# Search for periodicities in distribution of orbits of planets 

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#### Abstract

We investigate the distribution of extra-solar-system planets in order to search for periodicities in their distribution around their parent star. We use the data base of one hundred planets outside the solar system from www.exoplanets.org and compare it along with the solar system data. We show that distribution of planets is not arbitrary and has statistically significant peaks in the histogram of distance distribution. We then derive generalized parameters for planetary distribution around a strong central object. We show that there is a statistically significant indication that the distribution in planetary orbits depends on the $\mathrm{M}_{*} / A_{\text {planet }}^{2}$ where $\mathrm{M}_{*}$ is the mass of the star and $A_{\text {planet }}$ is the semi-major axis orbital distance of the planet.


Keywords : Extra-solar-system planets

## 1. Introduction

Gravitational field in a rotating cloud has an axial symmetry making it difficult to create periodicities in orbits of a proto-stellar and proto-planetary disk. Recent Hubble pictures of disks around stars also seem to indicate that the disks where planets can be formed seem to be symmetrical. As a result, it is often assumed that orbital distances are arbitrary. Empirical rules like the Titius Bode Law are taken to be more a mathematical exercise rather than a manifestation of specific physical processes. However the discovery of planets around other stars have once more brought the question of distribution of planets around a star into new focus.

Hayashi (1981) has theoretically analyzed the problems of magnetic fields in proto-planetary disk in thermal and gravitational equilibrium. He has shown that a surface density distribution

[^0]in the proto solar system disk to be of the form $k r^{-1.5}\left(\mathrm{~g} \mathrm{~cm}^{-2}\right)$ (where $k=7.1$ for $0.35 \mathrm{AU}<r$ $<2.7 \mathrm{AU}$ and $k=30$ for $2.7 \mathrm{AU}<r<36 \mathrm{AU}$ ) fits the mass distribution of the planets. He has shown that for the inner planets, the magnetic fields decay very rapidly since the high density of solid material prevents continuous ionization of the material by cosmic rays to sustain and magnify the magnetic field. However, contrary to this, for regions around Uranus ( $r>36 \mathrm{AU}$ ) there would be significant amplification of magnetic fields. Thommes et al. (2002) have suggested that Uranus and Neptune could not have been created at their present location but are a result of the interplay between giant planets originally close to the orbit of Jupiter.

Li (2002) has simulated the collapse of a proto-planetary disk and shown that self gravitating disks will have a certain embedded magnetic field which can fragment a disk into smaller rings from which planets can form. Ozernoy et al. (2000) have used numerical simulations to show that in the presence of a companion planet around a star, the proto-planetary disk will naturally break into definite resonant patterns which can again lead to the formation of planets. Patzold and Ravder (2002) and Kuchner and Lecar (2002) have argued that around any star there is an exclusion zone of the order of 0.1 AU . Orbit of any planet within this distance would rapidly decay in the central star.

Malhotra (2002) has argued that if two planets are in circular orbit, their close passage would induce an increase in eccentricity of planets forcing them to move in highly elliptical orbits even for a wide range of apparently stable initial conditions. Conversely Marcy et al. (2001) have shown that the 2 planets around GJ 876 are in a stable circular orbit of $2: 1$ resonance. Li et al. (2002) have simulated different coplanar and non planar configuration for GJ 876 and found that planets in $2: 1$ configuration have a very robust stable orbit.

In the present paper we attempt to empirically search for resonant orbits in planetary movement over the set of data of over 100 planets outside the solar system using distribution in planets and moons in the solar system.

## 2. Formulation of problem

The problem with proto-planetary disks breaking into bands that eventually coalesce into planets has proved difficult to crack in view of the strongly axi-symmetric nature of the problem. The most probable solutions seem to be to either assume an arm like structure similar to spiral galaxies or a very complex mixture of gravitational potential whose radial symmetry in the rotational plane is broken by magnetic field and radiation (ionization) field. Both are notoriously difficult to model in view of the high degree of sensitivity to several parameters such as composition, density, temperature gradients etc. The idea of assimilation of planetesimal boulders into planets also seems to disagree on the time scales required for forming the planets.

In the present study therefore, we take statistical approach in order to check if each planetary system is unique or if there is an underlying symmetry.

