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MEASUREMENT OF KODAIKANAL WHITE–LIGHT IMAGES: RELAXATION OF TILTS OF SPOT GROUPS AS INDICATOR OF SUBSURFACE DYNAMICS OF PARENT FLUX LOOPS

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ABSTRACT

We reexamine the evolution of the observed tilts θ of spot groups with life spans 2–7 days in the two latitude belts $<13°$ and $>13°$. Using an iterative procedure, we refine the linear fit between θ and the daily tilt angle changes $\delta\theta$ and obtain reliable estimates of the fit coefficients. We interpret our results in light of the scenario implied by the theoretical model of Longcope & Choudhuri for the subsurface dynamics of parent flux loops of bipolar magnetic regions and arrive at the following conclusions: (1) the parent flux tubes of spot groups possess a nonzero tilt at the onset of rise from the depths of their origin; these "inborn tilts" are $\sim$4°–11° in latitudes $<13°$ and $\sim$3°–15° in latitudes $>13°$; (2) during the rise the tilt of the omega loops of spot groups living 2–7 days get reduced to $\sim$2°–6° in both the latitude belts, and this calls for reexamination of the role of Coriolis force as understood so far; (3) after emergence of the top of the loop above the surface, magnetic tension in the legs tends to restore the tilt to the inborn tilt on timescales of $\sim$5 to 14 days; and (4) these timescales correspond to field strengths in the range $\sim$14–40 kG for the parent flux loops and are close to the limits set by Fan et al.

Subject headings: MHD — Sun: interior — Sun: magnetic fields — sunspots

1. INTRODUCTION

It has been known since the observations by Hale et al. (1919) that the axes of bipolar spot groups appear generally tilted with respect to the east–west direction, with the leader spot closer to the equator than the follower spot, and that the average tilt of spot groups in any latitude interval increases with the mean latitude of the interval. Models of the origin of the observed tilts of spot groups are based on the subsurface dynamics of the Ω-shaped loops of magnetic flux tubes (worked out by Choudhuri & Gilman 1987; Choudhuri 1989) that are supposed to rise buoyantly upwards through the convection zone and manifest on the surface as spot groups or bipolar magnetic regions (BMRs). It is widely accepted that such flux tubes originate in the Sun's strong toroidal magnetic fields that are generated in a layer of strong rotational shear, the tachocline, located at the base of the convection zone (Gilman et al. 1989; Rudiger & Brandenburg 1995; Fisher et al. 2000). According to the existing models of the origin of the tilts, the tops of the Ω loops would be deflected by the Coriolis force during their rise through the convection zone from an initial toroidal orientation (assumed to be along the east–west) to tilts in quantitative agreement with those measured on the surface (Wang & Sheeley 1991; D'Silva & Choudhuri 1993; Fan et al. 1993, 1994; Schussler et al. 1994; Caligari et al. 1995; Fisher et al. 2000, and the references therein). The broad scatter in the distribution, over and above the systematic tilt produced by the Coriolis force, has been explained by Longcope & Fisher (1996) as the result of the buffeting action on the flux loops by the turbulence during their rise through the convection zone.

To obtain insight into the subsurface dynamics of a flux tube, Howard (1996, his Figs. 3 and 4) plotted the observed daily changes in the tilt angles of spot groups against the existing values of the tilt angles. This plot showed that the axes of spot groups rotate in such a way that the spot groups with large tilt angles (+ or −) experience a large tilt angle change per day in the opposite sense (− or +), so as to bring the tilt angle back toward a specific value between $\sim$5° and 7°. Thus the tilt angles of spot groups seem to evolve with time under the action of a force trying to restore the tilt angles toward the average value. In the absence of information at that time on the ages of spot groups at the epochs of the measurements of the tilts, Howard (1996) could not follow the evolution of the tilt angle of individual spot groups. Instead, he examined the plots of the tilt angle versus tilt angle change per day of all spot groups in his data bank using various parameters as proxies for their age. He concluded that the linear correlation in the plot of tilt angle versus daily tilt angle change represents "a basic physical effect that is at work." According to the scenario based on his intuition, the flux tubes could be tilted at the average

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value at their subsurface source itself and the restoring force on the parent loop could rotate the axes of the spot groups to the preferred angle defined by the positions of their "fixed feet" at some specific depth beneath the solar surface. Howard (1996, his Fig. 3) also concluded that the parent flux loops would relax to a final equilibrium value of 5°–7° on a timescale of 4.37 days.

Prompted by this result, Longcope & Choudhuri (2002, hereafter LC02) developed a theoretical model to explain the way the tilt angles in BMRs (created by the Coriolis force as per the model of D'Silva & Choudhuri 1993) would relax to a final equilibrium value of 5°–7° on a timescale of 4.37 days obtained by Howard (1996). They argue that if a fully developed BMR continues to remain anchored to the toroidal field at the base of the convection zone, then the magnetic tension would take several weeks to align the BMR along the initial toroidal orientation, which they assume to be in the east–west direction. In the absence of any evidence that the flux loops eventually relax to the east–west direction (i.e., to tilt angle zero position of the toroidal field), LC02 consider the second option of a "dynamic disconnection," which would prevent the magnetic tension from aligning the upper parts of the flux loop along the east–west direction. Indeed, when Toth & Gerlei (2004) analyzed the tilt angle distribution and its evolution from a sample of 687 clearly oriented BMRs, graded according to their age, they found no evidence that the BMRs relax to the zero tilt angle position. This seems to confirm the occurrence of dynamic disconnection suggested by LC02. They also suggest that the disconnection should occur at a depth of ~75 Mm in the convection zone, so that the Alfvén wave traveling from the disconnection point to the surface can effect the relaxation of the bipole to an orientation of 5°–7° in 4.37 days, as in the observations. However, their simulations based on the disconnection depth at 75 Mm leads to relaxation time as large as 41.5 days (Fig. 10 of LC02). Thus, there remain considerable ambiguities in the understanding of the relaxation of the tilts of spot groups and BMRs.

