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Some statistical aspects of cometary orbits and their discoveries*

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Abstract. The study of the cometary orbital elements available in the tenth edition of the 'Catalogue of Cometary Orbits' (Marsden and Williams, 1995) which hereafter will be written as the 'Catalogue 1995' and the perpetual discoveries of comets reveal various interesting features.

The 'Catalogue 1995' contains 1472 orbital elements of 878 individual comets which were possible to observe for displaying 1444 apparitions during the period from 239 B.C. to 1994 A.D. Among these comets, 184 are designated as the Short Period (SP) comets, for having periods less than 200 years. The remaining 694 comets, are termed as the Long Period (LP) comets, since their periods are longer than 200 years. All the orbital elements have been studied thoroughly and found how the orbital inclinations, perihelion distances and aphelion distances are distributed.

The analysis of the proximity of aphelia distances of SP comet orbits from their respective nearby planetary orbits reveals that the orbits of major planets are closely visited by most of the SP comet aphelia. The set of comets whose aphelia closely visit a particular planetary orbit, are said to constitute a family of comets of the said planet. Classifying these comets, it has been noticed that the Jupiter Family of Comets (JFC) are the largest in number. Also the nodal distances of all the comets have been studied in respect of their distributions. It has been found that 91% of the nodes are within the orbital distance of the planet Neptune at 30 AU away from the Sun.

The consequences of the aphelia and nodes of the SP comets being closer to their nearby planetary orbits, have been studied. This study justifies the existence of a number of multiple nuclei comets as well as predicts the possible split of a few more SP cometary nuclei which may finally crash into any one of the perturbing major planetary globes, like the one C/1993 F2 (Shoemaker-Levy 9). Interestingly, it can

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be inferred from these studies that any group of comets having almost identical orbital elements, may be the outcome of the split suffered by their parent cometary nuclei at a distance far away from the Sun.

The discovery of comets was initiated by Gottfried Kirch in 1680 and the recovery of periodic comets was first done by J.G. Palitzsch in 1758. The rate at which the comets are discovered and also recovered, have been shown.

Key Words : cometary orbits, orbital elements, nodal distributions, split comets, cometary discovery.

1. Historical references

It is not known when the comets started to orbit around the Sun. Nor it is known from when man began to observe the comets even without realising its physical structure and orbital character. These peculiar kind of celestial visitors, in course of time, earned a special attention as the objects of awe, omen and superstition in human mind. The earliest available record of cometary observation dates back to 1000 B.C. by the Chinese. In the ancient Indian literature, the Vedas which came into being during the period from 3000 B.C. to 600 B.C., there is a reference to the comet in a hymn, viz. "Sṁ no mrtyurdhumketu" (Atherva Veda, 19-9-10), which is a prayer to get rid of its deadly menace.

However, the earliest available recorded data of any cometary orbit is of the comet of 239 B.C. This comet 1P/-239 K1 (Halley), as is now designated, was observed by Sir Edmund Halley (1656-1742) at its 26th recorded apparition in 1682. After examining the orbital elements of some two dozen comets, he found that the comets of 1456, 1531 and 1607 have nearly the same orbital elements as those of the comet 1682. He then made a prolonged study of these elements by the application of Sir Isaac Newton's (1642-1727) newly formulated Law of Gravitation. This study enabled him to know that these comets are the successive apparitions of the same comet 1P/1682 Q1. Finally in the year 1705, he predicted for the first time in the history of cometary studies that the comet of 1682 would return again in the year 1758. The comet did return in the year predicted by Halley. The comet was recovered by a well-to-do German farmer and amateur astronomer named Johann Goerg Palitzsch (1723-88) on 25 December, 1758. With the first recovery of the comet 1P/1758 Y1, not only the validity of prediction for the return of comet 1P/1682 Q1 made by Halley was proved, but also it conclusively demonstrated the practical verification of the Law of Gravitation. To honour the first ever successful prediction of cometary return, the comet was since named as the Halley's comet.

2. Classified orbital statistics

The 'Catalogue 1995' contains only 1472 sets of orbital elements of 878 individual comets. These comets were observed to make 1444 apparitions since 239 B.C. From the standpoint of different values of eccentricities and periods of comet orbits, these can be divided into two classes : The Short Period comets and the Long Period comets. The SP comet orbits are essentially elliptic. But the LP cometary orbits may be elongated elliptic, parabolic or hyperbolic. The 'Catalogue 1995' contains 33 defunct SP comet orbits of which 5 comets made multiple apparitions and 28 comets made single apparitions and all these comets have disappeared. Now these 33 comets are not known to exist.

The 1472 sets of orbital elements are of those 878 comets which have been observed since 239 B.C. by the watchful observers mostly from the night-side of the Earth's sky. There is no statistics of the innumerable other unobserved comets which might have visited solar arena even from the night-side of the sky prior to 239 B.C., and from day-side sky through the glare of the Sun, and which might have eluded observations by some other means, ever since the cometary studies started. So the statistical investigations on the cometary orbits made in this paper are restricted to those 1472 sets of orbital elements contained in the 'Catalogue 1995'.

Table 1. The classification of comet orbits and their statistics.