We use a generalized form of a functional relation between the mass of the central object and the distance of the planet. We assume the form:

$$
\begin{equation*}
a_{n}=a_{0} K^{n} \tag{1}
\end{equation*}
$$

Here $a_{n}$ is the orbital distance of $n^{\text {th }}$ planet and $a_{0}$ and $K$ are arbitrary constants. This is generally treated as a generalized form of the Titius Bode Law (Graner and Dubrulle, 1994, Dubrulle and Graner, 1994). This generalized derivation of this law is based on an assumption of rotational scale invariance. However, the derivation can be done in several ways with an arbitrary set of constants to fit the equation. The law is generally applied to various systems within the solar system including satellites of planets. In each case the constants of the equation 1 are different and hence the Titius Bode law is generally not assumed to indicate any underlying physical law.

The studies made so far have been from either the solar system or the satellites of planets. In that case, the number of variables such as the mass of the central object etc. available to check the statistical significance is small. Hence the problem has not been properly investigated. With new data now available, it is possible to investigate the possible periodicities in orbits of objects around massive objects.

We therefore try to investigate the planetary orbit distributions for extra-solar system planets through a generalized pattern search. The data available on planets outside the solar system is given in Tables 1 and 2. The data for the solar system bodies is given in the Tables 3 and 4.

We assume that the stability of any planetary system is controlled by gravity and that the mass of the star is fundamental to its stability. We therefore do not use the data where star mass is not known. We search for empirical parameters for possible resonance patterns keeping stellar mass as the central parameter. We also do not use the data for systems where the central object is a degenerate one since clearly, such objects have gone through far more evolution than solar system like objects.

## 3. Data Analysis

We have taken the data for 100 planets around main sequence stars from the web site www.exoplanets.org. From the data we have rejected 7 stars for which mass was not available. The full data used in the analysis is given in Tables 1-4. In Tables 1 and 2 the data is given for the extra-solar-system planets while in Tables 3 and 4 the data is given for the solar system planets and natural satellites around planets. In column 1 the star name is given along with its mass, and spectral type. We then give the various planet data such as planet mass $\mathrm{Mp} \sin (\mathrm{i})$, period $P$ (days) and distance $D$ in $A U$. In the Figure 1 we have ploted the orbital distance of planets against the mass of the parent star. The data for solar system objects and their satellites systems is also plotted.

Table 1. Planets outside the solar system with eccentricity less than 0.5 and stars with known mass.