Possible reasons for these ambiguities are as follows. First, the linear regression between the tilt angle change and tilt angle, which was used for deriving the relaxation time, expresses only the differential equation for the evolution of the systematically varying component of the observed tilt. To trace the evolution of this component unambiguously from this differential equation, one must know the initial value of this component, which can be determined only from the first-day values of the observed tilts. These values were not available at the time of the earlier studies (Howard 1996; LC02). Second, as we show in § 3.3, the intercept θ_x, of the regression line on the tilt angle axis defines not the mean tilt (as understood by these authors), but defines the "asymptotic tilt" θ_∞, i.e., the tilt to which the flux loop would approach after infinite time. Third, the data used for computing the linear regression between the tilt angle and the tilt angle change were heterogeneous, comprising *all* spot groups, irrespective of their life spans, sizes, and latitudes of occurrence.

In this paper we reexamine the problem of the long-term variation of the observed tilts using our measures of the tilts and the tilt angle changes at the surface. While doing so, we overcome the above difficulties through the following steps. We define in § 2 separate sets of spot groups according to their life spans and latitude of occurrence, assuming that the parent flux loops of spot groups in each such set have similar physical properties. We use the data sequences compiled by Sivaraman et al. (2003), from the sunspot data bank of the Kodaikanal observatory, which contains information on the ages of spot groups observed besides the value of the tilt angle and the daily tilt angle change of each spot group on each day of observation. From these we make a note of

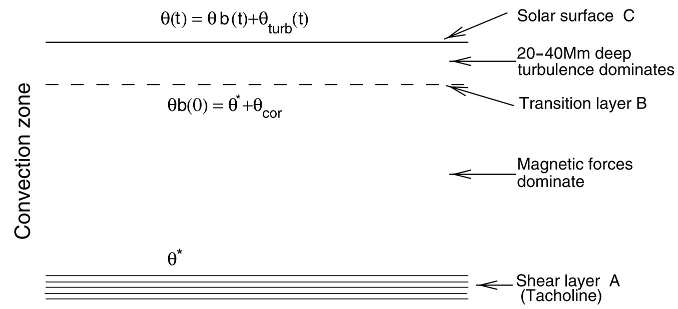

FIG. 1.—Schematic sketch showing the simplified scenario of the relative effects of magnetic, turbulent and Coriolis forces on the parent flux tube before and after its emergence on the surface based on Fig. 2 of Longcope & Choudhuri (2002). Symbols: θ^*, the "inborn tilt" i.e., the tilt of the parent flux tubes at the depths of their origin; $\theta_b(0)$, "the initial basic tilt", i.e., the tilt when the flux loop just crossed the layer B and before turbulence has acted on it; $\theta_{\rm cor}$, the tilt imparted by the Coriolis force during the rise of the flux loop from layer A to B; $\theta(t)$, observed tilt of the axis of spot group at time t; $\theta_b(t)$, the "basic tilt" of the Ω-shaped flux loop at time t; $\theta_{\rm turb}(t)$, cumulative effect of turbulence on the tilt.

the values of the "first-day tilt" (i.e., the observed tilt on the day the spot group is born), for all those spot groups whose first day of life could be identified (§ 3.1). In § 3.2 we show the correlation between the daily change of tilt angle versus the tilt angle for the combined data from all the spot groups in all the sets. We also show that the linear fit to such a correlation represents the differential equation for the smoothly varying component of the observed tilt and present the formal solution of this equation.

To provide a physical interpretation of the smoothly varying component of the observed tilt, we construct in § 3.3 a simplified scenario for the subsurface dynamics of the parent flux loop of a spot group using the dynamical model of LC02 as the template. According to this scenario (Fig. 1) the systematically varying component of the observed tilt is the tilt of the parent flux loop at the transition layer below which it is practically unaffected by turbulence. We term this the "basic tilt." In § 3.4 we use the values of the first-day tilts of § 3.1 and work backward to determine the "initial value," $\theta_b(0)$ of the basic tilt of the flux loop of a typical spot group in each set, i.e., the tilt when the flux loop has arrived at the layer B in Figure 1. In § 3.5, from the results of linear fit to the whole data set from each category of spot groups, we obtain preliminary estimates of the values of the "asymptotic" tilt angle θ_∞, to which the basic tilt of the flux loop of a typical spot group in the set would ultimately relax to, and τ the timescale of relaxation of the basic tilt. From these preliminary estimates we infer that each such data set contains substantial input from "inadmissible" data points that represent tilt variations containing hardly any systematic component. Hence, for determining reliable estimates of θ_∞ and τ, we employ the "iteratively weighted linear least-square fit" procedure described in Mosteller & Tukey (1977) to minimize the vitiating effects of the inadmissible data points and continue the iteration until the successive estimates of the fit parameters agree within about 10%.

The resulting reliable estimates of θ_∞, ~4°–11° in latitudes <13° and ~3°–15° in latitudes >13°, turn out to be generally *larger than* the corresponding estimates of the initial basic tilt $\theta_b(0)$ (~2°–6° in latitudes <13° and ~3°–6° in latitudes >13°). In § 4, we discuss the implications of the estimates of $\theta_b(0)$, θ_∞, and τ derived from our observations of tilts of spot groups for the scenario implied by the theory of LC02.

Our main conclusions (§ 5) are as follows. (1) The parent flux loops of spot groups (BMRs) possess tilts of at least ~4°–11° in latitudes <13° and ~3°–15° in latitudes >13°, at the very depths of their origin (i.e., these are the tilts of the toroidal field in the tachocline). (2) By the time the tops of the flux loops rise from level A to level B in the convection zone (Fig. 1), the Coriolis force *reduces* the tilt to values in the range $\sim 2^\circ$–6° for loops in latitudes $<13^\circ$ and $\sim 3^\circ$–6° for loops in latitudes $>13^\circ$. (3) After emergence of the top of the flux loop above the surface, the magnetic tension tends to restore the tilt of the flux loop to the original higher value with a relaxation timescale of $\sim$5–14 days.