Comet orbits	No. of comets	No. of apparitions	No. of orbits	Multiple Nuclei	
				Comets	No.
I. Short period comets Period < 200 yrs.					
a) Multiple apparitions :					
Defunct comets	5	19	21	3D/1846 A, B	2
				3D/1852 A, B	2
Periodic comets	111	663	663		
b) Single apparition :					
Defunct comets	28	28	48	D/1993 F2 - A, B, . . W*	19
				D/1994 P1 - A, B, C	3
Periodic comets	40	40	40		
TOTAL	184	750	772		
II. Long period comets Period > 200 yrs					
a) Elliptic orbits :					
Period < 1000 yrs.	47	47	50	C/1882 R1-A, B, C, D	4
				C/1965 S1-A	1
Period > 1000 yrs.	163	163	165	C/1947 X1-A,B	2
				C/1965 S1-B,	1
b) Parabolic orbits	347	347	348	C/1860 D1-A,B	2
c) Hyperbolic Orbits	137	137	137		
TOTAL	694	694	700		
GRAND TOTAL	878	1444	1472		

* I, J, M and 0 are omitted

3. Orbital elements

The cometary orbits are essentially conics having one of their focii at the Sun. For an unique identification of comet's orbit, it requires to know all the six of its orbital elements. The ecliptic plane AB containing the Earth's orbit and the cometary orbital plane EF are intersected along the line ML which is called the nodal line (Fig. 1). The Sun S lies on the nodal line ML, as also the ascending node N and the descending node N^1, the two points where the cometary orbit intersects the ecliptic plane from South to North and North to South hemispheres respectively. The elements of cometary orbits are determined with reference to

(i) the Sun S at one of its focii,
(ii) the direction of the first point of Aries $\overrightarrow{S\gamma}$ along the ecliptic plane,
(iii) the nodal line ML, and
(iv) the perihelion distance PS on the cometary orbital plane (Figure 1).

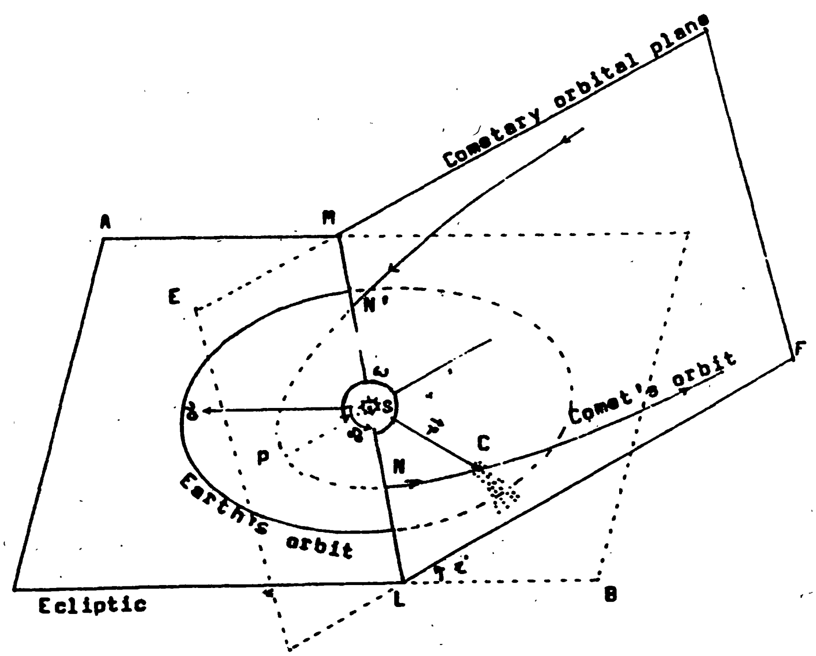

Figure 1. The elements of cometary orbit.

The six orbital elements of a comet are :

i - the inclination of the orbital plane EF with the ecliptic AB, ∠BLF;
q - the shortest distance of a cometary orbit from the Sun called the perihelion distance, PS;
Ω - the longitude of ascending node, ∠γ SN;
ω - the argument of perihelion, ∠NSP;
e - the eccentricity of the cometary orbit
T - the time of perihelion passage.

The elements of a cometary orbit can be determined by solving the following equation of motion obtained from the Two-Body Problem :

$$\ddot{\vec{r}} = \frac{GM}{r^2}\,\frac{\vec{r}}{r} \tag{1}$$

where, G is the constant of Gravitation,

M is the sum of the masses of the Sun M_0 and that of the comet m_c, i.e., $M=M_0+m_c$; and

$\vec{r}$ is the position vector of the comet at C, with respect to the pole at Sun S and the initial line along SP.

3.1 Eccentricity e

The 'Catalogue 1995' contains 1472 orbits of which 987 are elliptical ($0<e<1$), 348 parabolic ($e=1$) and 137 are hyperbolic ($e>1$) (Table 1). These 987 elliptical orbits are described by 184 SP and 210 LP individual comets. The elliptical orbit of the long period comet, C/1910 A1 (Great January Comet), has the largest value of eccentricity of 0.999995 and the elliptic orbit of short period comet, 29P/Schwassmann-Wachmann 1 has been observed to have the smallest value of eccentricity of 0.044660 at the time of its last perihelion passage in 1989. Among the 137 hyperbolic orbits, in the orbit of the comet C/1975 V2 (Bradfield) has an eccentricity of 1.000001 which is minimum for any such orbit, whereas the maximum value of eccentricity of a hyperbolic orbit is 1.057322 of the comet C/1980 E1 (Bowell). Thus the range of eccentricity of cometary orbits is from 0.044660, the lowest to 1.057322, the highest, as observed until 1994.

