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Approaching gravitation by quantum gravity with matter near four dimensions

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Abstract. The phase structure and infrared (IR) behaviour of higher-derivative quantum gravity (QG) near (but below) four dimensions, with a few different matter theories, shows remarkable properties. Models of this sort can be analyzed by renormalization group (RG) methods in $4-\varepsilon$ dimensions for the following matter sectors : $f\,\varphi^4$, $O(N)\varphi^4$, scalar electrodynamics, and $SU(2)$ with scalars. New fixed points for the scalar coupling are predicted and one of them turns out to be infrared (IR) stable. For the theory at nonzero temperature, the QG-perturbed IR stable fixed point leads then to a second order phase transition. Other noticeable effects of QG influence are changes in the shape of the RG improved effective potential, like in the $SU(2)$ theory, which can be viewed as the confining phase of the standard model.

1. Introduction

At present, no completely consistent QG theory is available. Fine as Einstein gravity may be in the classical realm, it is, regretfully, nonrenormalizable (t'Hooft & Veltman 1974; Deser & Nieuwenhuizen 1974; Goroff & Sagnotti 1986). String theory (Green et al. 1987) might lead to an effective gravity which could be eventually examined at classical level (being also, in general, nonrenormalizable). This may take, however, quite a long time yet. In the face of such a picture, there is still a chance of finding a short cut, by working with an effective QG model which mimics some essential features of the much coveted - but unknown - fully consistent theory. By lack of more ingenious thoughts, we can envisage this effective model as some expansion of the mysterious gravitational Lagrangian in powers of geometrical invariants. In principle, that would still be nonrenormalizable, unless one picks an exceptional example like four-dimensional higher-derivative gravity, which is multiplicatively renormalizable (Stelle 1977) (in a scheme, of course, of a propagator of fourth order). This theory is, furthermore, asymptotically free (Buchbinder et al. 1992; Julve & Tonin 1978; Fradkin & Tseytlin 1982; Antoniadis & Tomboulis 1986) and yields Einstein gravity as its low-energy limit, as should be required. Since the gravitational couplings in such a model are dimensionless, they might play a role in the renormalization program of its associated quantum field theory.

Yet, it would be naïve to overlook that such a QG model has ailments of its own. The main disadvantage of higher-derivative QG is surely the unitarity problem at perturbative level. Real chances of solving this problem by a nonperturbative approach exist (see e.g. the paper by Antoniadis and Tomboulis (1986). Since the nonunitarity disease is of dynamical nature, all possible quantum corrections should be included, perturbative and nonperturbative ones. In this respect, the string example (Antoniadis et al. 1989; Myers 1987), where negative-norm states decouple at the RG fixed point, may be worthy of attention. At any rate, when viewed as just an effective low-energy theory, lack of unitarity should not make such a heavy case against higher-derivative gravity. Another appealing aspect of this theory is that its critical dynamics can be studied up to almost the same level as in the absence of QG (for a study of different theories in relation to critical phenomena, the reader is addressed to (Zinn-Justin 1989). As a rule, when interacting with some asymptotically - free GUT at energy regions between the GUT and the Planck scales, higher-derivative gravity preserves asymptotic freedom. Thus, a totally asymptotically free theory for the unification of R^2-gravity with a GUT (Buchbinder et al. 1992; Julve & Tonin 1978; Fradkin & Tseytlin 1982; Antoniadis & Tomboulis 1986) can be built. Then, scalar and Yukawa couplings receive one-loop QG corrections that may influence the GUT in question in different ways.