The nonzero tilt of the toroidal field relieves the theoretical models of solar dynamo from an ad hoc constraint. The reduction in the tilt in the second conclusion calls for a modification in the existing model of the creation of tilts of spot groups by reviewing either the role of magnetic tension or the topology of the parent flux loops. The relaxation timescales estimated by us correspond to field strengths of parent flux loops in the range $\sim$14–40 kG and are close to the range $\sim$20–40 kG derived by Fan et al. (1994).

2. DATA AND ANALYSIS

The measurement of the Kodaikanal daily white-light photographs that gave rise to the unique data bank on positions and umbral areas of all spots for the 82 yr period (1906–1987) have been described in detail in an earlier paper (Gupta et al. 1999). The procedure by which the spots are grouped into spot groups, and their "returns" are identified from the first and second days when there are observations on consecutive days, follow exactly the procedure described by Howard et al. (1984). Thus, our analysis starts with data in which spot groups are listed as "day pairs." Each day pair refers to a single spot group that was sighted on two consecutive calendar days. The code now reads through the sequence of day-pair listings and generates for each spot group the first and last sightings of that spot group on the disk, and hence its life span. The method for determining the first sighting of a spot group on the disk is described in detail in Sivaraman et al. (2003).

The life span of a spot group as determined by the code is the time elapsed, measured in days, between the first and the last day when that spot group was sighted on the photoheliographs, but was not sighted on subsequent days in the vicinity of the expected location determined from knowledge of solar rotation. The photoheliographs are normally obtained only once a day, although for various reasons, there are some days of gaps in observations every year. Where there are days of gaps in observations between two consecutive sightings of a spot group, the code interpolates the existence of the spot group on the days of the gap. It is adequate for our present purpose to reckon the life spans of spot groups only in integral multiples of 1 day. The code thus reckons the life span of a spot group as L days if it was sighted consecutively on $L + 1$ days (from the day it was sighted first on the disk) but not sighted later, although it should have been sighted on the disk at least once had it continued to exist beyond $L + 1$ days. Since only spot groups within a longitude window of $\pm 60^\circ$, were measured on the white-light images to minimize the foreshortening errors in position and area measures, the procedure just described allows only nonrecurrent spot groups of life spans 1–7 days to be included in the present study.

The spot groups are now sorted into 7 bins, containing spot groups of life span 1 day, 2 days, etc., up to 7 days, respectively, in each of the two latitude zones, $<13^\circ$ and $>13^\circ$, with the north and the south hemispheres combined. We thus have 14 sets of spot groups defined by the life span and the latitude range.

For each of the spot groups, the tilt angle, defined as the angle formed by the line joining the centroids of the leading and following portions with the local parallel of latitude, was determined as described in Sivaraman et al. (1999). By convention, the tilt angle is positive for spot groups whose leading spots are located closer to the equator than the following spots. Since there is no magnetic polarity information for the spots in this data set, the measured values of tilt angles lie between $+90^\circ$ and -90°, and involve an error of 180° in the cases where the real tilt angle is beyond $\pm 90^\circ$. However, an examination of the plot of the tilt angle distribution for plages, which have polarity information (Howard 1996, his Fig. 1), shows that such cases are too few to cause a serious handicap.

In addition to the above, the code also computes the tilt angle change per day on each day, (except the last day), of the life of every spot group in each of the 14 sets defined above. Although the tilt angles are restricted to $\pm 90^\circ$, the tilt angle change per day for small and short-lived spot groups, can appear to vary as much as $\pm 90^\circ$, and most of it is caused by turbulence.

Thus, the data sequences used in the present investigation contain a listing of the spot groups identified on every day for which there is an observation, along with the following information pertaining to each spot group: (1) date and time of observation, (2) the days of the first and last sightings of the spot group and its life span, (3) the heliographic coordinates of the leader and the follower parts of the spot group, (4) the umbral area, (5) the tilt angle, and (6) the tilt angle change per day.

To study the evolution of the tilt angle of a spot group with respect to its latitude and life span (the latter as a proxy for the magnetic flux of the parent flux tube of the spot group) we introduce a symbol $\theta_{n,i,L,z}$ for the value of the tilt angle θ observed on the ith day of the life of the nth spot group in the set of spot groups specified by life span L and latitude zone z, where i varies from 1 to L. However, since we deal with each such set separately, the suffixes L and z are unnecessary, and we henceforth use the simpler symbol $\theta_{n,i}$ in the place of $\theta_{n,i,L,z}$, and similarly the symbol $\delta\theta_{n,i}$ for the change in the tilt angle of the nth spot group from day i to day $i + 1$.

3. EVOLUTION OF THE TILT AND ITS PHYSICAL INTERPRETATION

3.1. *First Day Tilt Angle*

We define the *first day tilt angle* of a spot group as its tilt measured on the surface on the day of its birth, i.e., $\theta_{n,1}$. This parameter is an important one for the study of the evolution of the tilt, and so we determine this parameter for all those bipolar and multipolar spot groups that are identified as having been born on the disk.

3.2. *Tilt Angle versus Tilt Angle Change Per Day*

Our data bank contains measures of the tilt angle on every day of the life of every spot group during its passage across the disk right from the day it is born and for all the days of observations. From these, we have derived the tilt angle change per day for each spot group. In Figure 2 we have plotted the daily tilt angle change $\delta\theta_{n,i}$ versus tilt angle $\theta_{n,i}$ for all spot groups of life spans 2–7 days, of all areas and over the whole Sun.

