ASYMMETRY PARAMETER AND EFFICIENCY FOR RADIATION PRESSURE AND SCATTERING OF ELECTROMAGNETIC RADIATION BY A VERY LARGE DIELECTRIC SPHERE

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Abstract. The asymmetry parameter of the scattering diagram and the efficiency for radiation pressure for scattering by a very large dielectric sphere conforming to geometrical optics and diffraction, have been investigated beyond the limit of existing literature. A new feature of shallow broad extremum (resonance) has been found in the asymmetry parameter as well as in the radiation pressure both as function of the index of refraction.

1. Introduction

The asymmetry parameter ($\langle \cos\theta \rangle$), defined as the average of the cosine of the scattering angle (θ) with the scattering phase function as the weighting function, occurs in many problems of scattering by a particle. For a homogeneous, isotropic, and smooth spherical particle with arbitrary radius = a, index of refraction = m and wavelength λ of the incident radiation, it is given by van de Hulst (1957) as

$$\langle \cos\theta \rangle = \frac{1}{x^2 Q_{\text{sca}}} \int_{-1}^{1} \{ |S_1|^2 + |S_2|^2 \} \mu \, d\mu \,, \tag{1}$$

where $x = 2\pi a/\lambda$, the size-to-wavelength parameter; Q_{sca} is the scattering efficiency; θ, the scattering angle; S_1 and S_2 are the scattering amplitude functions, for two orthogonal states of linear polarization, perpendicular and parallel, respectively, to the scattering plane, and $\mu = \cos\theta$.

For isotropic scattering (i.e., uniform scattered intensity in all latitudes and azimuths), $\langle \cos\theta \rangle = 0$. If the scattering diagram is symmetric with respect to the central line ($\theta = \pm 90°$) perpendicular to the direction of incident radiation ($\theta = 0°$), again $\langle \cos\theta \rangle = 0$. If the scattered light is more in the forward hemisphere compared to the backward hemisphere $\langle \cos\theta \rangle$ is positive. In the converse case, $\langle \cos\theta \rangle$ is negative. In general, $|\langle \cos\theta \rangle| \lesssim 1$.

For arbitrary size-to-wavelength parameter x and the index of refraction m (in general complex), one may use the Mie theory (see, for example, van de Hulst, 1957; Bohren and Huffman, 1983; Shah, 1977). However, for the restricted case of radiation scattered by a very large dielectric sphere, some simplification can be made (see, for example, van de Hulst, 1957; Irvine, 1963, and the references therein) on the basis of geometrical

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optics and diffraction (GOD). The essential conditions for GOD to be valid are that the size-to-wavelength parameter $x \gg 1$ and the central phase shift parameter $2x\,|m - 1| \gg 1$. Consequently, it is possible to separate the scattered radiation neatly into two distinct components due to (1) external and internal reflections as well as refractions at the interface between the sphere and the surrounding medium (assumed to be vacuum or air in the present case), and (2) diffraction around the edge of the sphere. Note that the phenomena of the ripple structure and the surface waves do not concern us here.

2. Analytical Formulation Based on Geometrical Optics and Diffraction

The recipe for calculating the asymmetry parameter and efficiency for radiation pressure for a dielectric sphere under the conditions of GOD has been given by van de Hulst (1957) and Irvine (1963). In what follows a modified version of the same expressions is given to facilitate computations. The subscripts/indices 1 and 2 refer to the states of polarization with the electric vectors perpendicular and parallel, respectively, to the plane of scattering.

Let s be the independent real variable in the closed interval $[0, 1]$. The index of refraction, another independent variable, is assumed to be real throughout and is denoted by m. Usually the range of m is $[1, \infty]$.