3.2 Semi-major axis a and period P

There are 348 parabolic orbits and 137 hyperbolic orbits listed in the 'Catalogue 1995' (Table 1). These 485 orbits are all non-periodic. Among the 394 periodic comets which have been observed to describe 987 elliptic orbits, the semi-major axis of the SP comet 2P/Encke measuring 2.209 A.U. and its orbital period of 3.28 years are the corresponding shortest values. The LP comet C/1992 J1 (Spacewatch) has the longest period of 25182543.47 years, as it orbits along an elongated elliptical path with the largest semi-major axis of 85914.5 A.U. (Table 2).

Table 2. List of longest periodic comets

Comet	Perihelion q A.U.	Eccentricity e	Semi-major axis a A.U.	Period P Yrs.	Aphelion Q A.U.
C/1992 J1	3.007007	0.999965	85914.49	25182543.47	171825.96
C/1937 N1	0.862744	0.999985	57516.27	13793866.93	115031.67
C/1972 X1	4.860748	0.999910	54008.30	12551363.15	108011.76
C/1910 A1	0.128975	0.999950	25795.00	4142889.00	51589.00
C/1958 D1	1.322689	0.999943	23205.07	3534877.18	46408.82
C/1889 G1	2.255596	0999818	12393.38	1379700.72	24784.51

3.3 Inclination i

The characteristics of the orbital inclination of SP comet orbits are entirely different from those of LP comets. While the SP comet orbits are rarely oblique to the extent of more than 30°, the LP comet orbits are inclined at all angles to the ecliptic (Figure 2). If we consider the orbits of SP comets alone, then it is observed that 87% of these orbits have inclination within 30° (Table 3). There are very few SP comet orbits which have inclinations more than 90°, whereas the measures of inclinations of the LP comet orbits are almost equally divided on the either sides of the normal to the ecliptic plane. Out of 694 LP comet orbits 47% have inclination less than 90°. It is remarkable that the distribution of LP comet orbits having lower angular inclinations to the ecliptic plane are comparatively less than those of higher ones.

Thus it can be concluded that the SP comet orbits are the product of perturbations exerted on the comets while they were passing close to the giant planets, in particular, Jupiter at the time of crossing their respective nodes.

Table 3. Distribution of orbital inclination.

Inclination i	No. of SP Comets	No. of LP Comets	No. of Total Comets
0° - 10°	83	20	103
10° - 20°	54	21	75
20° - 30°	23	22	45
30° - 40°	8	24	32
40° - 50°	4	52	56
50° - 60°	2	36	38
60° - 70°	2	52	54
70° - 80°	1	53	54
80° - 90°	2	44	46
90° - 100°	1	50	51
100° - 110°		47	47
110° - 120°	1	41	42
120° - 130°		52	52
130° - 140°	1	53	54
140° - 150°		67	67
150° - 160°		29	29
160° - 170°	2	19	21
170° - 180°		12	12
	184	694	878

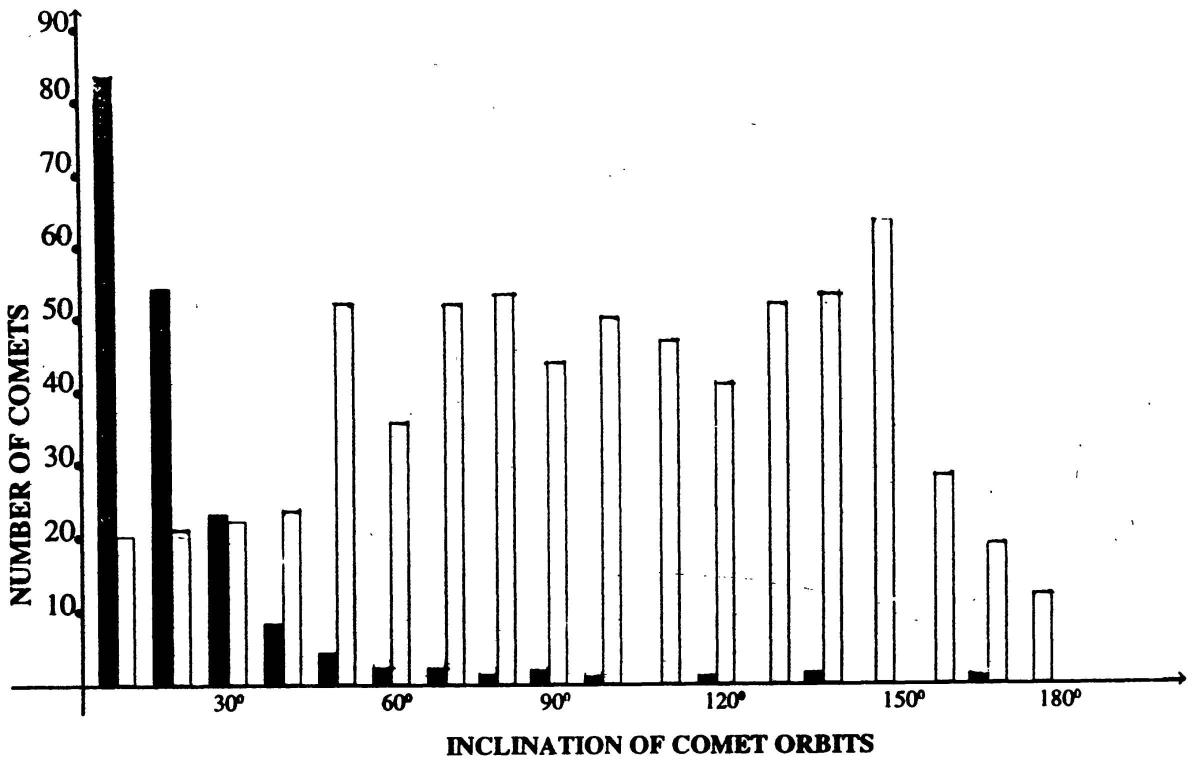

Figure 2. Comparative study of the distribution of inclinations of the SP comet orbits [hatched box] and that of the LP comet orbits [open box]