| STAR name | $\begin{array}{r} \text { Mass } \\ \left(\mathrm{M}_{\text {sun }}\right) \end{array}$ | $\begin{array}{r} M_{\mathrm{M} \cdot \sin (\mathrm{I})} \\ \left(\mathrm{M}_{\text {Jupiter }}\right) \end{array}$ | Semi Major axis <br> (AU) | Period <br> (days) | Eccentricity |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HD49674 | 1 | 0.12 | 0.0568 | 4.948 | 0 |
| HD76700 | 1 | 0.197 | 0.049 | 3.971 | 0 |
| HD 16141 | 1 | 0.215 | 0.35 | 75.82 | 0.28 |
| HD168746 | 0.92 | 0.23 | 0.065 | 6.403 | 0.081 |
| HD46375 | 1 | 0.249 | 0.041 | 3.024 | 0 |
| HD83443 | 0.79 | 0.35 | 0.038 | 2.9861 | 0.08 |
| HD108147 | 1.27 | 0.41 | 0.104 | 10.901 | 0.498 |
| HD75289 | 1.05 | 0.42 | 0.046 | 3.51 | 0.054 |
| 51 Peg | 1.05 | 0.47 | 0.05 | 4.2293 | 0 |
| BD 103166 | 1.1 | 0.48 | 0.046 | 3.487 | 0 |
| HD6434 | 1 | 0.48 | 0.15 | 22.09 | 0.3 |
| HD187123 | 1.06 | 0.52 | 0.042 | 3.097 | 0.03 |
| HD209458 | 1.05 | 0.69 | 0.045 | 3.524738 | 0 |
| Ups And | 1.3 | 0.71 | 0.059 | 6.617 | 0.034 |
| Ups And | 1.3 | 2.11 | 0.83 | 241.2 | 0.18 |
| Ups And | 1.3 | 4.61 | 2.5 | 1266.6 | 0.41 |
| Epsilon Eridani | 0.8 | 0.86 | 3.3 | 2502.1 | 0.608 |
| HD 38529 | 1.39 | 0.67 | 0.12 | 14.3007 | 0.312 |
| HD 38529 | 1.39 | 11.3 | 3.51 | 2189.5284 | 0.34 |
| HD 4208 | 0.93 | 0.81 | 1.69 | 829 | 0.04 |
| HD 179949 | 1.24 | 0.84 | 0.045 | 3.093 | 0.05 |
| 55 Cnc | 1.03 | 0.84 | 0.11 | 14.66 | 0.04 |
| $55 \mathrm{Cnc}{ }^{*}$ | 1.03 | 0.21 | 0.24 | 44.28 | 0.34 |
| 55 Cnc | 1.03 | 4.26 | 6.01 | 5300 | 0.16 |
| HD 82943 | 1.05 | 0.88 | 0.73 | 221.6 | 0.54 |
| HD 82943 | 1.05 | 1.63 | 1.16 | 444.6 | 0.41 |
| HD 121504 | 1 | 0.89 | 0.32 | 64.6 | 0.13 |
| HD 114783 | 0.92 | 0.9 | 1.2 | 501 | 0.1 |
| HD 114729 | 0.93 | 0.9 | 2.08 | 1136 | 0.33 |
| HD 37124 | 0.91 | 0.86 | 0.585 | 153 | 0.1 |
| HD 37124* | 0.91 | 1.01 | 2.95 | 1942 | 0.4 |
| HD 114386 |  | 0.99 | 1.62 | 872 | 0.28 |
| HD 150706 |  | 1 | 0.82 | 264.9 | 0.38 |
| HD 147513 | 0.92 | 1 | 1.26 | 540.4 | 0.52 |
| HD 20367 |  | 1.07 | 1.25 | 500 | 0.23 |
| HD 130322 | 0.79 | 1.08 | 0.088 | 10.724 | 0.048 |
| $\rho \mathrm{CrB}$ | 0.95 | 1.1 | 0.23 | 39.645 | 0.028 |
| HD 52265 | 1.13 | 1.13 | 0.49 | 118.96 | 0.29 |
| Gl 777A | 0.9 | 1.15 | 3.65 | 2613 | 0 |
| HD 223084 | 1.05 | 1.18 | 0.41 | 101.06 | 0.48 |
| HD 177830 | 1.17 | 1.28 |  | 391 | 0.43 |
| HD 217107 | 0.98 | 1.28 | 0.07 | 7.11 | 0.14 |
| HD 210277 | 0.99 | 1.28 | 1.097 | 437 | 0.45 |
| HD 142 | 1.1 | 1.36 | 0,98 | 338 | 0.37 |
| HD 27442 | 1.2 | 1.43 | 1,18 | 423 | 0.02 |
| 16 CygB | 1.01 | 1.5 | 1,7 | 804 | 0.67 |
| HD 74156 | 1.05 | 1.56 | 0.276 | 51.61 | 0.649 |
| HD 74156 | 1.05 | 7.5 | 4.47 | 2300 | 0.395 |
| HD 134987 | 1.05 | 1.58 | 0.78 | 260 | 0.25 |
| HID 4203 | 1.06 | 1.64 | 1.09 | 406 | 0.53 |

Table 2. Same as Table 1.