Define

$$\alpha = 1 - s\,, \tag{2}$$

$$\beta = m^2 - s\,, \tag{3}$$

$$\gamma = 2\,|\sqrt{\alpha\beta}|\,, \tag{4}$$

$$\delta = \left(\frac{2s}{m^2}\right) - 1\,. \tag{5}$$

The relevant Fresnel reflection coefficients are represented by the simple formulae

$$r_1 = \frac{\alpha + \beta - \gamma}{\alpha - \beta} \tag{6}$$

and

$$r_2 = \frac{m^4\alpha + \beta - m^2\gamma}{m^4\alpha - \beta}\,. \tag{7}$$

It is convenient to further define the auxiliary quantities A_i, B_i, and C_i as

$$m^2 A_i = 4r_i^4\beta(2s - 1)\,, \quad i = 1, 2\,; \tag{8}$$

$$B_i = (1 - r_i^2)^2\left[\delta(2s - 1) + \frac{2s\gamma}{m^2}\right], \quad i = 1, 2\,; \tag{9}$$

and

$$C_i = 1 - 2\delta r_i^2 + r_i^4\,, \quad i = 1, 2\,. \tag{10}$$

Now the integrals g_1 and g_2 in the notations of van de Hulst (1957) and Irvine (1963) are given by

$$g_i = \int_0^1 \{(A_i + B_i)/C_i\}\,\mathrm{d}s\,, \quad i = 1, 2\,. \tag{11}$$

The average of g_i's is obtained from

$$g = (g_1 + g_2)/2\,. \tag{12}$$

Finally, the asymmetry parameter becomes

$$\langle \cos\theta \rangle = 0.5(1 + g)\,. \tag{13}$$

In general, the efficiency for radiation pressure (Q_{pr}), for arbitrary x and complex m, is expressed by

$$Q_{pr} = Q_{\text{ext}} - Q_{\text{sca}}\langle \cos\theta \rangle\,, \tag{14}$$

where Q_{ext} is the extinction efficiency. But in the present case of very large dielectric sphere, this reduces to

$$Q_{pr} = 1 - g\,. \tag{15}$$

The significance of the efficiency for radiation pressure is that the scatterer is acted upon by the force in the direction of propagation of the incident radiation with intensity I_0. This force is given by

$$F = I_0(\pi a^2/c)Q_{pr}\,, \tag{16}$$

where c is the velocity of light. As mentioned by van de Hulst (1957), the scatterer can also experience a component of force perpendicular to this direction and possibly a torque also.

Note that, for a very large dielectric sphere with arbitrary m, as x tends to infinity in the asymptotic limit of geometrical optics, one obtains extinction and scattering efficiencies $Q_{\text{ext}} = Q_{\text{sca}} = 2$, absorption efficiency $Q_{\text{abs}} = 0$, and albedo = 1.0. However, according to Mie theory calculations, it turns out that the back-scattering efficiency Q_{back} for large dielectric sphere, may oscillate drastically with very large amplitudes depending on the size-to-wavelength parameters ($x \gg 1$) and the index of refraction (see, for example, Table IV). Initially, Q_{back} increases as m increases from $m = 1$. The maximum of Q_{back} is attained at the index of refraction $m \simeq 1.8$ (Kerker, 1969). As m goes on increasing still further, the trend is expected to reverse, finally, to conform to the case of perfectly reflecting/conducting sphere ($m = \infty$). It may be mentioned that the oscillations in the back-scattering efficiency and other scattering parameters are damped out considerably in the case of absorbing sphere with complex index of refraction. Consequently, the geometrical optics limit is approached at much smaller values of the size-to-wavelength parameter as the absorptivity increases.

Some useful special cases of $s = 0$ and 1 and $m = 1$ and ∞ can be treated analytically to obtain r_1, r_2, g_1, g_2, etc., without resort to computers. The resulting quantities are summarized in Table I.

TABLE I

The Fresnel reflection coefficients (r_1 and r_2), asymmetry parameter ($\langle \cos\theta \rangle$), efficiency for radiation pressure (Q_{pr}) and related quantities for the limiting cases of $s = 0$ and 1 as well as $m = 1$ and ∞ for very large dielectric spheres

Derived quantity	Arbitrary index of refraction m		Arbitrary $s = [0, 1]$	
	$s = 0$	$s = 1$	$m = 1$	$m = \infty$
r_1	$\frac{1-m}{1+m}$	-1	0	-1
r_2	$-r_1$	-1	0	1
$A_1 = A_2$	$-4r_1^4$	$4(m^2-1)/m^2$	0	$4(2s-1)$
$B_1 = B_2$	$(1-r_1^2)^2$	0	1	0
$C_1 = C_2$	$(1+r_1^2)^2$	A_1	1	4
$g_1 = g_2 = g$	–	–	1	0
$\langle \cos\theta \rangle$	–	–	1	0.5
Q_{pr}	–	–	0	1