3.4 Perihelion distance q

The study of perihelion distances q of comet orbits reveals a few interesting features. The value of q range from 0.00450 A.U. (C/1981 VI, SOLWIND 4), the smallest one, to 8.45394 A.U. (95P/Chiron), the longest one. Thus the values of q are spaced from well within the radius of Sun which is 0.00465 A.U. to a distance close to the orbital distance of Saturn at 9.56 A.U. The 25 comets which have crossed their respective perihelia at distances close to the Sun, are listed in Table 4. These comets have the smallest perihelia distances. There are 13 comets which have passed their respective perihelia beyond the distances equivalent to that of the orbit of Jupiter at 5.2 A.U. These 13 comets which are listed in Table 5, have the longest perihelia.

The comets whose perihelia are closed to the Sun, display all the cometary phenomena, such as, coma and tail formation. A fully developed comet offers a chance for prolonged observations followed by easier discovery. That is why the comets with shorter perhelion distances are discovered in higher numbers (Figure 5).

Table 4. List of comets with smallest perihelia and passed through the Roche's Limit of the Sun (0.01618 A.U.)

Sl. No.	Comet	Discoverer	Eccentricity e	Perihelion Distance q in A.U.	Perihelion Long. L°	Perihelion Lat. B°
1.	C/1981 V1	SOLWIND 4	1.0	0.00450	282.7	+ 35.2
2.	C/1989 N3	SMM 9	1.0	0.00462	282.7	+ 35.2
3.	C/1989 S1	SMM 10	1.0	0.00476	282.7	+ 35.2
4.	C/1979 Q1	SOLWIND	1.0	0.00480	282.7	+ 35.2
5.	C/1887 B1	Great Southern Comet	1.0	0.00483	282.5	+ 35.4
6.	C/1963 R1	Pereyra	0.999946	0.005065	282.6	+ 35.3
7.	C/1988 T1	SMM 5	1.0	0.00513	282.7	+ 35.2
8.	C/1988 M1	SMM 3	1.0	0.00516	282.7	+ 35.2
9.	C/1987 T2	SMM 1	1.0	0.00583	282.7	+ 35.2
10.	C/1880 C1	Great Southern Comet	1.0	0.00549	282.4	+ 35.2
11.	C/1843 D1	Great March Comet	0.999914	0.00553	282.6	+ 35.3
12.	C/1989 L1	SMM 8	1.0	0.00557	282.7	+ 35.2
13.	C/1988 U1	SMM 7	1.0	0.00579	282.7	+ 35.2
14.	C/1988 W1	SMM 6	1.0	0.00590	282.7	+ 35.2
15.	C/1988 Q1	SMM 4	1.0	0.00591	282.7	+ 35.2
16.	C/1981 O1	SOLWIND	1.0	0.00612	282.7	+ 35.2
17.	C/1680 V1	Gottfried Kirch	0.999986	0.00622	272.0	- 8.2
18.	C/1987 U4	SMM 2	1.0	0.00627	282.7	+ 35.2
19.	C/1945 X1	du Toit	1.0	0.00752	283.0	+ 36.0
20.	C/1983 S2	SOLWIND 6	1.0	0.00753	282.7	+ 35.2
21.	C/1882 R1	Great				
	- A	September	0.999899	0.007750	282.9	+ 35.2
	- B	Comet	0.999907	0.007751		
	- C		0.999915	0.007751		
	- D		0.999920	0.007749		
22.	C/1965 S1	Ikeya-Seki				
	- A		0.999915	0.007786	283.0	+ 35.2
	- B		0.999925	0.007778		
23.	C/1981 B1	SOLWIND 2	1.0	0.00792	282.7	+ 35.2
24.	C/1970 K1	White-Ortiz-Bolelli	1.0	0.008879	283.0	+ 35.1
25.	C/1984 O2	SOLWIND 5	1.0	0.01541	282.7	+ 35.2

The Table 6 and Figure 3, show that almost 93% comet orbits have perihelia spaced within the heliocentric sphere of 3 A.U. radius with the pick at around 1 A.U.

Table 5. List of longest perihelia comets with q>5 A.U.

Sl. No.	Comet	Perihelion Distance in A.U.
1.	95P/Chiron	8.453942
2.	C/1991 R1	6.986340
3.	C/1976 D2	6.880674
4.	C/1978 G2	6.282837
5.	C/1974 V1	6.018928
6.	C/1993 F1	5.900821
7.	C/1976 U1	5.857415
8.	29P/Schwassmann-Wachmann 1	5.757565
9.	C/1977 D1	5.715207
10.	C/1978 A1	5.606362
11.	39P/Oterma	5.499987
12.	C/1989 U1	5.489159
13.	C/1987 H1	5.457548

Except the comet C/1729 P1, whose perihelion is at 4.05 A.U. from the Sun, 64 other comets with perihelia, q > 3 A.U. were discovered in the 20th century, mostly during its second half when more sophisticated observational techniques were available. This is because of the fact that the cometary nucleus remains dormant and eludes observations when it is at a distance beyond 6 A.U. from the Sun. Occasionally, the comets with long perihelion distances are observed when they became active suddenly and display fluctuation in their brightness. Such possibilities are rare. From the distance around 3 A.U. most cometary nucleus begins to display coma formation followed by the extension of tail, as it approaches the Sun and enter into the ever increasing intense solar radiation sphere. The full cometary display with its coma and tail formation takes place when it orbits approximately within the heliocentric distances from 1.5 to 1.0 A.U.