We find that such a plot, for *each* of the sets defined in § 2, fits the linear regression

$$\delta\theta = a - k\theta,$$

which can be approximated, on timescales >1 day, to the differential equation,

$$\frac{d\theta}{dt} = -k(\theta - \theta_x), \qquad (1)$$

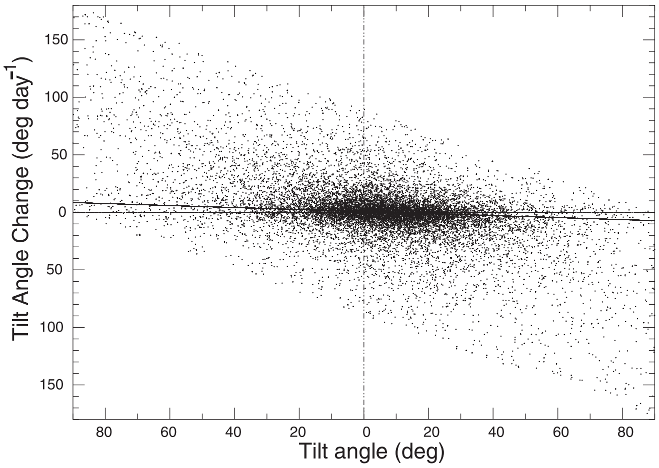

FIG. 2.—Plot of tilt angle (deg) vs. daily tilt angle change (in deg day^{-1}; i.e., rotation rates of the axes of spot groups) of spot groups of life spans 2–7 days, of all areas and over the whole Sun. Number of data points is 23,004. The straight line represents the least-square fit to the subsample reached after five iterations according to the procedure described in § 3.5.

where $\theta_x(=a/k)$ is the "intercept" on the θ-axis. This in principle is similar to what Howard (1996) did.

This linear fit for each set shows that the tilt angle θ of a typical spot group in the set evolves according to the differential equation

$$\frac{d}{dt}(\theta - \theta_x) = -k(\theta - \theta_x), \tag{2}$$

which has the solution

$$\theta(t) = \theta_x + (\theta_0 - \theta_x)\exp(-kt), \tag{3}$$

where

$$\theta = \theta_0 \quad \text{at} \quad t = 0 \tag{4a}$$

and

$$\theta \quad \rightarrow \quad \theta_x, \quad \text{the asymptotic tilt, as} \quad t \rightarrow \infty. \tag{4b}$$

The observed tilt on the surface consists of two components: a systematic or smoothly varying component and a random component caused by turbulence. Equations (1), (2), (3), (4a), and (4b) pertain to the smoothly varying component.

3.3. *Implication of the Linear Regression as Applied to the Dynamics of the Parent Ω Loop of a Typical Spot Group*

3.3.1. *Scenario for Subsurface Dynamics of the Parent Flux Loop of a Spot Group*

To bring out the physical interpretation of the smoothly varying component we construct a simplified scenario for the subsurface dynamics of the parent flux loop of a spot group using the dynamical model of LC02 as the template. These authors pointed out that in the top most region of 20– 40 Mm thick of the convection zone the forces exerted by convective turbulence on the parent flux tubes of spot groups (BMRs) exceed the magnetic forces within the flux tubes and below this region the magnetic forces overwhelm the forces exerted by the turbulence. We assume that the transition from the relative dominance of turbulence to that of magnetic forces occurs in a thin layer, which, for convenience in further discussion, we term the "transition layer" (layer B in Fig. 1). Thus, the model of LC02 together with the emergence models for the creation of tilt (references in § 1) would imply that during the rise of a parent flux loop from the tachocline to the transition layer (i.e., from layer A to layer B in Fig. 1), the magnetic and the Coriolis forces overwhelm the forces of turbulence. However, during the rise of the loop from the layer B to the surface S, and also after it has emerged above the surface S, the effect of the Coriolis force on the top portion of the loop would be negligible. Consequently, the random component of the observed variation of the tilt of a spot group is controlled mainly by the forces exerted by turbulence on the legs of the loop above the layer B, and the smoothly varying component is controlled by the forces exerted by magnetic tension in the legs of the loop in the region below the layer B (Fig. 1).

3.3.2. *Physical Interpretation of the Smoothly Varying Component of the Observed Tilt and the Linear Regression*

According to the above scenario the temporal variation of the tilt angle, $\theta(t)$, observed at the surface S, can be expressed as

$$\theta(t) = \theta_b(t) + \theta_{\rm turb}(t), \tag{5}$$

where $\theta_b(t)$ is the tilt of the line joining the two points on the flux loop, where it intersects the transition layer and $\theta_{\rm turb}(t)$ represents the additional effective contribution to the tilt by the day-to-day random buffeting by the turbulence above the transition layer. Here $\theta_b(t)$ constitutes the smoothly varying component of the observed tilt, and we call this the "basic tilt."

The amplitude of the day-to-day random changes in the observed tilt $\theta(t)$ of a spot group is comparable to or (often) larger than the average value of the tilt. Although the maximum possible life span in this study is 7 days, the vast majority of the spot groups are too short lived to bring out the trend in the evolution of $\theta_b(t)$. These factors do not permit us to separate $\theta_b(t)$ from $\theta_{\rm turb}(t)$ for the flux loop of any individual spot group from a plot of the daily observations of its tilt. Hence, we adopt a statistical approach to estimate the value of $\theta_b(t)$ from $\theta(t)$. For an ensemble of spot groups having parent flux loops of similar physical properties and similar depths of origin, the linear regression in the plot of $\delta\theta$ versus θ obtained from observations will sample the relation between $d\theta/dt$ and θ for those spot groups up to the linear order (as in § 3.2). The evolution of the basic tilt $\theta_b(t)$ of the parent flux loop of a *typical* spot group in such a sample can be represented by equations similar to equations (3), (4a), and (4b), and the values of k and θ_x can be determined from the regression.

Thus, the basic tilt θ_b *evolves* according to the equation

$$\theta_b(t) = \theta_x + [\theta_b(0) - \theta_x]\exp(-kt), \tag{6}$$

from its "initial value", $\theta_b(0)$ and *approaches* its "asymptotic value", θ_∞, as $t \rightarrow \infty$; the latter is given by

$$\theta_\infty = \theta_x, \tag{7}$$

and the timescale of evolution is

$$\tau = 1/k. \tag{8}$$

In fact, τ is the timescale of relaxation of the tilt due to the magnetic tension in the legs of the loop.

Equations (7) and (8) can be used to determine θ_∞ and τ for flux tubes of spot groups in any specified set, by determining θ_x and k from the linear fit to the sample of data points (θ, $\delta\theta$) obtained from those spot groups.

