affect the calculations of physical observables if the calculations are done perturbatively, and, therefore, in order to minimize the dependence on μ_0, the renormalization should be chosen to be close to the natural energy scale, which could be either $\mu_0 \simeq B^{1/4}$ or the average kinetic energy of quarks in the bag, in which case $\mu_0 \simeq 313$ MeV. In the present study, our choice of μ_0 is dictated by requiring that strange matter be more stable than nucleon matter at zero temperature and pressure with a positive baryon electric charge. This leads to the following representative choice of the parameter values:

set 1:

$$B = 56 \text{ MeV fm}^{-3}, \quad m_s = 150 \text{ MeV}, \quad \mu_0 = 150 \text{ MeV} ,$$

set 2:

$$B = 67 \text{ MeV fm}^{-3}, \quad m_s = 150 \text{ MeV}, \quad \mu_0 = 80 \text{ MeV} ,$$

set 3:

$$B = 45 \text{ MeV fm}^{-3}, \quad m_s = 150 \text{ MeV}, \quad \mu_0 = 150 \text{ MeV} .$$

The thermodynamically favored phase of matter at a given pressure is the one which has lower chemical potential. *For all the sets of parameter values cited above, we find that a plot of chemical potential versus pressure (Maxwell construction) does not indicate a phase transition to quark matter.*

6. SUMMARY AND CONCLUSIONS

Using the chiral sigma model and adopting the approach that the isoscalar vector field, needed to provide saturating binding energy of degenerate nuclear matter, is generated dynamically, we have obtained an EOS of degenerate nuclear and neutron-rich matter at high densities.

Table 6 lists neutron star structure parameters as predicted by our EOS for neutron-rich matter. The maximum gravitational mass for a stable nonrotating neutron star predicted by our model is 2.58 $M_\odot$. This occurs for a central density of 1.4×10^{15} g cm^{-3}. The corresponding radius of the star is 13.55 km. Figures 2 and 3 shows the plots of gravitational mass versus central density and moment of inertia versus the gravitational mass. The maximum moment of inertia is 4.69×10^{45} g cm^2.

A comparison with the maximum mass of (nonrotating) neutron stars predicted by other, recent field theoretical EOS models is given in Table 7. The choice of the EOS models in Table 7 is representative but by no means exhaustive. Included in this comparison are EOS models due to Alonso & Cabanell (1985) and Prakash & Ainsworth (1987), which are also based

TABLE 6

NEUTRON STAR BULK PARAMETERS FOR THE PRESENT EOS

ρ_c (g cm^{-3}) (1)	R (km) (2)	$M/M_\odot$ (3)	I (g cm^2) (4)	ϕ (5)
3.5E14	12.10	0.95	1.15E45	0.876
4.0E14	13.08	1.31	1.89E45	0.839
4.5E14	13.62	1.59	2.57E45	0.809
5.0E14	13.98	1.82	3.15E45	0.784
6.0E14	14.25	2.12	3.92E45	0.748
8.0E14	14.21	2.41	4.57E45	0.706
1.0E15	13.99	2.53	4.69E45	0.683
1.2E15	13.72	2.56	4.60E45	0.669
1.4E15	13.55	2.58	4.51E45	0.661

NOTE.—Columns (1)–(5), respectively, give the central density, the radius, the gravitational mass, the moment of inertia calculated for angular velocity $(GM_G/R^3)^{1/2}$, and the surface redshift ratio ϕ.

on the sigma model but differ from our model in the details. The EOS of Alonso & Cabanell (1985), model II, comes from a determination of the free parameters of the linear sigma model with an explicit symmetry-breaking term and is coupled to ω and ρ mesons, in a renormalizable way. Prakash & Ainsworth (1987) included the σ meson one-loop contributions, but the isoscalar vector field was not generated dynamically, so that its role is reduced to an empirical one, allowing for arbitrary variations in its coupling constant (thereby making it possible to obtain any desired value of the nuclear matter compressibility modulus). The vector field plays no role in determining the value of the effective mass of the nucleon in such an approach. A comparison of this EOS with our model for neutron-rich matter and pure neutron matter is shown in Figure 4. The EOS of Serot (1979) is calculated in the mean-field approximation to the Walecka model including σ, ω, and ρ mesons and that of Chin (1977) with one-loop corrections to the σ-ω model. The EOS due to Glendenning (1989) is along