IR properties of QG are relevant for applications to particle physics and cosmology at energy scales below the Planck mass, and epsilon expansion techniques are often useful for investigating the IR properties of a theory, as happens in critical phenomena (Wilson & Kogut 1974; Brezin et al. 1989). For renormalizable theories, this method supplies a standard way of analyzing the critical behaviour and the nature of phase transitions at nonzero temperature (Wilson 1972; Chen et al. 1978; Ginsparg 1980; Appelquist & Pisarski 1981; Wetterich & Reuter 1993). Moreover, the IR behaviour of QG is closely related to its thermal features (G.W.Gibbson et al. 1978; Taylor & Veneziano 1990). In QG, the ε - expansion technique has been applied to $(2 + \varepsilon)$-dimensional theories (Weinberg 1979; Gastmans et al. 1978; Christensen & Duff 1978; Kojima et al 1994), (Jack & Jones 1991). However, the nonrenormalizability of standard Einstein gravity persists even in $2 + \varepsilon$ dimensions (Jack & Jones 1991). Elizalde & Odintsov (1995), Elizalde & Odintsov (1996) and Sakai's paper (Kojima et al. 1994) include the suggestion that one should consider dilaton gravity with matter in $D = 2 + \varepsilon$. As shown in those works, an asymptotically safe QG theory may be formulated and Weinberg's program (Weinberg 1979; Gastmans et al. 1978; Christensen & Duff 1978; Kojima et al. 1994) may be completed for such models. Nevertheless, it is not clear that these $D = 2 + \varepsilon$ theories admit an analytic continuations to $\varepsilon = 2$, thus taking us to the $D = 4$ world. This is why it is quite interesting to consider a multiplicatively renormalizable QG in $D = 4 - \varepsilon$ dimensions, as one can try to manage without the special properties of two-dimensional models and, hopefully, take a more realistic approach to QG than in $D = 2 + \varepsilon$ theories.

2. QG Corrections

We have investigated the influence of higher-derivative QG corrections on the phase transitions of scalar-gauge theories (Elizalde et al. 1996), and on their infrared properties, using the ε-expansion technique near four dimensions. The starting Lagrangian for all these cases in $D = 4 - \varepsilon$ has the form

$$\mathcal{L} = \mu^{-\varepsilon}\left(\frac{1}{\lambda}W - \frac{\omega}{3\lambda}R^2 + \chi R + \Lambda\right) + \frac{1}{2}\xi R\varphi^2 + \mathcal{L}_m(\varphi, A^a_\mu) \tag{1}$$

where $W = C_{\mu\nu\alpha\beta}\, C^{\mu\nu\alpha\beta}$ is the square of the Weyl tensor, R the Riemann scalar curvature, Λ the cosmological constant, and μ a mass parameter used to make λ dimensionless in $4 - \varepsilon$ dimensions. Further, φ is a matter scalar field coupled with the curvature through ξ, and $\mathcal{L}_m(\varphi, A^a_\mu)$ stands for the matter Lagrangian, which includes pure matter, φ, as well as matter gauge fields, A^a_μ. In order to study the critical behaviour, the RG equations have been written for $D = 4 - \varepsilon$ with different choices of $\mathcal{L}_m(\varphi, A^a_\mu)$, and the fixed points of these equations have been found. (For details of the quantization of such a theory, calculation of the corresponding ghost terms and the one-loop effective action, one can consult (Buchbinder et al. 1992; Julve & Tonin 1978; Fradkin & Tseytlin 1982; Antoniadis & Tomboulis 1986). Note that there are mass-like terms in the theory under discussion, i.e. Einstein and cosmological ones. Without these terms higher derivative QG would still be multiplicatively renormalizable. As was shown some time ago (Wilson 1972; Chen et al. 1978; Ginsparg 1980; Appelquist & Pisarski 1981; Wetterich & Reuter 1993), mass-like terms play no role in the study of IR stability and fixed points.

Taking $f\varphi^4$ with QG we have seen, after numerically solving those equations, that the model offers a richer phase structure with new fixed points for the scalar coupling which were absent from the theory without QG. Unluckily enough, no new IR stable fixed point has arisen in that particular model as a result of introducing gravity. However, the IR stable fixed point for the scalar coupling, which was already present in the absence of QG, is perturbed. And near this fixed point, the prediction is that the theory at nonzero temperature will undergo a second-order phase transition.