TABLE 1
ESTIMATES OF INITIAL BASIC TILT, ASYMTOTIC TILT, AND RELAXATION TIMESCALE

Life Span (days) (1)	No. of data points (2)	$\theta_b(0) \pm \sigma\theta_b(0)$ (deg) (3)	$\theta\infty \pm \sigma\theta\infty$ (deg) (4)	$k \pm \sigma k$ (day^{-1}) (5)	Timescale ($T=k^{-1}$day) (6)	Correlation Coefficient (7)	Standard Error (deg) (8)
			For Spot Groups in Latitudes $<13°$				
2	2119 (2380)	4.62 ± 0.24	4.06 ± 0.34	0.109 ± 0.022	9.17	0.28	9
3	1372 (1512)	2.43 ± 0.56	6.92 ± 0.37	0.106 ± 0.026	9.43	0.26	10
4	777 (872)	3.05 ± 1.21	5.21 ± 0.56	0.103 ± 0.036	9.73	0.28	9
5	475 (533)	1.06 ± 1.23*	5.70 ± 0.77	0.092 ± 0.046	10.9	0.24	9
6	294 (316)	0.40 ± 3.20*	5.57 ± 0.50	0.186 ± 0.059	5.37	0.37	12
7	199 (221)	6.25 ± 3.60	10.77 ± 0.69	0.137 ± 0.075	7.30	0.37	10
1–7 Combined	8936 (9930)	3.55 ± 0.05	5.53 ± 0.14	0.116 ± 0.011	8.62	0.30	9
			For Spot Groups in Latitudes $<13°$				
2	2526 (2732)	5.24 ± 0.18	7.60 ± 0.15	0.193 ± 0.020	5.18	0.39	12
3	1433 (1552)	4.63 ± 0.45	9.47 ± 0.21	0.166 ± 0.027	6.02	0.38	10
4	777 (872)	4.44 ± 0.94	3.10 ± 0.74	0.084 ± 0.035	11.9	0.23	10
5	475 (533)	2.81 ± 1.31	8.99 ± 0.63	0.094 ± 0.046	10.64	0.24	9
6	294 (316)	5.58 ± 2.27	8.58 ± 1.15	0.076 ± 0.062	13.16	0.25	8
7	199 (221)	4.14 ± 4.10	14.64 ± 0.29	0.198 ± 0.030	5.05	0.36	12
1–7 Combined	8936 (9930)	5.53 ± 0.03	6.72 ± 0.09	0.159 ± 0.010	6.28	0.36	11

NOTES.—Initial basic tilt $\theta_b(0)$ determined in § 3.4; asymptotic tilt θ_∞, and time scales of relaxation τ, determined by the final fit in the weighted iterative linear-fit procedure described in § 3.5. In column 2 are the number of data points with nonzero weights in the final fit and the figures within the brackets are the number of data points with which the iterative procedure was started. Values of $\theta_b(0)$ where the error $\sigma\theta_b(0)$ exceeds $\theta_b(0)$ are marked with an asterisk (*) and have been omitted while defining the range of $\theta_b(0)$.

3.4. *Determination of the Initial Basic Tilt $\theta_b(0)$ from Observations*

We now proceed to determine the typical value of the "initial basic tilt" $\theta_b(0)$ for the parent flux loops of spot groups in each of the 14 sets defined in § 2.

As pointed out by LC02, the buffeting effect of turbulence on the observed tilt will be considerable after the top of the Ω loop of a flux tube has crossed the transition layer and before it appears on the solar surface. The effects of the Coriolis force will be negligible after the top of the loop has crossed the transition layer. Hence, $\theta_{n,1}$, the observed *first-day tilt* of a spot group n at the surface will be related to the *initial basic tilt* $\theta_b(0)$, of its parent flux tube by

$$\theta_b(0) = \theta_{n,1} - \Delta\theta_n, \quad (9)$$

where the $\theta_b(0)$ will depend mainly on the typical physical properties of the parent flux tubes of spot groups in the selected set (and much less on individual spot group n), and $\Delta\theta_n$ is the *random tilt* imposed by turbulence on the top parts of the parent loop of the spot group n, *before* the top emerges on the surface. The ensemble mean $\langle\Delta\theta_n\rangle$, over the selected set, will be ≈ 0. Hence, equation (9) reduces to

$$\theta_b(0) = \langle\theta_{n,1}\rangle, \quad (10)$$

where $\langle\theta_{n,1}\rangle$ is the average of the observed *first-day* tilts, $\theta_{n,1}$ with respect to n.

This procedure for estimating $\theta_b(0)$ is possible since in each set the uncertainty $\sigma\theta_b(0)$, in $\theta_b(0)$, as given by the standard deviation of $\theta_{n,1}$, is much smaller than $\langle\theta_{n,1}\rangle$ except when $\theta_b(0)$: itself is small, as can be seen in Table 1. The values of the initial basic tilt $\theta_b(0)$ so evaluated for spot groups of given life spans in the two latitude belts are presented in Table 1.

3.5. *Determination of Asymptotic Basic Tilt θ_∞ and Timescale of Relaxation τ*

3.5.1. *Presence of "Inadmissible Points" in the Data Sets*

Linear fits to the data sets defined by the life span and latitude of spot groups as described in § 3.3.2 (eqs. [7] and [8]) yield values of θ_∞ and the slope. Values of θ_∞ so obtained are larger than the corresponding values of $\theta_b(0)$ determined in § 3.4, and values of the slope k lie between 0.34 and 0.44, for spot groups of all categories irrespective of the life span.

These preliminary results warrant further scrutiny for the following reasons:

1. The values of θ_∞ larger than the corresponding values of $\theta_b(0)$ imply that the magnetic relaxation increases the tilts of the flux tubes instead of decreasing as expected conventionally.
2. The slope $k > 0.34$ corresponds to relaxation time $\tau < 3$ days.