3. Numerical Results and Discussion

The results of calculations, based on Equations (2)–(15) using Simpson's rule with $n = 1001$ integration points at uniform step size of $\Delta s = 0.001$ in the interval $s = [0, 1]$ and index of refraction in the interval $m = [1, 5]$, are found to be in exact agreement with those of Irvine (1963). Figure 1 shows the variation of the asymmetry parameter ($\langle \cos\theta \rangle$) and the efficiency for radiation pressure (Q_{pr}) with index of refraction in the range [1, 5]. Apparently $\langle \cos\theta \rangle$ decreases monotonically from 1 to about 0.5 and Q_{pr} increases monotonically from 0 to about 1 as m increases from 1 to 5. It looks as though both $\langle \cos\theta \rangle$ and Q_{pr} would continue to show the monotonic trends for $m > 5$. To verify this aspect, the calculations were extended for $m > 5$ out of curiosity. Some of these results for $m = 1$ to 1000 at selected intervals are presented in Table II. These results were obtained by considering uniform integration step-size $\Delta s = 0.00002$. Note that Asym in the tables represents asymmetry parameter $\langle \cos\theta \rangle$. Table II indicates that a resonance feature occurs in the asymmetry parameter as well as in the efficiency for radiation pressure. One can say roughly that $\langle \cos\theta \rangle$ attains a minimum value in the neighbourhood of $m \simeq 11.0$. Correspondingly Q_{pr} reaches a maximum value. Figure 2 displays the plots for $\langle \cos\theta \rangle$ and Q_{pr} as function of m. These results were obtained by considering the index of refraction in the interval $m = 5(1)50$. The shallow broad resonance features in $\langle \cos\theta \rangle$ and Q_{pr} are now conspicuous.