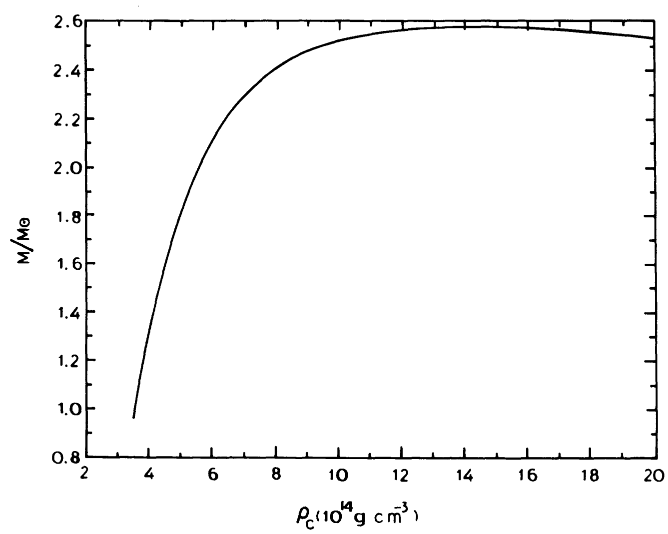

FIG. 2.—Gravitational mass (M_G) of nonrotating neutron stars vs. central density (ρ_c) as predicted by the present model EOS (neutron-rich matter). The maximum stable mass is 2.59 $M_\odot$. The corresponding central density, ρ_c, is 1.5×10^{15} g cm^{-3}, and the radius is 13.9 km.

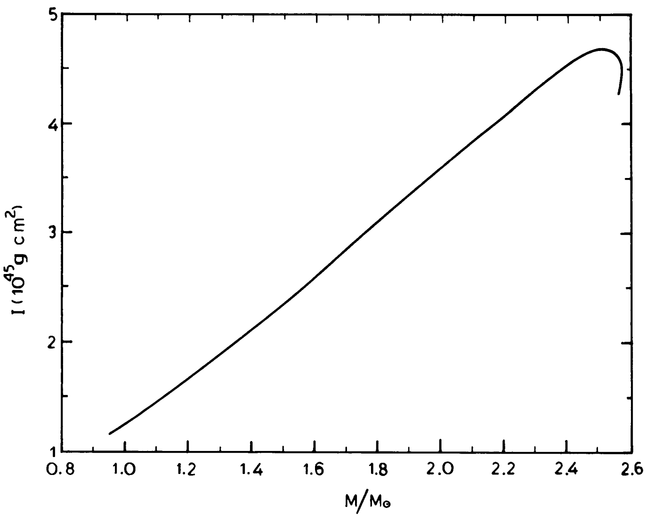

FIG. 3.—Neutron star moment of inertia (I) vs. gravitational mass (M_G), as predicted by the present model.

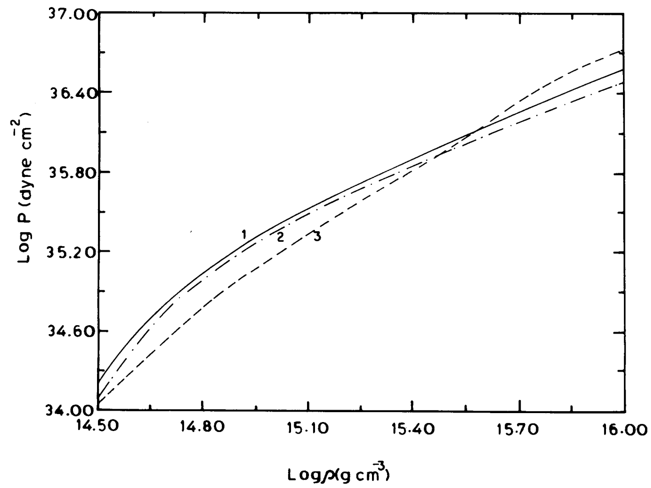

FIG. 4.—Pressure vs. total mass-energy density. Curve 1 corresponds to neutron-rich matter in β equilibrium (including the contribution from ρ meson exchange): present model. Curve 2 corresponds to pure neutron matter: present model. Curve 3 (*dashed curve*) is due to Prakash & Ainsworth (1987) (see text).

similar lines, and includes one-loop corrections and also scalar self-interactions (up to quartic order), whose magnitudes are adjusted to reproduce empirical saturation properties. This work takes into account the effect of hyperons in β equilibrium in addition to electrons and muons.