Table 6. Distribution of perihelion distance in cometary orbits.

Perihelion	No. of Comets		Total	Perihelion	No. of Comets		Total	Perihelion	No. of Comets		Total
q in A.U.	SP	LP		q in A.U.	SP	LP		q in A.U.	SP	LP	
0.0 - 0.1		43	43	2.9 - 3.0	3		3	5.8 - 5.9		1	1
0.1 - 0.2	2	26	28	3.0 - 3.1	4	3	7	5.9 - 6.0		1	1
0.2 - 0.3		25	25	3.1 - 3.2		1	1	6.0 - 6.1		1	1
0.3 - 0.4	1	32	33	3.2 - 3.3		2	2	6.1 - 6.2			
0.4 - 0.5	3	37	40	3.3 - 3.4	1	6	7	6.2 - 6.3		1	1
0.5 - 0.6	3	48	51	3.4 - 3.5	2	1	3	6.3 - 6.4			
0.6 - 0.7	3	44	47	3.5 - 3.6	1	2	3	6.4 - 6.5			
0.7 - 0.8	5	51	56	3.6 - 3.7		3	3	6.5 - 6.6			
0.8 - 0.9	3	56	59	3.7 - 3.8	1	1	2	6.6 - 6.7			
0.9 - 1.0	8	46	54	3.8 - 3.9		2	2	6.7 - 6.8			
1.0 - 1.1	4	43	47	3.9 - 4.0	1		1	6.8 - 6.9		1	1
1.1 - 1.2	10	34	44	4.0 - 4.1		4	4	6.9 - 7.0		1	1
1.2 - 1.3	11	24	35	4.1 - 4.2		2	2	7.0 - 7.1			
1.3 - 1.4	8	18	26	4.2 - 4.3	1	2	3	7.1 - 7.2			
1.4 - 1.5	14	13	27	4.3 - 4.4				7.2 - 7.3			
1.5 - 1.6	15	14	29	4.4 - 4.5		2	2	7.3 - 7.4			
1.6 - 1.7	10	13	23	4.5 - 4.6				7.4 - 7.5			
1.7 - 1.8	8	11	19	4.6 - 4.7	1	1	2	7.5 - 7.6			
1.8 - 1.9	10	6	16	4.7 - 4.8		3	3	7.6 - 7.7			
1.9 - 2.0	8	9	17	4.8 - 4.9		4	4	7.7 - 7.8			
2.0 - 2.1	7	11	18	4.9 - 5.0				7.8 - 7.9			
2.1 - 2.2	5	7	12	5.0 - 5.1		1	1	7.9 - 8.0			
2.2 - 2.3	4	8	12	5.1 - 5.2				8.0 - 8.1			
2.3 - 2.4	7	6	13	5.2 - 5.3				8.1 - 8.2			
2.4 - 2.5	5	5	10	5.3 - 5.4	1		1	8.2 - 8.3			
2.5 - 2.6	4	6	10	5.4 - 5.5		2	2	8.3 - 8.4			
2.6 - 2.7	3	5	8	5.5 - 5.6				8.4 - 8.5	1		1
2.7 - 2.8	3	2	5	5.6 - 5.7							
2.8 - 2.9	2	1	3	5.7 - 5.8	1	1	2				

The most distant perihelion at 8.45 A.U. is of the comet 95P/Chiron which is also designated as the asteroid 2060 Chiron. Incidentally, it is one of the 30 largest asteroids with its diameter of 220 km. Such a size is at least ten times larger than the size of any known comet. But its orbital characteristics are more similar to those of a comet. The practice of naming an object