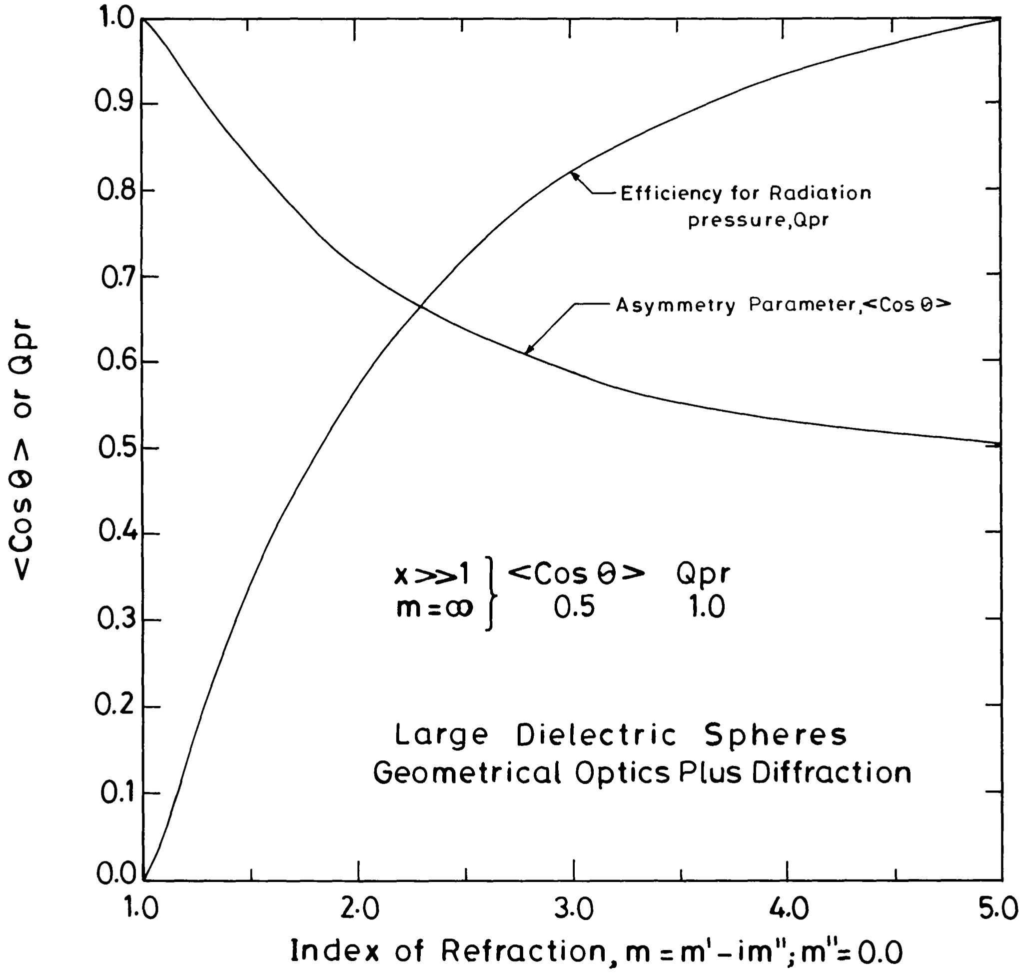

Fig. 1. Asymmetry parameter ($\langle\cos\theta\rangle$) and efficiency for radiation pressure (Q_{pr}) versus index of refraction (m) of homogeneous, isotropic, and smooth dielectric spheres. The range of m is $[1, 5]$.

In order to pinpoint the exact value of m at which the minimum of $\langle\cos\theta\rangle$ and the maximum of Q_{pr} occur, a series of detailed calculations were undertaken by varying the number of integration points, viz., $n = 1001$, 10001, 50001, and 100001. This implies uniform step-sizes of $\Delta s = 0.001$, 0.0001, 0.00002, and 0.00001, respectively. The indices of refraction were chosen in the interval 11.0(0.001)11.5. The results of calculations for $\langle\cos\theta\rangle_{\min}$ and $\{Q_{pr}\}_{\max}$ are summarized in Table III. Note that $\langle\cos\theta\rangle_{\min}$ maintains the same extremum value for a range of m which depends on the step-size of integration. The same is true for $\{Q_{pr}\}_{\max}$. The accuracy of the results improves obviously with increase of n, i.e., decrease of the step-size in the numerical integration of Equations (11). For example, comparison of the cases with $n = 1001$ and $n = 10001$ shows that the results for the former case are accurate to 4 significant digits. Similarly the comparison between the cases with $n = 50001$ and 100001 indicate that, in the former case, the results for $\langle\cos\theta\rangle_{\min}$ and $\{Q_{pr}\}_{\max}$ are accurate to 7 and 6 digits, respectively.

TABLE II

Scattering parameters for very large dielectric spheres based on geometrical optics and diffraction. Number of integration ponits = 50001. Asym = asymmetry parameter, Q_{pr} = efficiency for radiation pressure, index of refraction = m, and g_1, g_2, g as defined in the text

