Bull. Astr. Soc. India (2001) **29**, 439–442

A family of triaxial mass models : Analytical study of its projected properties

Mousumi Das and D.K. Chakraborty[1]

School of Studies in Physics, Pt. Ravishankar Shukla University, Raipur 492010, India
[1]IUCAA, Post Bag 4, Ganeshkhind, Pune 411007, India

Abstract. Triaxial modified Hubble mass models are studied. The models are more general than those studied by earlier workers. The potential and the projected surface density of the mass models can be calculated analytically. The shapes of the isophotes and the profiles of ellipticity and position angles are studied. The inclusion of large number of terms in density function, allows one to produce a large variety of these profiles, which can be compared with observations.

1. Introduction

Schwarzschild (1979) proposed a triaxial generalisation of a modified Hubble mass model. Later, deZeeuw and Merritt (1983) casted it into an analytical form. Chakraborty and Thakur (2000) (hereafter, CT2K), studied the projected properties of these models, following the apporach of deZeeuw and Carollo (1996). The models have density $\rho(r,\theta,\phi) = f(r) - g(r)Y_2^\circ(\theta) + h(r)Y_2^2(\theta,\phi)$ and potential $V(r,\theta,\phi) = u(r) - v(r)Y_2^\circ + w(r)Y_2^2$. We modified the model of CT2K by adding more number of terms to it and studied its projected properties analytically. The potential and the projected surface density of the resultant mass model can be calculated analytically. The inclusion of large number of terms in density function, allows one to produce a large variety of the profiles of projected parameters, which can be compared with observations.

2. Formulations

It was mentioned by deZeeuw and Carollo (1996), that terms with spherical harmonics of higher order may be added. As a preclude to such studies, we added terms

$$g_1(r) = \frac{M}{4\pi b_o^3} 4a_1^4 \frac{r^8 + 12a_2^2r^6 - 9a_2^4r^4}{(a_2^2 + r^2)^6}$$

$$h_1(r) = \frac{M}{4\pi b_o^3} 4a_3^4 \frac{r^8 + 12a_4^2r^6 - 9a_4^4r^4}{(a_4^2 + r^2)^6} \qquad (1)$$

e-mail : ircrsu@bom6.vsnl.net.in

respectively to $g(r)$ and $h(r)$ of the density. The potential gets modified and the terms

$$v_1(r) = \frac{-a_1 r^6}{(1 + a_2 r^2)^4}$$

$$w_1 = \frac{-a_3 r^6}{(1 + a_4 r^2)^4} \qquad (2)$$

should be added respectively to $v(r)$ and $w(r)$ of the potential. The radial dependences of the "extra-terms" g_1, h_1, v_1 and w_1 are chosen such that they are effective in intermediate range of r only. Therefore, at small and at large radii, the terms $f(r)$, $g(r)$ and $h(r)$ are dominating and the parameters appearing in these radial functions can be fixed by chosen axis ratios of the approximate ellipsoidal constant ρ surfaces, at small and at large radii. The parameters a_1..... a_4 are chosen such that $|g_1(r)| \ll |g(r)|$ and $|h_1(r)| \ll |h(r)|$, at all r.

The projected surface density is given by $\Sigma(R, \Theta) = \Sigma_o(R) + \Sigma_2(R) \cos 2(\Theta - \Theta_*)$ where Σ_0 and Σ_2 are given by certain r - integrals of the radial functions appearing in ρ, and viewing angles (θ', ϕ'). These, in turn, can be used to find position angles Θ_* of the major axis and the axis ratio $\frac{b}{a}$ of the projected isodensity contours (cf: CT2K). The additional terms g_1 and h_1, modify the integrals G_1, G_2, H_1 and H_2 of CT2K : $G_1 \to G_1 + G_{11}$, $G_2 \to G_2 + G_{12}$, $H_1 \to H_1 + H_{11}$, and $H_2 \to H_2 + H_{12}$. The integrals G_{11}, G_{12}, H_{11} and H_{12} can be calculated analytically for the chosen functions. We found

$$G_{11} = 4a_1^4 I_3 + 12a_2^2 I_2 - 9a_2^4 I_1$$

$$G_{12} = 4a_1^4 R^2 I_2 + 12a_2^2 I_1 - 9a_2^4 I_4 \qquad (3)$$

where

$$a^2 + R^2 = \alpha^2 \qquad \beta = \frac{\pi}{512\alpha^{11}}$$

$$I_1 = \beta(63R^4 + 14R^2\alpha^2 + 3\alpha^4) \qquad I_2 = \beta(63R^6 + 21R^4\alpha^2 + 9R^2\alpha^4 + 3\alpha^6)$$

$$I_3 = \beta(63R^8 + 28R^6\alpha^2 + 18R^4\alpha^4 + 12R^2\alpha^6 + 7\alpha^8) \qquad I_4 = \beta(63R^2 + 7\alpha^2) \qquad (4)$$

and for H_{11} (R) and H_{12} (R) in terms of a_3 and a_4, in place of a_1 and a_2 respectively.