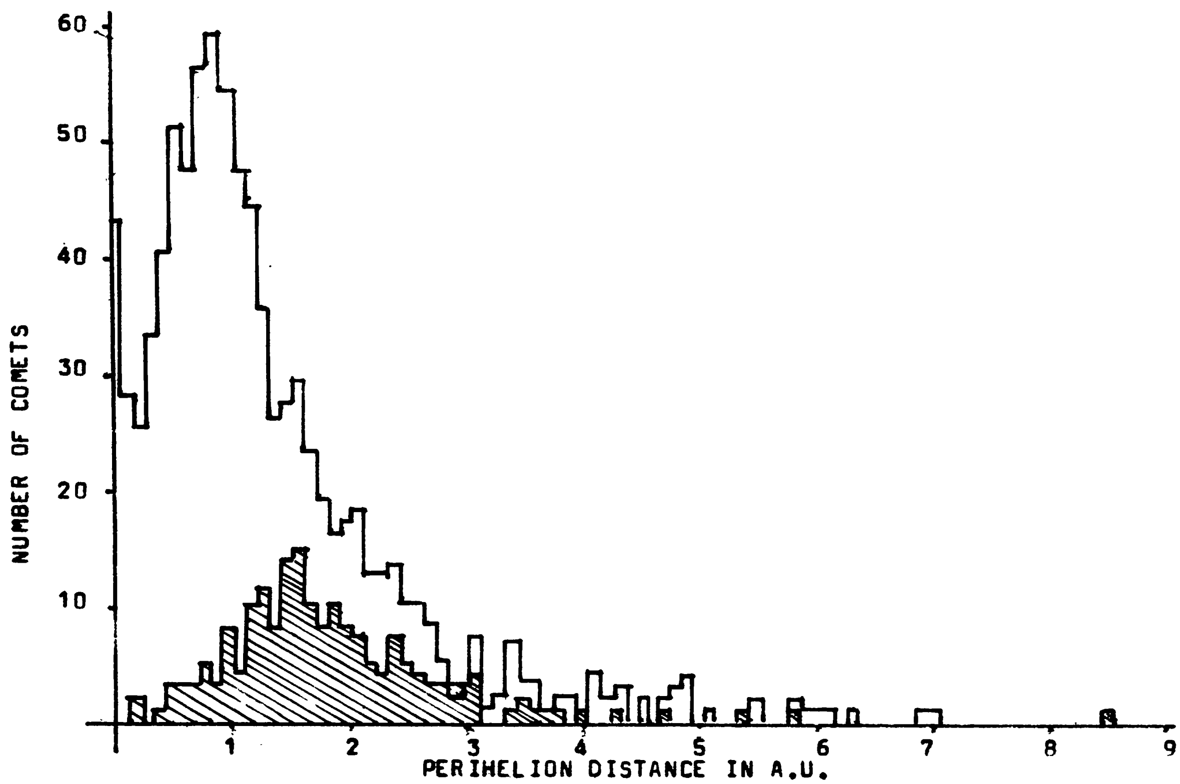

Figure 3. The distribution of perihelion distance in cometary orbits. The deep lined areas represent perihelion distribution of SP comets.

by its discoverer is in vogue for asteroids only. Accordingly, the name Chiron for this object was suggested by its discoverer Charles Kowel himself after its discovery in 1977. Such a practice is again contrary to naming a comet which is usually named after its discoverer by the International Astronomical Union (I.A.U.).

There are four SP comets 95P, 29P, 99P and P/1993 W which have perihelion distances beyond 4 A.U. Also the comet D/1993 F2-K which had a perihelion distance of 5.38 A.U., crashed into the globe of the planet Jupiter in 1994 and exists no longer. Omitting these five cases, if the perihelion distances q, of the remaining 179 SP comet orbits are arranged against their respective eccentricities e, then a very good correlation between these two orbital elements could be seen (Table 7).

Table 7. Correlation between q in A.U. and e of SP comets.

PERIHELION DISTANCE q in A.U. \ ECCENTRICITY e	0.0–0.1	0.1–0.2	0.2–0.3	0.3–0.4	0.4–0.5	0.5–0.6	0.6–0.7	0.7–0.8	0.8–0.9	0.9–1.0	
0.0–0.4									1	3	4
0.4–0.8								2	4	7	13
0.8–1.2							12	6	1	6	25
1.2–1.6						28	10	4	3	4	49
1.6–2.0					10	16	8	1			35
2.0–2.4				10	10	3					23
2.4–2.8			2	6	1	5	1				15
2.8–3.2			5	2		2					9
3.2–3.6		4									4
3.6–4.0		1		1							2
		5	7	19	21	54	31	13	9	20	179

The heliocentric ecliptic longitude L and latitude B of the perihelion of a comet orbit are determined by :

$L = \Omega + \mathrm{Tan}^{-1}$ (Tanω Cos$i$) and $B = \mathrm{Sin}^{-1}$ (Sinω Sin$i$).

The perihelia of SP comet orbits have a marked tendency for crowding around the ecliptic. Out of 184 SP comet perihelia, as many as 149 have their latitudes less than 10° (Table-8). Except the comets P/1983 V1 ($37^{\circ}.5$, + $46^{\circ}.8$), 13P ($142^{\circ}.5$ + $39^{\circ}.4$) and 20D ($355^{\circ}.4$, + $33^{\circ}.3$) all the perihelia of these comets are spaced within 30° of the either sides of the ecliptic plane.

The longitudes of perihelia of SP comet orbits are distributed at random all over the ecliptic. Even then, the maximum congestion of perihelia is conspicuous near 0° longitude, and it is minimum near 180° longitude.

Table 8. Distribution of the perihelia of SP comets.

ECLIPTIC LONGITUDE L

ECLIPTIC LATITUDE B	•	•	60°	•	•	120°	•	•	180°	•	•	240°	•	•	300°	•	•	360°	•
+ 50°																			
+ 40°																		1	1
+ 30°		1						1											2
+ 20°	1								1	1	1						2		6
+ 10°	1	1	3		3	1	1	1					1		1	2			15
0°	7	9	5	4	6	8	3	1	4	2	4	5	4	2	3	2	3	6	78
- 10°	6	7	6	7	5	5	3	2	2	2	4	1	1	4	4	3	2	7	71
- 20°	2				1			1			1		1	1	1			1	9
- 30°						1					1								2
- 40°																			
- 50°																			
	17	18	14	11	15	15	7	6	7	5	11	6	7	7	9	7	7	15	184

Table 9. Distribution of the perihelia of LP comets.