m	g_1	g_2	g	Asym	Q_{pr}
0.1010E+01	0.998548E+00	0.998759E+00	0.998653E+00	0.999327E+00	0.134654E−02
0.1020E+01	0.995153E+00	0.996010E+00	0.995582E+00	0.997791E+00	0.441832E−02
0.1030E+01	0.990396E+00	0.992322E+00	0.991359E+00	0.995680E+00	0.864081E−02
0.1040E+01	0.984592E+00	0.987981E+00	0.986286E+00	0.993143E+00	0.137136E−01
0.1050E+01	0.977958E+00	0.983167E+00	0.980562E+00	0.990281E+00	0.194379E−01
0.1100E+01	0.936994E+00	0.955299E+00	0.946146E+00	0.973073E+00	−0.538538E−01
0.1150E+01	0.889985E+00	0.925010E+00	0.907497E+00	0.953749E+00	0.925027E−01
0.1200E+01	0.841915E+00	0.894332E+00	0.868123E+00	0.934062E+00	0.131877E+00
0.1250E+01	0.795109E+00	0.863866E+00	0.829488E+00	0.914744E+00	0.170512E+00
0.1300E+01	0.750663E+00	0.833789E+00	0.792226E+00	0.896113E+00	0.207774E+00
0.1330E+01	0.725338E+00	0.815950E+00	0.770644E+00	0.885322E+00	0.229356E+00
0.1333E+01	0.722863E+00	0.814175E+00	0.768519E+00	0.884260E+00	0.231481E+00
0.1350E+01	0.709042E+00	0.804147E+00	0.756594E+00	0.878297E+00	0.243406E+00
0.1400E+01	0.670378E+00	0.774952E+00	0.722665E+00	0.861332E+00	0.277335E+00
0.1450E+01	0.634623E+00	0.746211E+00	0.690417E+00	0.845209E+00	0.309583E+00
0.1500E+01	0.601640E+00	0.717933E+00	0.659787E+00	0.829893E+00	0.340213E+00
0.1600E+01	0.543239E+00	0.662840E+00	0.603040E+00	0.801520E+00	0.396960E+00
0.1700E+01	0.493602E+00	0.609838E+00	0.551720E+00	0.775860E+00	0.448280E+00
0.1800E+01	0.451245E+00	0.559099E+00	0.505172E+00	0.752586E+00	0.494828E+00
0.1900E+01	0.414896E+00	0.510768E+00	0.462832E+00	0.731416E+00	0.537168E+00
0.2000E+01	0.383506E+00	0.464941E+00	0.424223E+00	0.712112E+00	0.575777E+00
0.3000E+01	0.215105E+00	0.138140E+00	0.176623E+00	0.588311E+00	0.823377E+00
0.4000E+01	0.149482E+00	−0.193489E−01	0.650664E−01	0.532533E+00	0.934934E+00
0.5000E+01	0.115030E+00	−0.933501E−01	0.108399E−01	0.505420E+00	0.989160E+00
0.6000E+01	0.937587E−01	−0.127852E+00	−0.170467E−01	0.491477E+00	0.101705E+01
0.7000E+01	0.792686E−01	−0.143014E+00	−0.318728E−01	0.484064E+00	0.103187E+01
0.8000E+01	0.687337E−01	−0.148336E+00	−0.398011E−01	0.480099E+00	0.103980E+01
0.9000E+01	0.607134E−01	−0.148519E+00	−0.439026E−01	0.478049E+00	0.104390E+01
0.1000E+02	0.543947E−01	−0.145999E+00	−0.458020E−01	0.477099E+00	0.104580E+01
0.1100E+02	0.492829E−01	−0.142087E+00	−0.464021E−01	0.476799E+00	0.104640E+01
0.1200E+02	0.450596E−01	−0.137509E+00	−0.462248E−01	0.476888E+00	0.104622E+01
0.2000E+02	0.267998E−01	−0.102311E+00	−0.377558E−01	0.481122E+00	0.103776E+01
0.3000E+02	0.178173E−01	−0.750174E−01	−0.286001E−01	0.485700E+00	0.102860E+01
0.4000E+02	0.133500E−01	−0.588450E−01	−0.227475E−01	0.488626E+00	0.102275E+01
0.5000E+02	0.106752E−01	−0.483208E−01	−0.188228E−01	0.490589E+00	0.101882E+01
0.6000E+02	0.889383E−02	−0.409603E−01	−0.160332E−01	0.491983E+00	0.101603E+01
0.7000E+02	0.762216E−02	−0.355335E−01	−0.139557E−01	0.493022E+00	0.101396E+01
0.8000E+02	0.666875E−02	−0.313706E−01	−0.123509E−01	0.493825E+00	0.101235E+01
0.9000E+02	0.592739E−02	−0.280777E−01	−0.110751E−01	0.494462E+00	0.101108E+01
0.1000E+03	0.533440E−02	−0.254085E−01	−0.100371E−01	0.494981E+00	0.101004E+01
0.2000E+03	0.266680E−02	−0.130127E−01	−0.517294E−02	0.497414E+00	0.100517E+01
0.3000E+03	0.177782E−02	−0.873928E−02	−0.348073E−02	0.498260E+00	0.100348E+01
0.4000E+03	0.133335E−02	−0.657593E−02	−0.262129E−02	0.498689E+00	0.100262E+01
0.5000E+03	0.106668E−02	−0.526978E−02	−0.210155E−02	0.498949E+00	0.100210E+01
0.6000E+03	0.888894E−03	−0.439588E−02	−0.175350E−02	0.499123E+00	0.100175E+01
0.7000E+03	0.761908E−03	−0.377028E−02	−0.150419E−02	0.499248E+00	0.100150E+01
0.8000E+03	0.666669E−03	−0.330039E−02	−0.131686E−02	0.499342E+00	0.100132E+01
0.9000E+03	0.592594E−03	−0.293455E−02	−0.117098E−02	0.499415E+00	0.100117E+01
0.1000E+04	0.533334E−03	−0.264166E−02	−0.105416E−02	0.499473E+00	0.100105E+01