| STAR name | $\begin{gathered} \text { Mass } \\ \left(\mathrm{M}_{\text {sun }}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \text { M.sin(I) } \\ \left(\mathrm{M}_{\text {Jppter }}\right) \\ \hline \end{gathered}$ | Semi Major axis <br> (AU) | $\begin{aligned} & \text { Period } \\ & \text { (days) } \end{aligned}$ | Eccentricity |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HD 108874 | 1 | 1.65 | 1.07 | 401 | 0.2 |
| HD 68988 | 1.2 | 1.9 | 0.071 | 6.276 | 0.14 |
| HD 160691 | 1.08 | 1.7 | 1.5 | 638 | 0.31 |
| HD 160691 | 1.08 | 1 | 2.3 | 0 | 0.8 |
| HD 19994 | 1.35 | 2 | 1.3 | 454 | 0.2 |
| HD 216437 | 1.07 | 2.1 | 2.7 | 1294 | 0.34 |
| Gliese 876 | 0.32 | 1.98 | 0.21 | 61.02 | 0.27 |
| Gliese 876 | 0.32 | 0.56 | 0.13 | 30.1 | 0.12 |
| HD 8574 |  | 2.23 | 0.76 | 228.8 | 0.4 |
| HR 810 |  | 2.26 | 0.925 | 320.1 | 0.161 |
| 47 Uma | 1.03 | 2.41 | 2.110950 .96 |  |  |
| 47 Uma | 1.03 | 0.76 | 3.73 | 2594 | 0.1 |
| HD 23079 | 1.1 | 2.54 | 1.48 | 627.3 | 0.06 |
| HD 12661 | 1.07 | 2.47 | 0.85 | 263.558 | 0.347 |
| HD 12661 | 1.07 | 1.86 | 2.61 | 1407 | 0.22 |
| HD 72659 | 0.95 | 2.55 | 3.24 | 2185 | 0.18 |
| HD 128311 | 0.8 | 2.63 | 1.06 | 414 | 0.21 |
| HD 169830 | 1.4 | 2.96 | 0.823 | 230.4 | 0.34 |
| HD 196050 | 1.1 | 3 | 2.5 | 1289 | 0.28 |
| HD 73526 | 1.02 | 3 | 0.66 | 190.5 | 0.34 |
| HD 40979 | 1.08 | 3.16 | 0.818 | 260 | 0.42 |
| 14 Her | 0.79 | 3.3 | 2.5 | 1619 | 0.3537 |
| GJ 3021 | 0.9 | 3.31 | 0.49 | 133.82 | 0.505 |
| HD 80606 | 0.9 | 3.41 | 0.439 | 111.78 | 0.927 |
| HD 195019 | 1.02 | 3.43 | 0.14 | 18.3 | 0.05 |
| HD 13507 |  | 3.46 | 2.39 | 1318 | 0.13 |
| HD 92788 | 1.06 | 3.8 | 0.94 | 340 | 0.36 |
| $\tau$ Boo | 1.3 | 3.87 | 0.0462 | 3.3128 | 0.018 |
| Gl 86 | 0.79 | 4 | 0.11 | 15.78 | 0.046 |
| HD 213240 | 1.22 | 4.5 | 2.03 | 951 | 0.45 |
| HD 50554 | 1.1 | 4.9 | 2.38 | 1279 | 0.42 |
| HD 190228 | 1.3 | 4.99 | 2.31 | 1127 | 0.43 |
| HD 2039 | 0.98 | 5.1 | 2.2 | 1190 | 0.69 |
| HD 222582 | 15.4 | 1.35 | 576 | 0.71 |  |
| HD 28185 | 0.99 | 5.6 | 1 | 385 | 0.06 |
| HD 178911 | 0.87 | 6.292 | 0.32 | 71.487 | 0.1243 |
| HD 10697 | 1.1 | 6.59 | 2 | 1083 | 0.12 |
| 70 Vir | 1.1 | 6.6 | 0.43 | 116.6 | 0.4 |
| HD 106252 | 1.05 | 6.81 | 2.61 | 1500 | 0.54 |
| HD 23596 |  | 7.19 | 2.72 | 1558 | 0.314 |
| HD 89744 | 1.4 | 7.2 | 0.88 | 256 | 0.7 |
| HD 30177 | 0.95 | 7.7 | 2.6 | 1620 | 0.22 |
| HD 168443 | 1.01 | 7.7 | 0.29 | 58.116 | 0.529 |
| HD 168443 | 1.01 | 16.9 | 2.85 | 1739.5 | 0.228 |
| HD 33636 | 0.99 | 7.71 | 2.62 | 1553 | 0.39 |
| HIP 75458 | 1.05 | 8.64 | 1.34 | 550.651 | 0.71 |
| HD 141937 | 1 | 9.7 | 1.52 | 653.22 | 0.41 |
| HD 39091 | 1.1 | 10.37 | 3.34 | 2083 | 0,62 |
| HD 114762 | 0.82 | 11 | 0.3 | 84.03 | 0.334 |
| HD 136118 | 1.24 | 11.9 | 2.335 | 1209.6 | 0.366 |
| HD v162020 | 0.7 | 13.75 | 0.072 | 8.428198 | 0.277 |

Table 3. Table distribution of solar system objects.