Table 7 implies that the present neutron-rich matter EOS is comparatively "stiff" as far as neutron stars are concerned. This is reflected in the value of the maximum mass of neutron stars, which is the largest for the present model. It may be mentioned here that observational evidence in favor of a stiff EOS comes from the identification by Trümper et al. (1986) of the 35 day cycle of the pulsating X-ray source Hercules X-1 as originating in free precession of the rotating neutron star (see Pines 1987 for a discussion).

Observationally, masses of neutron stars are estimated from compact binary systems, one member of which is a pulsar. The most precise estimate comes from the pulsar PSR 1913+16, which gives $1.442 \pm 0.003\ M_\odot$. A recent compilation of the estimated masses by X-ray pulsars (Nagase 1989) gives the maximum mass (corresponding to the Vela X-1 pulsar) to be $1.77 \pm 0.21\ M_\odot$. Stable neutron star masses predicted by the present EOS are thus compatible with the observational estimates. The surface redshift factor provides a probe for neutron star structure, if one presumes that observed γ-ray bursts are gravitationally redshifted e^+e^- annihilation lines produced near their surface. The surface redshift ratio ϕ, defined as

$$\phi = (1 - 2GM_G/Rc^2)^{1/2} , \tag{44}$$

is expected to be 0.78 ± 0.02 on the basis of observed data (see Friedman & Pandharipande 1981 for a discussion). The present neutron-rich matter EOS gives, for a 1.4 $M_\odot$ neutron star, $R = 13.28$ km, $I = 2.11 \times 10^{45}$ g cm^2, and the redshift ratio (at the surface) = 0.83. The corresponding central density is 4.15×10^{14} g cm^{-3}.

In constructing the neutron star matter EOS, we have restricted ourselves to (n, p, e^-) matter in β equilibrium. At high densities, hyperons can also appear (Glendenning 1989; Kapusta & Olive 1990); this is expected to reduce the "stiffness" in the EOS, and produce a consequent reduction in the maximum mass of neutron stars. Another point that we have not investigated here is the possible role of the ρ meson tensor interaction as far as the symmetry energy is concerned.

An interesting feature of the nuclear matter EOS derived by us is the following: up to a density of $4n_s$, the energy per nucleon is in very good agreement with estimates from heavy-ion collision data. The present model does not favor a strict first-order transition to (u, d, s) quark matter at high densities. The finite-temperature extension of the nuclear matter EOS (up to $k_B T = 15$ MeV considered here) shows little temperature dependence of the effective nucleon mass. The entropy

TABLE 7

MAXIMUM MASS NEUTRON STARS FOR FIELD-THEORETICAL EOS MODELS

EOS Reference[a]	n_S (fm^{-3})	K (MeV)	$M_{max}/M_\odot$	R (km)
Serot 1979	0.193	540	2.54	12.28
Chin 1977	0.193	471	2.10	10.57
Alonso & Cabanell 1985 EOS II	0.172	225	1.94	10.90
Prakash & Ainsworth 1987 ($g_\omega^2 = 16.27$)	0.160	225	1.83	9.89
Glendenning 1989	0.153	300	1.79	11.18
Present model	0.153	700	2.58	13.55

NOTE.—n_s and K are nuclear matter saturation density and compression modulus.
[a] See text.

per nucleon (S/A) is found to be low, ≤ 1 for $k_B T$ up to 10 MeV. These features and the EOS presented in Tables 3–5 are expected to find applications in stellar collapse modeling leading to Type II supernova bounce.

P. K. S. thanks Professor L. Satpathy for helpful discussions. B. D. thanks Professor R. Ramachandran for hospitality at the Institute of Mathematical Sciences, where part of this work was done.