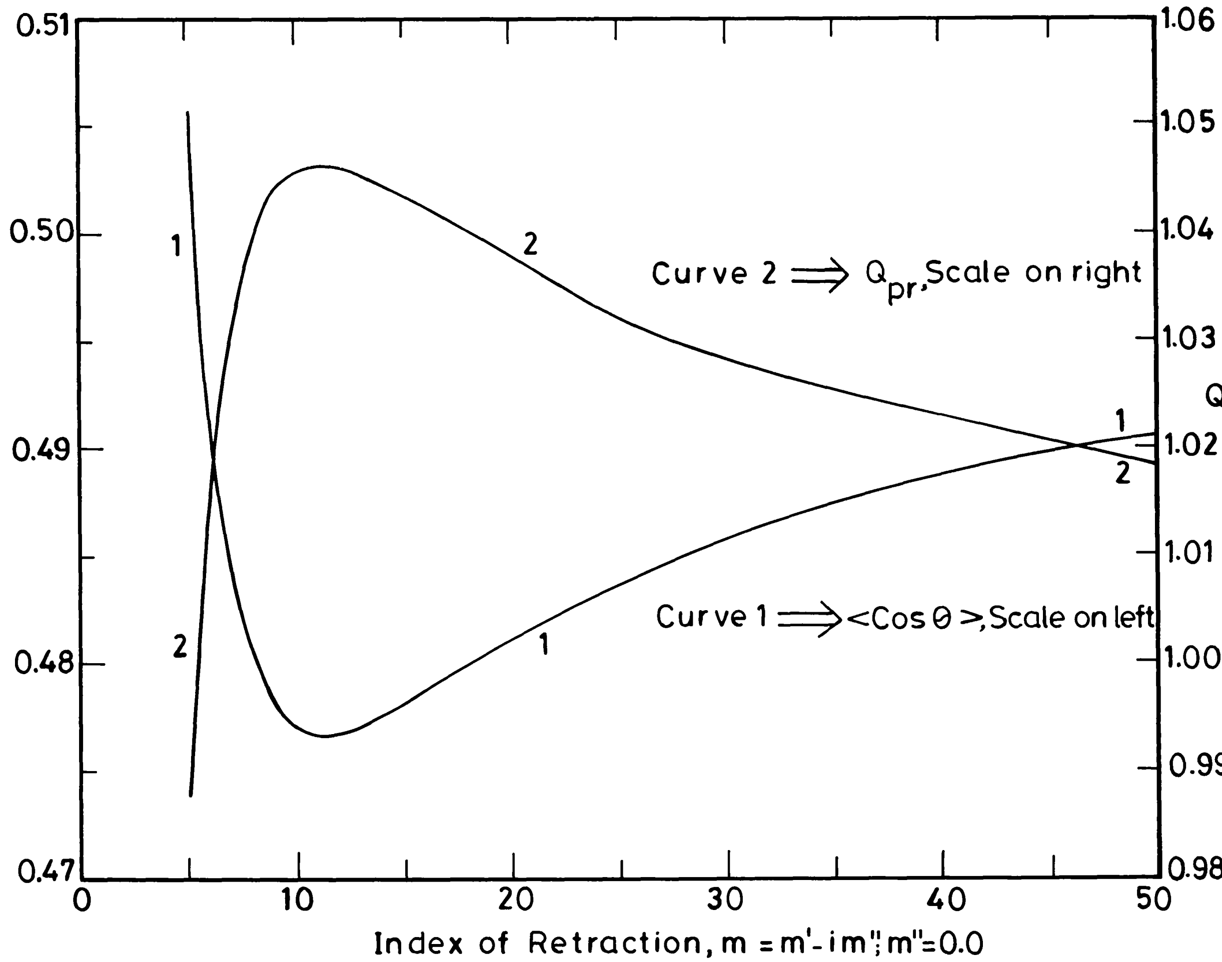

Fig. 2. The same as Figure 1 but the interval of m is [5, 50]. Curve 1 is for $\langle \cos\theta \rangle$ with vertical scale on the left-hand side. Curve 2 corresponds to Q_{pr} with vertical scale on the right-hand side.