| Source | Object name | $\begin{array}{r} \text { Mass } \\ \left(\mathrm{M}_{\text {sun }}\right) \end{array}$ | Semi Major axis <br> (AU) |
| :---: | :---: | :---: | :---: |
| Solar System | Mercury | 1 | 0.3859 |
|  | Venus | 1 | 0.7213 |
|  | Earth | 1 | 1 |
|  | Mars | 1 | 1.5186 |
|  | Jupiter | 1 | 5.1893 |
|  | Satum | 1 | 9.5133 |
|  | Uranus | 1 | 19.14 |
|  | Neptune | 1 | 29.99 |
|  | Pluto | 1 | 39.413 |
| Jupiter | Metis | $9.5010^{-4}$ | $8.5310^{-4}$ |
|  | Adrastea | $9.5010^{-4}$ | $8.5310^{-4}$ |
|  | Amalthea | $9.5010^{-4}$ | $1.2110^{-3}$ |
|  | Theb | $9.5010^{-4}$ | $1.4710^{-3}$ |
|  | Leda | $9.5010^{-4}$ | $7.4110^{-2}$ |
|  | Himalia | $9.5010^{-4}$ | $7.6510^{-2}$ |
|  | Lysithea | $9.5010^{-4}$ | $7.8110^{-2}$ |
|  | Ealara | $9.5010^{-4}$ | $7.8310^{-2}$ |
|  | Ananke | $9.5010^{-4}$ | $1.3810^{-1}$ |
|  | Carme | $9.5010^{-4}$ | $1.4910^{-1}$ |
|  | Pasiphae | $9.5010^{-4}$ | $1.5510^{-1}$ |
|  | Sinope | $9.5010^{-4}$ | $1.5810^{-1}$ |
|  | Io | $9.5010^{-4}$ | $2.8110^{-3}$ |
|  | Eurropa | $9.5010^{-4}$ | $4.4710^{-3}$ |
|  | Ganymede | $9.5010^{-4}$ | $7.1310^{-3}$ |
|  | Callisto | $9.5010^{-4}$ | $1.2510^{-2}$ |
| Saturo | Allas | $2.8410^{-4}$ | $9.1310^{-4}$ |
|  | Inner $G$. | $2.8410^{-4}$ | $9.2910^{-4}$ |
|  | Outer G. | $2.8410^{-4}$ | $9.4510^{-4}$ |
|  | Epimetheus | $2.8410^{-4}$ | $1.0110^{-3}$ |
|  | Janus | $2.8410^{-4}$ | $1.0010^{-3}$ |
|  | Mimas | $2.8410^{-4}$ | $1.2410^{-3}$ |
|  | Enceladus | $2.8410^{-4}$ | $1.5910^{-3}$ |
|  | Tethys | $2.8410^{-4}$ | $1.9710^{-3}$ |
|  | Telesto | $2.8410^{-4}$ | $1.9710^{-3}$ |
|  | Calypso | $2.8410^{-4}$ | $1.9710^{-3}$ |
|  | Dione | $2.8410^{-4}$ | $2.5110^{-3}$ |

While trying to solve equation 1 for the various systems, it is clearly not possible to ensure that all the planets of a given star have been observed and hence it is not possible to derive the exact fits to equation 1. We have therefore searched for a general fit to the relation

$$
\begin{equation*}
\log \left(M_{\text {star }}\right)=n \log \left(A_{\text {planet }}\right)+C \tag{2}
\end{equation*}
$$

Table 4. Same as Table 3.