ECLIPTIC LONGITUDE L

ECLIPTIC LATITUDE B			60°			120°			180°			240°			300°			360°	•
90°																			
+ 80°						1					1	2	1						5
+ 70°		1	1	3	2	1	4		1	2		1				1	1	1	19
+ 60°		1	1	3	5	3	1	1	2	1			3	1	1	2	1	2	28
+ 50°	3	3	1	4	6	5	4	2		2	2	1	8	2	2			1	46
+ 40°	3			5	5	5	3	2	1	2	5	3	2	2	5	1	2	4	50
+ 30°	4	4	3	4	2	3	3	3	2	1	1	4	6	4	**28**	6	4	2	84
+ 20°	2	4	7		6	1	2	1	1	1	1	1	1	5	5	4	2	1	45
+ 10°	5	3	2	1	2	8	2	2	2	1	2	3	6	4	5	5	5	1	59
0°	4	2	4	3	4	3	4	3	2	1	3	1	7	7	8	7	2	2	67
- 10°	4	3	2	2	7	4	5	7	2	5	5	4	2	8	4	3	5		72
- 20°	1	2	3	3	3	3	4	6	3	4	6	2		3	2	3	1	2	51
- 30°	2	2	4	2	3	2	4	5	7	2	10	4	4	5	2		1		59
- 40°	1		2	1	3	1	4	1	2		6	1		4	2	1	1		30
- 50°	1	1	1	2		1			2	2	1	1	2	6	4	2	1	1	28
- 60°	1	1	1	2	1	1				2			3	2	2	2	1		19
- 70°	2		1		1				1		1		1	1	1	1		2	12
- 80°	2	2	1	2			2	1						3	2	1			16
- 90°		2			1								1						4
	35	31	34	37	51	42	42	34	28	26	44	28	47	57	73	39	27	19	694

The distribution of both the SP and LP comet perihelia show that the perihelia of Northern latitudes are numerically superior to those of Southern latitudes (Tables 8 and 9). Specifically, 102 perihelia of SP comets lie in the North latitudes as againat 82 in the South and 403 perihelia of LP comets lie in the North latitudes compared to 291 in the South.

The perihelia of LP comet orbits are spaced in all directions of the helieocentric sphere (Table 9), very much unlike the distributions of the SP comet perihelia. There is preponderance of perihelia distribution in the lower latitudes compared to the higher ones with a few exceptions. But there is an exception in the interval from 30^0 to 40^0 North latitudes where the distribution of 84 perihelia is the highest in number. Of those 84 perihelia, 28 are located in the longitudes from 280^0 to 300^0. It is very interesting to note that except the perihelia of the comet C/1680 V1, all 24 other comets with smallest perihelia listed in Table 4, are included in this group of 28 perihelia. The congregation of the LP comet perihelia is minimum around 0^0 and 180^0 longitudes; whereas their condensation is maximum around 280^0 longitude (Fig. 4). The Figure 4 obtained from Table 10 also shows that both the SP and LP comet aphelia are distributed almost at random.

Table 10. Distribution of perihelion of the SP and LP comet orbits along the longitudes.

Longitudes	No. of Peri		Longitudes	No. of Peri		Longitudes	No. of Peri	
	SP	LP		SP	LP		SP	LP
0^0 - 10^0	9	15	120^0 - 130^0	3	26	240^0 - 250^0	4	23
10^0 - 20^0	8	20	130^0 - 140^0	4	16	250^0 - 260^0	3	24
20^0 - 30^0	4	14	140^0 - 150^0	3	18	260^0 - 270^0	4	21
30^0 - 40^0	14	17	150^0 - 160^0	3	16	270^0 - 280^0	3	36
40^0 - 50^0	9	20	160^0 - 170^0	6	22	280^0 - 290^0	5	51
50^0 - 60^0	5	14	170^0 - 180^0	1	6	290^0 - 300^0	4	22
60^0 - 70^0	6	13	180^0 - 190^0	4	11	300^0 - 310^0	4	22
70^0 - 80^0	5	24	190^0 - 200^0	1	15	310^0 - 320^0	3	17
80^0 - 90^0	12	19	200^0 - 210^0	5	19	320^0 - 330^0	2	18
90^0 - 100^0	3	32	210^0 - 220^0	6	25	330^0 - 340^0	5	9
100^0 - 110^0	8	26	220^0 - 230^0	2	11	340^0 - 350^0	3	7
110^0 - 120^0	7	16	230^0 - 240^0	4	17	350^0 - 360^0	12	12