In the flux tube of a typical spot, the Alfvén speed falls steeply from $\sim$10 km s^{-1} at the surface to $\sim$0.1 km s^{-1} at the critical layer (Fig. 1). From this we estimate that in 3 days the Alfvén wave would travel only a distance of <40 Mm. Hence, the relaxation time <3 days pertains to flux tubes anchored at depths <40 Mm. Thus, in the context of §§ 3.3.1 and 3.3.2, result 2 means that each data set contains a substantial contribution from spot groups after the anchorings of their flux tubes have risen above the critical layer. The data points from such flux tubes will represent tilt variations that contain hardly any systematic part and should therefore be considered inadmissible in the determination of θ_∞ and k. Inclusion of such data points in the linear fits will vitiate the estimates of the parameters θ_∞ and k for flux tubes anchored below the critical layer, which are the ones we are looking for.

3.5.2. *Minimization of the Effects of Inadmissible Points by Weighted Iterative Linear-Fit Procedure*

It is known from earlier studies on rotation rates of spot groups that during last 1 or 2 days of the their lives the anchoring of their flux tubes should rise to layers <40 Mm (Javaraiah & Gokhale 1997; Sivaraman et al. 2003). From this we roughly estimate that the percentage of the above described inadmissible data points in our data sets will vary from >50% for spot groups of life span $\sim$1 day, to $\sim$28% for spot groups of life span 7 days. Their exclusion through individual identification is not practicable.

Their undesirable effect on the determination of θ_∞ and k can, however, be minimized *statistically* using the iteratively weighted linear least-square fit procedure described by Mosteller & Tukey (1977), on the following rationale. Presumably the residuals of admissible and inadmissible points with respect to the linear fit will both have zero-mean normal distributions. The distribution of the inadmissible points will be wider than that of the admissible points, and in all categories with a life span of 2 or more days, the number of admissible points will exceed that of the inadmissible points. As a result, the center of the overall distribution will contain a larger number of the admissible data points than the inadmissible ones, and in the tail the situation will be reversed.

3.5.3. *Weighted Iterative Linear-Fit Procedure*

In this procedure the effects of the inadmissible data points are minimized systematically through a series of "weighted" linear fits. The first linear fit is calculated by attaching equal statistical weights to all the data points. In the second and subsequent fits each data point is assigned a new weight that decreases (first slowly and then rapidly) with the magnitude of the residual. The new weight is zero for points with residuals far out in the tail. After each Kth linear fit, new statistical weights for the $K+1$th fit are assigned as

$$W_{K+1}(i) = \begin{cases} \left(1 - U_K^2\right)^2 & \text{for } U_K^2 < 1 \\ 0 & \text{for } U_K^2 \geq 1 \end{cases} \tag{11}$$

where

$$U_K(i) = \frac{y(i) - Y_K(i)}{cS_K}, \tag{12}$$

where $y(i)$ is the ordinate of the ith data point, $Y_K(i)$ is the corresponding value given by the Kth linear fit, S_K is the median of the distribution of the residuals $[y(i) - Y_K(i)]$ from the Kth, and c is a constant that needs to be chosen suitably (according to Mosteller & Tukey, $c = 6$–9 is found adequate for most applications). Even by this procedure the effect of inadmissible data points is not eliminated totally, because the distribution of these points and that of the admissible points both peak near the zero value. The procedure has to be halted when the effect of the inadmissible data points is so minimized that the estimates of the coefficients can be accepted as adequately reliable (e.g., when the differences between successive values of each fitting coefficient are sufficiently small).

3.5.4. *Results Obtained Using the Above Procedure*

We applied the above iterative linear-fit procedure to each of the sets of data. While doing so we chose $c = 6$, and halted the iterations (accepted the values of θ_∞ and k as adequately reliable) when the values of θ_∞ and k given by successive linear fits differed by less than 10%. The accepted values of k and θ_∞ given by the final fit (where the iterative procedure was halted), and the corresponding values of the relaxation time τ are presented in Table 1 for the two latitude belts.

4. RESULTS AND THEIR INTERPRETATION

4.1. *Dependence of the Initial Basic Tilt, the Asymptotic Tilt, and the Relaxation Timescale on the Properties of Spot Groups*

4.1.1. *On the Life Span of Spot Groups*

Neither the initial basic tilts $\theta_b(0)$, nor the asymptotic tilts θ_∞ and the relaxation timescales τ in Table 1 show any systematic dependence on the life span of the spot groups. This suggests that the sets of spot groups in each life span range might still be heterogeneous with respect to some other physical property of their parent flux tubes, such as, for example, the magnetic flux content and age. To bring out the dependence of $\theta_b(0)$, θ_∞, and τ on such properties, the data sets may have to be further divided with respect to such new parameters. We propose to undertake this in a future study. In the remaining sections of this paper we discuss the more fundamental issues that arise out of the results from Table 1.

4.1.2. *On the Latitude of Spot Groups*

The values of $\theta_b(0)$ and θ_∞ are in general higher in higher latitudes >13° than in the lower latitudes <13° (Table 1). The relaxation timescale τ does not seem to depend significantly on latitude.

4.2. *Nonzero Inborn Tilts*

Based on the theoretical models of the creation of the tilt on the top part of a flux loop by the Coriolis force (see references in § 1), we can relate the initial basic tilt $\theta_b(0)$ of our observational model to the inborn tilt θ^*, which is the tilt the parent flux loop possesses at the depths of its origin (at the layer A in Fig. 1) by the relation,

$$\theta_b(0) = \theta^* + \theta_{\rm cor}. \tag{13}$$

TABLE 2

Ranges of Values of $\theta_b(0)$, θ_∞, and τ

Estimated Parameter	Latitudes <13°	Latitudes >13°
$\theta_b(0)$ (deg)	2.43–6.25	2.81–5.58
θ_∞ (deg)	4–11	3–15
τ (day)	5–11	5–14

Note.—As found in the two latitude zones from Table 1. Estimates exceeded by uncertainties are not taken into account while determining these ranges.