REFERENCES

Ainsworth, T. L., Baron, E., Brown, G. E., Coopersten, J., & Prakash, M. 1987, Nucl. Phys. A, 464, 740
Alcock, C., Farhi, E., & Olinto, A. 1986, ApJ, 310, 261
Alonso, J. D., & Cabanell, J. M. 1985, ApJ, 291, 305
Barranco, M., & Treiner, J. 1981, Nucl. Phys. A, 351, 269
Baym, G. 1991, in Neutron Stars: Theory and Observation, ed. J. Ventura and D. Pines (Dordrecht: Kluwer), 26
Baym, G., & Chin, S. A. 1976, Phys. Lett. B, 62, 241
Baym, G., Pethick, C. J., & Sutherland, P. G. 1971, ApJ, 170, 299
Bethe, H. A., Brown, G. E., & Cooperstein, J. 1987, Nucl. Phys. A, 462, 791
Boguta, J. 1983, Phys. Lett. B, 128, 19
Brown, F. R., Butler, F. P., Chen, H., Christ, N. H., Dong, Z., Schaffer, W., Unger, L. I., & Vaccarino, A. 1990, Phys. Rev. Lett., 65, 2491
Brown, G. E., Bethe, H. A., & Pizzochero, P. M. 1991, Phys. Lett. B, 263, 337
Chapline, G., & Nauenberg, M. 1977, Phys. Rev. D, 16, 450
Chin, S. A. 1977, Ann. Phys., 108, 301
Datta, B., Raha, S., & Sinha, B. 1988, Mod. Phys. Lett. A, 3, 1385
Ellis, J., Kapusta, J. I., & Olive, K. A. 1991, Nucl. Phys. B, 348, 345
Farhi, E., & Jaffe, R. L. 1984, Phys. Rev. D, 30, 2379
Feynman, R. P., Metropolis, N., & Teller, E. 1949, Phys. Rev., 75, 1561
Friedman, B., & Pandharipande, V. R. 1981, Nucl. Phys. A, 361, 502
Gell-Mann, M., & Levy, M. 1960, Nuovo Cimento, 16, 705
Glendenning, N. K. 1986, Ann. Phys., 168, 246
———. 1988, Nucl. Phys. A, 480, 597
———. 1989, Nucl. Phys. A, 493, 521
Glendenning, N. K., Banerjee, B., & Gyulassy, M. 1983, Ann. Phys., 149, 1
Glendenning, N. K., Weber, F., & Moszkowski, S. A. 1991, Phys. Rev. C, 45, 844
Goyal, A., & Anand, J. D. 1990, Phys. Rev. D, 17, 1109
Haensel, P., Zdunick, J. L., & Schaeffer, R. 1986, A&A, 160, 121
Horowitz, C., & Serot, B. 1987, Nucl. Phys. A, 464, 613
Jackson, A. D., Rho, M., & Krotscheck, E. 1985, Nucl. Phys. A, 407, 495
Kapusta, J. I., & Olive, K. A. 1990, Phys. Rev. Lett., 64, 13
Lee, T. D., & Wick, G. C. 1974, Phys. Rev. D, 9, 2291
Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1970, Gravitation (San Francisco: Freeman)
Möller, P., Myers, W. D., Swiatecki, M. J., & Treiner, J. 1988, Atomic Data Nucl. Data Tables, 39, 225
Nagase, F. 1989, PASP, 41, 1
Negele, J. W., & Vautherin, D. 1973, Nucl. Phys. A, 207, 298
Pines, D. 1987, in High Energy Phenomena around Collapsed Stars, ed. F. Pacini (Dordrecht: Reidel), 193
Prakash, M., & Ainsworth, T. L. 1987, Phys. Rev. C, 36, 346
Rosenhauer, A., Staubo, E. F., Csernia, L. P., Øvergård, T., & Østgaard, E. 1992, Nucl. Phys. A, 540, 630
Serot, B. D. 1979, Phys. Lett. B, 86, 146
Stock, R. 1986, Phys. Rep., 135, 259
———. 1989, Nature, 337, 319
Trümper, J., Kahabka, P., Ögelman, H., Pietsch, W., & Voges, W. 1986, ApJ, 300, L63
Witten, E. 1984, Phys. Rev. D, 30, 272