| Source | Object name | Mass <br> $\left(\mathrm{M}_{\text {sun }}\right)$ | Semi Major axis <br> $(\mathrm{AU})$ |
| :--- | :--- | ---: | ---: |
| Saturn | DioneB | $2.8410^{-4}$ | $2.5110^{-3}$ |
|  | Rhea | $2.8410^{-4}$ | $3.5110^{-3}$ |
|  | Titan | $2.8410^{-4}$ | $8.1510^{-3}$ |
|  | Hyperion | $2.8410^{-4}$ | $9.8710^{-3}$ |
|  | Iapetus | $2.8410^{-4}$ | $2.3710^{-2}$ |
|  | Phoebe | $2.8410^{-4}$ | $8.6410^{-2}$ |
|  |  |  |  |
|  | Cordelia | $4.3510^{-5}$ | $3.3310^{-4}$ |
|  | Ophelia | $4.3510^{-5}$ | $3.6010^{-4}$ |
|  | Bianca | $4.3510^{-5}$ | $3.9610^{-4}$ |
|  | Cressida | $4.3510^{-5}$ | $4.1310^{-4}$ |
|  | Desdemona | $4.3510^{-5}$ | $4.1910^{-4}$ |
|  | Juliet | $4.3510^{-5}$ | $4.3010^{-4}$ |
|  | Portia | $4.3510^{-5}$ | $4.4210^{-4}$ |
|  | Rosalind | $4.3510^{-5}$ | $4.6810^{-4}$ |
|  | Belinda | $4.3510^{-5}$ | $5.0310^{-4}$ |
|  | Puck | $4.3510^{-5}$ | $5.7510^{-4}$ |
|  | Miranda | $4.3510^{-5}$ | $8.6510^{-4}$ |
|  | Ariel | $4.3510^{-5}$ | $1.2810^{-3}$ |
|  | Umbriel | $4.3510^{-5}$ | $1.7810^{-3}$ |
|  | Titania | $4.3510^{-5}$ | $2.9110^{-3}$ |
|  | Oberon | $4.3510^{-5}$ | $3.9010^{-3}$ |
|  |  |  |  |
| Neptune | Naiad | $5.1610^{-5}$ | $3.2210^{-4}$ |
|  | Thalassa | $5.1610^{-5}$ | $3.3510^{-4}$ |
|  | Despina | $5.1610^{-5}$ | $3.5110^{-4}$ |
|  | Galatea | $5.1610^{-5}$ | $4.1410^{-4}$ |
|  | Larissa | $5.1610^{-5}$ | $4.9110^{-4}$ |
|  | Proteus | $5.1610^{-5}$ | $7.8610^{-4}$ |
|  | Triton | $5.1610^{-5}$ | $2.3710^{-3}$ |
|  | Nereid | $5.1610^{-5}$ | $3.6810^{-3}$ |
|  |  |  |  |

Here $C$ is an arbitrary constant and $n$ is an integer. In Figure 1 we have plotted the lines for $n=2$ in equation 2 and different values of $C$ to give a good eye fit to the data set. Since $n=2$ seems to give a good fit, we have calculated a quantity X given by equation 3 .

$$
\begin{equation*}
X=\log \left(\frac{M_{\text {star }}}{A_{\text {planet }}^{2}}\right) \tag{3}
\end{equation*}
$$

We have plotted a histogram of $X$ in Figure 2. If the distribution plotted by us was random, we

Table 5. List of Stars with multiple planets.

| STAR name | Mass <br> Msun) | M.sin(I) <br> $\left(M_{\text {jupiter })}\right.$ | Semi Major axis <br> (AU) | Period <br> $($ days $)$ | Eccentricity |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Ups And | 1.3 | 0.71 | 0.059 | 6.6 | 0.0 |
| Ups And | 1.3 | 2.11 | 0.83 | 241.2 | 0.2 |
| Ups And | 1.3 | 4.61 | 2.5 | 1266.6 | 0.4 |
| HD 38529 | 1.39 | 0.67 | 0.12 | 14.3 | 0.3 |
| HD 38529 | 1.39 | 11.3 | 3.51 | 2189.5 | 0.3 |
| 55 Cnc | 1.03 | 0.84 | 0.11 | 14.7 | 0.0 |
| 55 Cnc | 1.03 | 0.21 | 0.24 | 44.3 | 0.3 |
| 55 Cnc | 1.03 | 4.26 | 6.01 | 5300 | 0.2 |
| HD 82943 | 1.05 | 0.88 | 0.73 | 221.6 | 0.5 |
| HD 82943 | 1.05 | 1.63 | 1.16 | 444.6 | 0.4 |
| HD 114783 | 0.92 | 0.9 | 1.2 | 501 | 0.1 |
| HD 114729 | 0.93 | 0.9 | 2.08 | 1136 | 0.3 |
| HD 37124 | 0.91 | 0.86 | 0.585 | 153 | 0.1 |
| HD 37124 | 0.91 | 1.01 | 2.95 | 1942 | 0.4 |
| HD 217107 | 0.98 | 1.28 | 0.07 | 7.1 | 0.1 |
| HD 210277 | 0.99 | 1.28 | 1.097 | 437 | 0.5 |
| HD 74156 | 1.05 | 1.56 | 0.276 | 51.6 | 0.6 |
| HD 74156 | 1.05 | 7.5 | 4.47 | 2300 | 0.4 |
| HD 160691 | 1.08 | 1.7 | 1.5 | 638 | 0.3 |
| HD 160691 | 1.08 | 1 | 2.3 | 0 | 0.8 |
| Gliese 876 | 0.32 | 0.56 | 0.13 | 30.1 | 0.1 |
| Gliese 876 | 0.32 | 1.98 | 0.21 | 61.0 | 0.3 |
| HD 12661 | 1.07 | 2.47 | 0.85 | 263.6 | 0.3 |
| HD 12661 | 1.07 | 1.86 | 2.61 | 1407 | 0.2 |
| HD 168443 | 1.01 | 7.7 | 0.29 | 58.1 | 0.5 |
| HD 168443 | 1.01 | 16.9 | 2.85 | 1739.5 | 0.2 |
|  |  |  |  |  |  |