Here $\theta_{\rm cor}$ is the tilt imparted to the top of the flux loop by the Coriolis force over and above the tilt θ^* during its rise from the depth of origin to the transition layer. The effect of turbulence during this rise can be neglected.

From Table 1 we note that $\theta_b(0)$ and θ_∞ generally satisfy

$$\theta_b(0) < \theta_\infty, \tag{14}$$

except in one set (life span of 4 days and latitude of >13°), where also it is probably satisfied in view of the uncertainties. The general validity of the inequality in (eq. [14]) is clear also from Table 2.

According to our observational model described in § 3.3, the long-term relaxation of the basic tilt θ_b takes place from its initial value $\theta_b(0)$ to its asymptotic value θ_∞, and according to LC02 this asymptotic value would be the inborn tilt when the relaxation is complete. Hence, one must have either

$$\theta_\infty = \theta^*, \tag{15a}$$

in which case the relaxation will be *complete*, or

$$\theta_\infty < \theta^*, \tag{15b}$$

in which case the relaxation will remain *incomplete*.

In either case, inequality (14) implies

$$\theta_b(0) < \theta^*, \tag{16}$$

i.e., the tilt $\theta_b(0)$ at the layer B *is generally less than* the tilt θ^* at the depths of origin of the flux loops. Inequality (16) might appear perplexing if one is guided by the perception that the relaxation by magnetic tension should lead only to a reduction in the tilt. However, the magnetic tension makes short-term restoring contributions to the observed tilt, with signs opposite to the random perturbations due to turbulence, and in the long run this short-term process will provide an overall relaxation of either sign. Thus, the magnetic relaxation can effect either an *increase or decrease* of the basic tilt θ_b, which can only be inferred by checking whether the observational data yields $\theta_b(0) < \theta_x$, or $\theta_b(0) > \theta_x$. Thus, the inequality (16) does not conflict with the basic principles in the model of LC02 for the subsurface dynamics of the flux loops, but it does contradict their ad hoc assumption that the tilts of the flux loops are parallel to the equator at the depths of their origin, adopted from the dynamo model of Choudhuri & Gilman (1987). From equations (14), (15a), and (15b), which follow from the scenario in § 3.3, along with the values of $\theta_b(0)$ and θ_∞ from Table 1, it is clear that the values of the *inborn tilts* (i.e., the tilts with which the parent flux tubes start to rise from the shear layer) lie in the range ~4°–11° in latitudes <13° and ~3°-15° in latitudes >13°, and *not zero*.

4.3. *Negative $\theta_{\rm cor}$ and Its Implication*

According to inequality (16) the values of the *initial basic tilt* $\theta_b(0)$ acquired by the loop during its rise to the transition layer are lower than the *inborn tilt* θ^*. Taken along with equation (13), this shows that the tilt imparted by the Coriolis force on the top part of the omega loop as it rises from the depth of origin to the transition layer ($\theta_{\rm cor}$) is *negative*. It is this negative contribution that *reduces* the tilt from the higher inborn value to the lower value of the initial basic tilt. This result, although surprising deserves acceptance as true, since it comes from the analysis of the same reliable data set that was used for earlier studies (Sivaraman et al. 1999; Howard et al. 2000) and by a similar method, but with several improvements listed in § 1.

The negative contribution from Coriolis force will be possible if the lower parts of the flux loop's legs are bent inwards (e.g., as in the real Ω shape, or as in a topologically closed loop), where the down flows in these parts will have *converging* horizontal components in contrast to the *diverging* horizontal components of down flows in the upper parts. The Coriolis force would in that case create a *negative* tilt in the lower parts of the loops, and this (by virtue of their stronger magnetic field) could overwhelm the positive tilt contributed by the Coriolis force on the weaker upper parts.

4.4. *Timescales of Relaxation τ*

According to Table 1, the timescales of relaxation, τ of the tilt of a spot group of life span between 2 to 7 days toward the inborn tilt θ^* lies in the range ~5–14 days. To provide magnetic relaxation in a flux loop in 5 to 14 days, an Alfvén wave traveling along the loop will have to make *at least* one return trip between the top of the loop and the anchoring of the flux loop where the wave spends major part of the travel time. For this to happen in the case of flux tubes anchored at the tachocline, the field strength $\langle B \rangle$ in the anchoring will have to be *at least* ~14–40 kG. This range is close to that derived by Fan et al. (1994) in their model of the rise of a flux tube. In the case of loops that are topologically disconnected from the tachocline (a possibility suggested in § 4.3), the relaxation timescale of ~5–14 days would still be consistent with the limits ~14–40 kG on $\langle B \rangle$, if the magnetic field in the subsurface parts of the loops weakens sufficiently during the rise. It would be interesting to note that if the loops are topologically disconnected, the whole loop system will eventually rise to very shallow depths. This will ultimately bring their anchoring to depths <40 Mm (i.e., above the critical layer), which will facilitate the transport of the residual magnetic flux of the low-latitude active regions to the high latitudes as envisaged in the model of Wang et al. (1989).

5. CONCLUSIONS AND DISCUSSIONS

We have analyzed the data from the Kodaikanal observatory on tilts of spot groups, compiled by Sivaraman et al. (2003), in sequences with values of tilt of each spot group as a function of its age and for each day of observation. From the analysis of these sequences we have obtained reliable estimates of the initial basic tilts $\theta_b(0)$ (i.e., the tilt just below the layer B in Fig. 1), the asymptotic tilts θ_∞, and the relaxation timescales τ, of the parent magnetic flux loops of typical spot groups of specific life spans in the range 2–7 days occurring in latitude zones <13°, and >13°. These estimates are presented in Table 1. We interpret our results in the light of the scenario implied by the dynamical model of LC02 and arrive at the following conclusions. (1) Parent flux tubes of spot groups possess *inborn* tilts θ^* (i.e., tilts of the flux tubes at the depths of their origin) of ~4°–11° in latitudes < 13° and of ~ 3°–15° in latitudes >13°. This is the tilt of the toroidal field in the tachocline. (2) The values of the initial basic tilts $\theta_b(0)$ (i.e., tilts of the flux loops just below the layer B in Fig. 1) are in the range of ~2°–6° in latitudes <13° and ~3°– 6° in latitudes >13° and are thus lower than the inborn tilts θ^* in the two latitude belts. This would imply that the Coriolis force acting on the flux loop during its rise through the convection zone tends to *reduce* the tilt of its top parts from the inborn tilt to the initial basic tilt. (3) After the emergence of the top of the flux loop above the surface, magnetic tension *increases* the basic tilt and tends to restore it back toward the inborn tilt on relaxation timescale of ~5–14 days. These timescales are possible if the field strengths $\langle B \rangle$ of the Ω loops at the anchoring depths are in the range ~14–40 kG, which is close to the limits of ~20–40 kG derived by Fan et al. (1994).