Moreover, we have also considered the $O(N)$ φ^4 - theory where, as expected, gravitational corrections to fixed points decrease when increasing the number of scalars. In this example, the IR stable fixed point perturbed by QG has been found. This is illustrated in the table below, where the numerical values of the QG-perturbed RG fixed points are shown :

N	λ	ω	ξ	f	type of point
10000	0.88	-2.354	0	0	saddle point
	0.88	-1.141	0.036	0.0001	"
	0.88	-0.003	0.163	0.0084	"
	0.88	-0.003	0.164	0.0354	IR stable fixed point
	0.88	-0.031	0.185	0.0049	saddle point
	0.88	-0.094	0.200	0.0041	"
	0.88	-53.135	0	0	"
	0.88	-54.896	0.082	0.0455	"
100000	-0.094	-2.485	0	0	saddle point
	-0.094	-1.450	0.039	0.000001	"
	-0.094	-0.002	0.162	0.000216	"
	-0.094	-0.001	0.164	0.004044	IT stable fixed point
	-0.094	-0.003	0.184	0.000051	saddle point
	-0.094	-0.125	0.204	0.000041	"
	-0.094	-503.005	0	0	"
	-0.094	-504.868	0.083	0.004717	"
1000000	-0.0095	-2.4985	0	0	saddle point
	-0.0095	-1.4540	0.039	0.000000	"
	-0.0095	-0.0020	0.162	0.000002	"
	-0.0095	-0.0001	0.165	0.000439	IR stable fixed point
	-0.0095	-0.0266	0.183	0.000001	saddle point
	-0.0095	-0.1278	0.204	0.000000	"
	-0.0095	-5002.9915	0	0	"
	-0.0095	-5004.8753	0.083	0.000474	"

The IR stable fixed point for N = 1000000 has $f \simeq$ 0.00044 while, in the same conditions without QG, it would yield $f \simeq$ 0.00047. Such a perturbation, by about a 7%, is a measurable correction. It is remarkable that even for such a large N the perturbation is not very essential. This shows that, unlike the case of QCD, one has to be very careful with the large-N limit in QG. By general arguments, for large N one expects to be back to the theory of quantized fields in curved spacetime, or in other words one may neglect QG corrections. Our study shows that while in QCD large N means 'of order ten', in higher derivative QG large N means "of the order of a few billion" !

For higher-derivative QG with scalar QED or nonabelian gauge fields, we have obtained only unstable fixed points. A theory we have studied in detail is the $SU(2)$ gauge model with scalars interacting with higher-derivative QG. We have calculated the RG-improved effective potential at nonzero temperature, which approximately describes the confining phase of the standard model. Numerical estimates indicate that the QG corrections can actually add some non-essential contributions to this potential, which may become sizeable near the Landau pole, at least for some specific values of the gravitational coupling. It is also interesting to note that unlike pure higher derivative QG, the theory under discussion here has the correct Newtonian limit (t'Hooft & Veltman 1974).

3. Conclusions

We conclude that in systems made of renormalizable R^2 - gravity plus a scalar gauge theory near four dimensions, the ε-expansion technique can be applied with reasonable success to the study of the critical behaviour and phase transitions-in a number of realistic theories-showing that these models provide amenable examples of how four-dimensional QG might alter several apparently well established results (Weinberg 1979; Gastmans et al. 1978; Christensen & Duff 1978; Kojima et al. 1994) on temperature phase transitions and IR fixed points. As a last outcome of our work, after a careful analysis of what we have obtained, we definitely claim to have shown that the case of the influence of Quantum Gravity on high energy theories with matter cannot be overlooked any longer. It would be also very interesting to study higher loops effects there. However, even one-loop calculations in QG are so complicated that it would be naive to expect that higher loops may be taken into account in higher-derivative QG in the near future.

We see also that there might even be a chance of trying a combination of the $(4-\varepsilon)$ – and $(2+\varepsilon)$ – approaches in R^2 - gravity, so as to seek some agreement at $\varepsilon = 1$. If in some appropriate region the result of such a synthesis were born out, perhaps our present understanding of these methods could widen.

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