TABLE III

$\langle \cos\theta \rangle_{\min}$ and $\{Q_{pr}\}_{\max}$ with corresponding ranges of index of refraction (m) of very large dielectric spheres as function of step size (Δs) in the numerical integration of Equation (11)

Number of integration points n	Step size of integration Δs	$\langle \cos\theta \rangle_{\min}$	Range of m for $\langle \cos\theta \rangle_{\min}$	$\{Q_{pr}\}_{\max}$	Range of m for $\{Q_{pr}\}_{\max}$
1000	0.001	0.47682153	$11.189 \le m \le 11.198$	1.0463569	$11.178 \le m \le 11.209$
10001	0.0001	0.47679257	$11.196 \le m \le 11.207$	1.0464149	$11.196 \le m \le 11.207$
50001	0.00002	0.47679173	$11.196 \le m \le 11.208$	1.0464165	$11.186 \le m \le 11.218$
100001	0.00001	0.47679167	$11.201 \le m \le 11.203$	1.0464167	$11.201 \le m \le 11.203$

TABLE IV

Extinction efficiency (Q_{ext}), asymmetry parameter (Asym), efficiency for radiation pressure (Q_{pr}), and backscattering efficiency (Q_{back}) as function of the size-to-wavelength parameter (x) according to the Mie theory.
NN = the number of Mie coefficients considered from 1 to NN

INDEX OF REFRACTION: REAL PART=11.202, IMAGINARY PART=0.000

x	Qext	Asym	Qpr	Qback	NN
.100E-04	.254263E-19	.429969E-09	.254263E-19	.381394E-19	2
.100E-03	.254264E-15	.429969E-07	.254264E-15	.381396E-15	2
.100E-02	.254264E-11	.429974E-05	.254263E-11	.381393E-11	2
.100E-01	.254293E-07	.430442E-03	.254184E-07	.381109E-07	3
.100E+00	.257884E-03	.483078E-01	.245426E-03	.349243E-03	3
.100E+01	.964813E+00	-.257175E+00	.121294E+01	.153832E+01	5
.100E+02	.218623E+01	.472743E+00	.115271E+01	.122852E+01	18
.200E+02	.210169E+01	.425986E+00	.120640E+01	.533258E+01	29
.400E+02	.215436E+01	.494377E+00	.108930E+01	.218724E+01	51
.600E+02	.199765E+01	.475164E+00	.104844E+01	.285320E+01	72
.800E+02	.204210E+01	.456581E+00	.110972E+01	.197594E+02	93
.100E+03	.201349E+01	.458644E+00	.109001E+01	.371644E+02	115
.200E+03	.207116E+01	.483329E+00	.107011E+01	.365024E+01	218
.300E+03	.202455E+01	.475554E+00	.106177E+01	.113077E+02	320
.400E+03	.203496E+01	.476138E+00	.106604E+01	.207613E+01	422
.500E+03	.203797E+01	.474793E+00	.107036E+01	.249550E+02	524
.600E+03	.201797E+01	.476039E+00	.105734E+01	.560037E+02	624
.700E+03	.200615E+01	.474993E+00	.105324E+01	.491730E+01	725
.800E+03	.202058E+01	.477915E+00	.105491E+01	.696662E+02	827
.100E+04	.201515E+01	.475310E+00	.105733E+01	.310072E+00	1029
.110E+04	.200227E+01	.472733E+00	.105573E+01	.163338E+03	1129
.120E+04	.201167E+01	.475878E+00	.105436E+01	.511452E+02	1231
.130E+04	.201357E+01	.473308E+00	.106053E+01	.167949E+03	1331
.140E+04	.201163E+01	.471955E+00	.106223E+01	.768452E+02	1431
.150E+04	.201383E+01	.474465E+00	.105834E+01	.111071E+03	1533
.160E+04	.201658E+01	.476526E+00	.105563E+01	.151731E+03	1634
.170E+04	.202184E+01	.479894E+00	.105157E+01	.197070E+03	1734
.180E+04	.201657E+01	.477395E+00	.105387E+01	.166809E+03	1834
.190E+04	.200317E+01	.473162E+00	.105535E+01	.141271E+03	1934
.200E+04	.200737E+01	.473724E+00	.105643E+01	.166487E+03	2034
.250E+04	.200046E+01	.473886E+00	.105247E+01	.380850E+03	2538
.300E+04	.200639E+01	.475084E+00	.105319E+01	.113837E+03	3040
.350E+04	.200630E+01	.475345E+00	.105262E+01	.787164E+02	3542
.400E+04	.200115E+01	.473677E+00	.105325E+01	.214076E+03	4043
.450E+04	.200531E+01	.477093E+00	.104859E+01	.306128E+03	4545
.500E+04	.200152E+01	.475534E+00	.104973E+01	.248476E+03	5045
.600E+04	.200593E+01	.477286E+00	.104853E+01	.152996E+03	6050
.700E+04	.200333E+01	.476491E+00	.104876E+01	.296640E+03	7050
.800E+04	.200723E+01	.476911E+00	.104996E+01	.642665E+03	8052
.900E+04	.200782E+01	.477359E+00	.104937E+01	.112426E+03	9055
.100E+05	.200423E+01	.477505E+00	.104720E+01	.719043E+03	10055
.150E+05	.200439E+01	.476789E+00	.104872E+01	.829807E+03	15065
.200E+05	.200333E+01	.477379E+00	.104698E+01	.691258E+03	20068
.250E+05	.200185E+01	.476429E+00	.104811E+01	.449494E+03	25073
.300E+05	.200104E+01	.476916E+00	.104671E+01	.101654E+04	30077
.400E+05	.200197E+01	.477118E+00	.104679E+01	.297154E+04	40083
.500E+05	.200285E+01	.477293E+00	.104690E+01	.445212E+04	50088
.600E+05	.200147E+01	.476847E+00	.104707E+01	.303510E+04	60095
.700E+05	.200069E+01	.476696E+00	.104697E+01	.183848E+04	70097
.800E+05	.200163E+01	.477125E+00	.104660E+01	.524984E+04	80102
.900E+05	.200178E+01	.477183E+00	.104656E+01	.920077E+04	90104
.100E+06	.200040E+01	.476760E+00	.104669E+01	.673339E+04	100109