Conclusion (1) contradicts the assumption made in the theoretical models that the tilt of the toroidal field in the tachocline is zero or that the inborn tilts of the parent loops are zero. Conclusion (2) contradicts the *sign of* the effect of the Coriolis force on the parent loop during its rise as inferred in all emergence models. As explained in § 4.3, these contradictions are results that have been brought to light by the improvements introduced by us in the data analysis (listed in § 1), as well as improvements in the interpretation of the results derived.

The possibility that the initial flux tubes could be tilted even before they enter the convection zone has been pointed out, although intuitively, by Longcope & Fisher (1996). According to them, there is no reason to believe that the solar dynamo produces toroidal field that is perfectly straight (i.e., oriented along the latitudes). Furthermore, in Gilman's (2000) view, one of the unanswered questions pertaining to the toroidal flux residing in the tachocline is whether this field is purely toroidal or has a poloidal component also. According to Gilman, the dynamo action by differential rotation shearing a broad poloidal field into a toroidal field would contain some poloidal component as well. Our first conclusion validates the intuitive guesses by Longcope & Fisher (1996) and by Gilman (2000) and relieves the theoretical models of the solar dynamo of an ad hoc constraint.

Our second conclusion ($\theta_{\rm cor} < 0$) suggests that the parent Ω loops of spot groups (BMRs) should have the lower parts of their legs bent inwards (e.g., as in the real Ω shape, or as in a topologically closed loop). We believe that such bends could be possible during the rise of the loops when the magnetic tension becomes comparable to the buoyancy and Coriolis forces, as in the case of loops that have lengths comparable to the observed polarity separation of the spot groups (BMRs).

Hence, models for the origin of tilts of spot groups (BMRs) like that of D'Silva & Choudhuri (1993) should be calculated using loop lengths <400 Mm (corresponding to azimuthal wavenumbers $m > 8$). Loops of lengths <400 Mm will have the trajectories of their tops and rise timescales in disagreement with the dynamo model of Choudhuri & Gilman (1987), which assumes toroidal flux ropes in the tachocline to be strictly parallel to the local latitude. But now that this assumption is shown to be no longer applicable, being inconsistent with the values of θ^* and $\theta_b(0)$ derived from the observed tilt angles and their daily changes, a reexamination of the problem of the creation of the parent flux loops is called for. Thus, our inferences $\theta^* \neq 0$ and $\theta_b(0) < \theta^*$ (i.e., $\theta_{\rm cor} < 0$), obtained from the analysis of observational data, provide new challenges for theoretical modeling of the dynamics of flux tubes in the Sun and for the theory of solar dynamo.

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REFERENCES

Caligari, P. F., Moreno-Insertis, F., & Schussler, M. 1995, ApJ, 441, 886
Choudhuri, A. R. 1989, Sol. Phys., 123, 217
Choudhuri, A. R., & Gilman, P. A. 1987, ApJ, 316, 788
D'Silva, S., & Choudhuri, A. R. 1993, A&A, 272, 621
Fan, Y., Fisher, G. H., & De Luca, E. E. 1993, ApJ, 405, 390
Fan, Y., Fisher, G. H., & Mc Clymont, A. N. 1994, ApJ, 436, 907
Fisher, G. H., Fan, Y., Longcope, D. W., Linton, M. G., & Pevstov, A. A 2000, Sol. Phys., 192, 119
Gilman, P. A. 2000, Sol. Phys., 192, 27
Gilman, P. A., Morrow, C. A., & De Luca, E. E. 1989, ApJ, 338, 528
Gupta, S. S., Sivaraman, K. R., & Howard. R. F. 1999, Sol. Phys., 188, 225
Hale, G. E., Ellermann, F., Nicholson, S. B., & Joy, A. H. 1919, ApJ, 49, 153
Howard, R. F. 1996, Sol. Phys., 167, 95
Howard, R. F., Gilman, P. A., & Gilman, P. I. 1984, ApJ, 283, 373
Howard, R. F., Sivaraman, K. R., & Gupta, S. S. 2000, Sol. Phys., 196, 333
Javaraiah, J., & Gokhale, M. H. 1997, A&A, 327, 795
Longcope, D. W., & Choudhuri, A. R. 2002, Sol. Phys., 205, 63 (LC02)
Longcope, D. W., & Fisher, G. H. 1996, ApJ, 458, 380
Mosteller, F., & Tukey, J. W. 1977, Data Analysis and Regression: A Second Course in Statistics (Reading: Addison-Wesley)
Rudiger, G., & Brandenburg, A. 1995, A&A, 296, 557
Schussler, M., Caligari, P., Ferriz-Mas, A., & Moreno-Insertis, F. 1994, A&A, 281, L69
Sivaraman, K. R., Gupta, S. S., & Howard, R. F. 1999, Sol. Phys., 189, 69
Sivaraman, K. R., Sivaraman, H., Gupta, S. S., & Howard, R. F. 2003, Sol. Phys., 214, 65
Toth, L., & Gerlei, O. 2004, Sol. Phys., 220, 43
Wang, Y.-M., Nash, A. G., & Sheeley, N. R. 1989, ApJ, 347, 529
Wang, Y.-M., & Sheeley, N. R. 1991, ApJ, 375, 761