Figure 1. Distance distribution of planets outside the solar system. $\times$ indicates extra-solar-system planets and moons of solar system objects. $\circ$ indicates the solar system planets.


Figure 2. $M_{\text {star }} /(\text { Planet Distance })^{2}$ distribution of extra-solar-system planets and solar system objects.
would expect an average of 2.6 objects per bin for the 159 objects distributed in 61 bins. However, we obtain a distinct peak with 8 points and 5 peaks of 7 objects in a clear broad maximum. The probability of finding 1 bin with 8 objects is $3.810^{-3}$ and the probability of finding 1 peak of 7 objects is 0.01 . However, each of these probabilities and hence the probability of finding 1 bin with 8 objects and 5 bins of 7 objects is less than $10^{-10}$. We therefore conclude that the relation given by equation 4 (where $C_{1}$ and $k$ are constants) represents a physically significant condition of resonance in planet distribution around stars.

$$
\begin{equation*}
A_{p l a n e t}^{2}=C_{l} M_{\text {sar }}+k \tag{4}
\end{equation*}
$$

Recent studies on the formation of gaps between planet locations triggered by disk planet tidal interactions (Rafikov, 2002) Iwasaki et al (2002) use Hill's radius which relates the disk scale height to the Toomre stability Parameter. Under these assumptions, the Hill's radius between two adjacent proto-planets is given by the equation 5 .

$$
\begin{equation*}
r_{H}=\frac{a_{1}+a_{2}}{2}\left[\frac{2 m}{3 M_{*}}\right]^{1 / 3} \tag{5}
\end{equation*}
$$

where $a_{1}$ and $a_{2}$ are the initial semi-major axes of the first and second planet and the second fraction is the reduced Hill radius where $m$ is the mass of the protoplanet and $M_{*}$ is the mass of the parent star. In this assumption, the orbits of the neighbouring planets must be separated by the cube of the distance. In Figure 3 we have plotted the distribution as a function of $M_{*} / d^{3}$ where $d$ is the distance of the planet. As can be seen from the figure, there is only one peak in the distribution with a statistical significance of $7.510^{-4}$


Figure 3. $M_{*} /(\text { Planet Distance })^{3}$ distribution of planets outside the solar system and solar system objects

## 4. Discussion and Conclusion

We have searched for possible patterns in distance distribution of planets around massive objects in light of the data from extra-solar-system planets as well as the solar system and the natural satellites of the Solar System planets. Our analysis shows that there is a periodicity and the location of the planet orbit will depend on the mass of the parent star and the quantity $\mathrm{M}_{\text {star }} / A_{\text {planet }}^{2}$ is constant. If this result is confirmed with more data, it should be possible to predict the possible location of extra-solar-system planets around stars and may also indicate significant resonance effects in condensation of planets from protoplanetary disks.

While the approach taken by us is purely statistical, our results indicate that there seems to be an underlying symmetry and though coming from statistics rather than physical principles it seems to indicate some universal underiying principle.

It is possible to interpret the results to indicate that the results imply that the further out you go, the wider the annulus that can be dominated by a single condensing core over tidal forces of the central object. This interpretation seems to be simplistic. Amongst the hundred extra-solarsystems listed in Tables 1 and 2, twelve have more than one planet. For three stars, the inner planets are heavier than the outer planets and for two systems the inner and outer planets have similar masses in spite of their large separation in distances indicating a more complex interplay between the planet condensing forces and the density of the protoplanetary disks.

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