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An Extended Matched Filtering Method to Detect Periodicities in a Rough Grating for Extremely Large Roughness

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Abstract. Matched filtering algorithm has beenshown by us to be an effective method to detect hidden periodicities in a rough grating, extending detectability to regimes of higher roughness than was hitherto known. The above matched filtering is further extended here to even 'rougher' regimes by introducing a new filtering function $\chi_a(\nu_x)$,-demonstrating improved detectability for different wavelength of light.

1. Introduction

Detection of phase structures, hidden behind large randomness in a challenging problem in physics and technology. If the random phase fluctuations of the object in the XY-plane follow a Gaussian correlation over a distance r_0 then it is known to be impossible to detect a hidden phase grating of wavelength Λ, for $r_0/\Lambda \leq 0.33$ from purely intensity measurements, detection is even worse for Cauchy or Lorentz type correlation of the phase randomness.

Our recent matched filtering method work enables detection of Λ and the amplitude 'a' of the periodic part even for $r_0/\Lambda \sim 0.16$ from purely intensity observations, i.e. way below the Baltes limit (see for example, Dainty and Newman 1984). In the present paper we present an extended matched filtering method which greatly improves the detection of both Λ and 'a'.

The intensity of light scattered in any direction (θ_2, θ_3) where the incident direction is $(\theta_1, 0)$ is given by (Chatterjee, 2000)

$$< \rho\rho^* >= J_0^2(a\nu_z) + \Sigma J_n^2(a\nu_z)[f(\nu_x + nQ) + f(\nu_x - nQ)]$$

where the elevations in the grating are

$$\xi(x, y) = a \cos Qx + \delta\xi(x, y),$$

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while the function $f(\nu_x)$ is given by

$$f(\nu_x = (2\pi/\lambda)\int \exp\{-\nu_z^2[1-f(r)]\}J(\nu_x r)rdr$$

with $<\delta\xi(x_1,y_1)\delta\xi(x_2,y_2)>=\sigma f(r)$ and the scattering wave vectors are, $\nu_X = k(\sin\theta_1 - \sin\theta_2\cos\theta_3)$, $\cdot\nu_y = k\sin\theta_1\sin\theta_3\cdot\nu_z = -k(\cos\theta_1+\cos\theta_2)$ with $k=2\pi/\lambda$, λ being the wavelength of light. We shall assume the correlation function to be $f(r)=\exp[(-r/l)^\theta]$ where r is the distance between the points (x_1,y_1) and (x_2,y_2).

2. Implementation of Matched Filtering

It is seen that reduction of r_0 broadens the central peak, submerging the peaks for $n=\pm 1,2,3\ldots$. Considering the peaks to have the shape $f_a(\nu_x)=[1+c'\nu_x^2/2y]^y$ and defining

$$Z(\nu_x = \{<\rho^*(\nu_x)\rho(\nu_x)>/<\rho^*(0)\rho(0)>\}-f_a(\nu_x)$$

and $\chi(\nu_x)=Z(\nu_x)/Z_{\max}$, it is seen that

$$\chi(\nu_x)=\chi_N/\chi_D,$$

where

$$\chi_N = J_0^2[f(\nu_x)-f_a(\nu_x)]+J_1^2[f(\nu_x+Q)+f(\nu_x Q)2f_a(\nu_x)f(Q)]$$

and

$$X_D = J_0^2[f(Q)-f_a(Q^*)]+J_1^2[f(Q+Q^*)-2f_a(Q^*)f(Q)]$$

where $Z_{\max}$ occurs at the point $\nu_x=Q^*$. By seeping c' and y when $f_a(\nu_x)$ comes very close to $f(\nu_x)$ we shall have

$$\chi(\nu_x)\approx\chi(\nu_x)=\{f(\nu_x+Q)+f(\nu_x-Q)2f_a(\nu_x)f(Q)\}/[d(Q+Q^*)-2fa(Q^*)fQ)].$$

The matched filtering procedure involves matching c' and y in such a way that $\int[\chi(\nu_x)-\chi_z(\nu_x)]^2 d\nu_x$ is a minimum. We have implemented the above for $a = 0.056\ \lambda$, $\Lambda = 6.25$ microns, i.e. $Q = 10040\ \mathrm{cm}^{-1}$, $\sigma = 0.\ 16$ microns, $\theta = 1.0$, for different values of λ. The present extended matched filtering method yields the following results.

Wave length (λ)	$.r_0/\lambda$	C' in 10^8 cm^2	y	Q in 10^4 cm^2	a in Å	a (cal) in Å	Error in Q	Error in a
6943A	0.1949	4.10	1.55	1.02	395	343	1.6%	13%
631A	0.1619	2.85	1.55	1.05	360	304	4.5%	15%
5700A	0.1314	2.00	1.55	0.99	324	304	−1.5%	10%

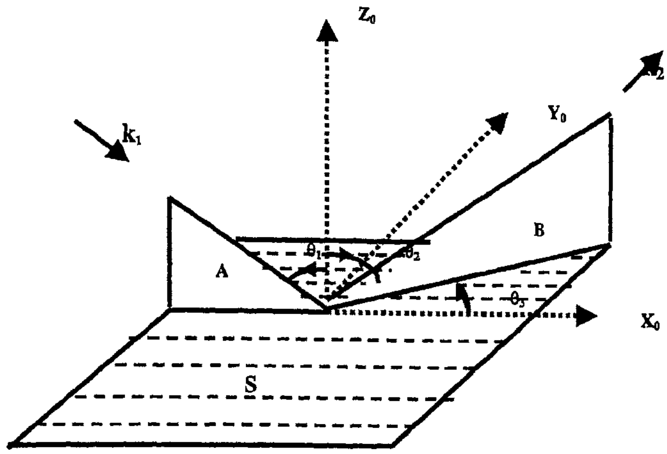

Figure 1. t_{l0} as a function of t_c for cases (a1), (a2), (a3), (c) and (d). Both times are in units of t_{ff}.

Numerical work has been done at three different wavelengths, in orer to examine the feasibility of the matched filter method at different wavelengths as also to see effectiveness of the method as r_0 decreases with decreasing λ but is also accompanined by the increasing amplitudes of the $n \neq 0$ peaks. It is seen that the present 'extended matched filtering' improves the detectability when compared to the matched filtering given earlier. Though with decrease in λ (also in r_0) the scatter increases, the method is capable of detecting even at $r_0/\Lambda \sim 0.13$, from purely intensity profile measurements to a limit about 1/3 of what was known to be feasible. In the case of astronomical seeing $\theta = 5/3$. The situation is more favorable than the $\theta = 1$ case considered above, detectability of structure in turbulence degraded images may be even less than $r_0/\Lambda \sim 0.13$ if the extended matched filtering method be employed.

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