In terms of these expressions, position angles Θ_* and the axis ratio $\frac{b}{a}$ can be written down, which at small and at large radii can be expressed in terms of elementary functions.

3. Results

Projected properties $\frac{b}{a}$ and Θ_* are studied. The inclusion of g_1 and h_1 terms enables us to generate varieties of ellipticity and position angle profiles which may be compared with observations.

We found an interesting property. The axis ratio $\frac{b}{a}$ at small and large R are strongly correlated, when a model is viewed in all possible orientations. This property is also exhibited by other triaxial models, such as the triaxial version of γ - models of Dehnen (1993). We found

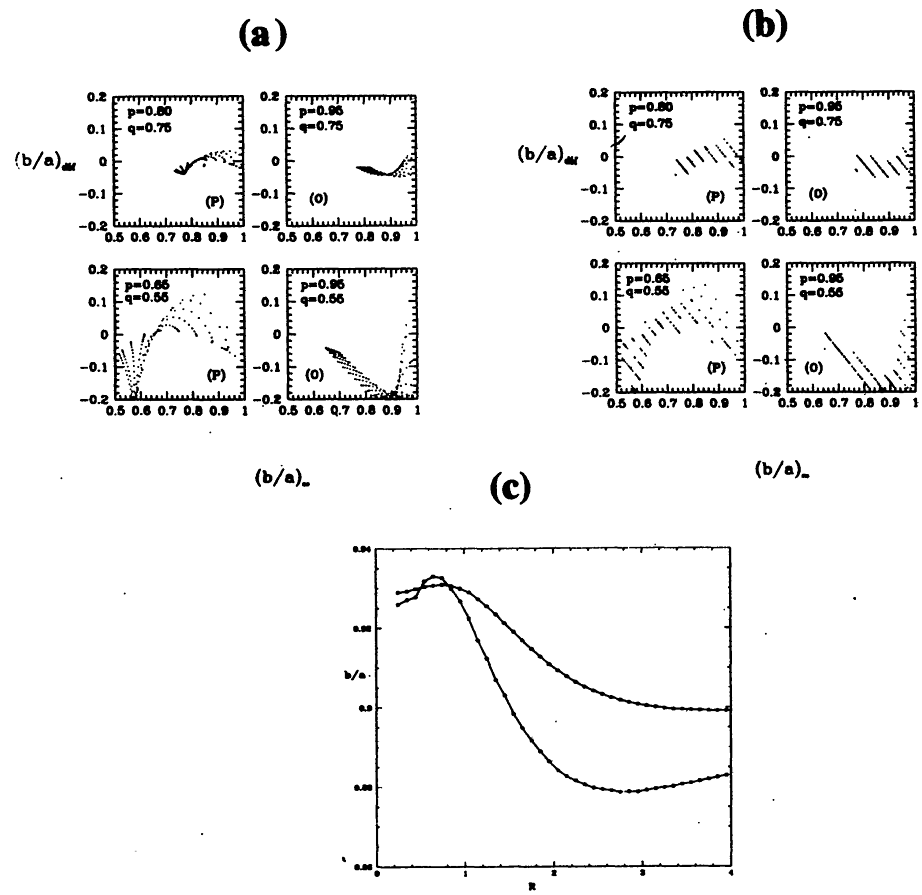

Figure 1. (a) Correlation between $\left(\frac{b}{a}\right)_O$ and $\left(\frac{b}{a}\right)_\infty$ when the model of a chosen (p, q) is projected in all possible viewing angles in absence of "extra terms", (b) Same with "extra terms", (c) Plot with crosses (filled circles) show $\frac{b}{a}$ as a function of R with (without) g_1 and h_1 terms.

that the correlations patterns are qualitatively similar for models with or without the g_1 and h_1 terms. (Compare fig 1(a). and fig 1(b).). It will be interesting to see if the correlation patterns carry the signature of intrinsic axis ratio of the mass models (cf: Statler and Fry 1994).

The "extra terms" also produce small scale variations in the profiles of photometric parameters, which are observed in many elliptical galaxies. Fig 1(c). presents $\frac{b}{a}$ profiles with and without the "extra terms". We find the inclusion of g_1 and h_1 terms introduces a small scale variations in the profile.

The study of triaxial mass models with fourth order spherical harmonics are in progress.

Acknowledgements

DKC expresses his sincere thanks to CSIR, New Delhi India for providing support through the project grant No. 03(0807)/97/EMR-II. We are also thankful to IUCAA, Pune, India for providing local hospitality and support during our visits.

References

Chakraborty D.K., Thakur P., 2000, MNRAS, 318, 1273
de Zeeuw P.T., Caroloo C.M., 1996, MNRAS, 281, 1333
Dehnen W., 1993, MNRAS, 265, 250
de Zeeuw T., Merritt D., 1983, ApJ., 267, 571
Schwarzschild M., 1979, ApJ., 232, 236