The minimum in the asymmetry parameter $\langle \cos\theta \rangle_{\min}$ and the maximum in the efficiency for radiation pressure $\{Q_{pr}\rangle_{\max}$, on the basis of the last line in Table III can be said to occur in the interval $11.201 \lesssim m \lesssim 11.203$. The results correct to six significant digits can be set at $\langle \cos\theta \rangle_{\min} = 0.476792$ and $\{Q_{pr}\}_{\max} = 1.04642$.

Incidentally, it is worthwhile to compare these results with the exact calculations based on the Mie theory for index of refraction $m = 11.202$ and arbitrary size-to-wavelength parameter (x). The resulting scattering parameters according to the Mie theory are listed in Table IV for $x = 10^{-5}$ to 10^5 at selected intervals. This table shows somewhat asymptotic but exact values of extinction efficiency (Q_{ext}), asymmetry parameter (Asym $\equiv \langle \cos\theta \rangle$), efficiency for radiation pressure (Q_{pr}) and back-scattering efficiency (Q_{back}) all for x up to 10^5. Such results are useful in estimating the minimum value of x, say $x_{\min}$, at which geometrical optics begins to be applicable for each of the scattering parameter. Note that the precise value of $x_{\min}$ for dielectric spheres is difficult to determine because of the oscillations in Q_{ext}, $\langle \cos\theta \rangle$, Q_{pr}, Q_{back}, etc. For example, consider the particular case of the dielectric sphere in Table IV and set the tolerance of accuracy at 3 significant digits. From Table IV, one can infer roughly that $x_{\min} \simeq 2500$, 10000, and 30000 for Q_{pr}, Q_{ext}, and $\langle \cos\theta \rangle$, respectively. However, as mentioned earlier, Q_{back} does not seem to attain an asymptotic value even for x as large as 100000.

4. Conclusions

It has been found that the asymmetry parameter and the efficiency for radiation pressure, for scattering by a very large dielectric sphere, do not follow monotonic pattern as function of the index of refraction m. Rather they show shallow broad resonances with extrema in the interval of index of refraction defined by $11.201 \lesssim m \lesssim 11.203$. The minimum of $\langle \cos\theta \rangle$ is equal to 0.476792 and the maximum of Q_{pr} is equal to 1.04642. It would be an interesting feat if one can prove these results analytically. Ultimately as m tends to infinity, the results for $\langle \cos\theta \rangle$ and Q_{pr} are expected to conform to the limiting case of perfectly conducting sphere.

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