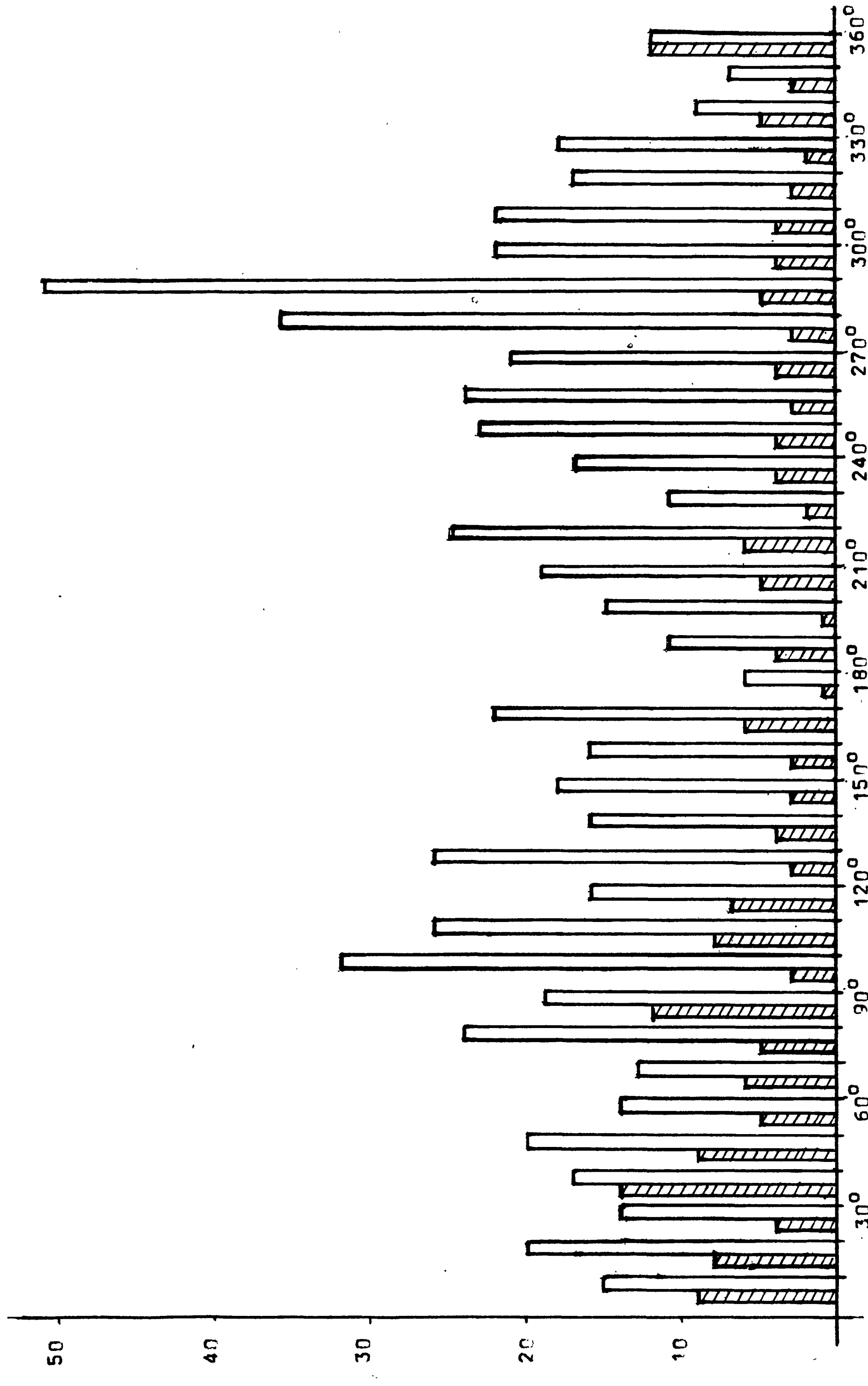

Figure 4. The comparative study of the distribution of perihelion of SP comet orbits [hatched] relative to that of LP comet orbits [open] along the Ecliptic Longitude.

4. Family of comets

The aphelion distances of all the 184 SP comets listed in 'Catalogue 195' have been arranged in the higher order of their values for the studies of their proximity of cometary aphelion from the nearby planetary orbit (Appendix-II). The closest distance d_{PC} for each of the comet aphelion from nearby planetary mean orbital path has been calculated by the formula

$$d_{PC} = \left[Q^2+r_p^2-2Qr_p\text{Cos}(i_c - i_p)\right]^{1/2} \tag{2}$$

Where, Q is the aphelion distance of the comet at C from the Sun at S, SC;

r_p is the radius vector of the nearby planet at P along the projection of aphelion on the plane of planetary orbit, SP;
i_c is the inclination of cometary orbit with the ecliptic plane, $\angle$ CSE; and
i_p is the inclination of planetary orbit with the ecliptic plane, $\angle$ PSE; (Fig. 5).

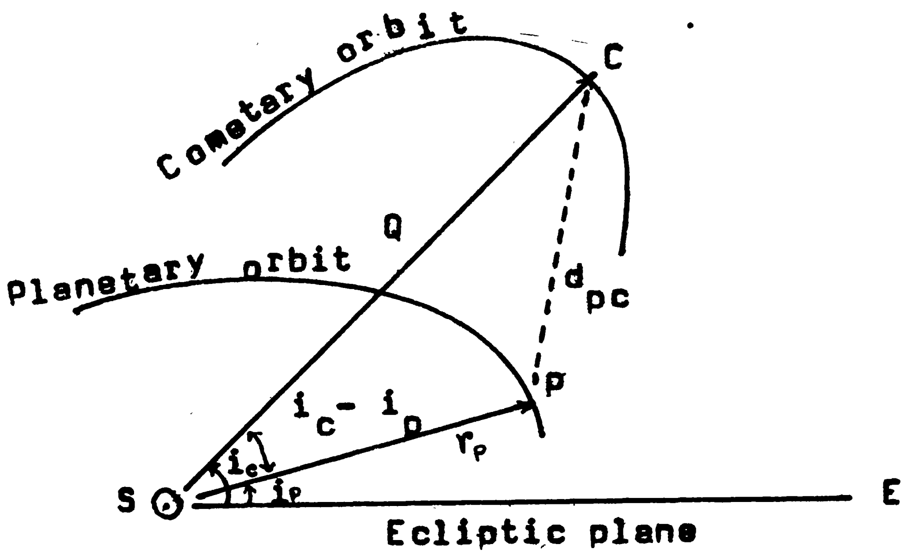

Figure 5. The calculation of the shortest distance of cometary aphelion from nearby planetary orbit, d_{pc}

For determining the proximity of cometary aphelia at C from the orbit of Jupiter d_{JC} that of Saturn d_{SC} that of Uranus d_{UC} and that of Neptune d_{NC} the following relations obtained from the equation (2) have been used :

$$d_{JC} = \left[Q^2+5.2^2 - 2Q5.2\text{Cos}(i_c - 1.3)\right]^{1/2},$$

$$d_{SC} = \left[Q^2+9.55^2 - 2Q9.55\text{Cos}(i_c - 2.49)\right]^{1/2},$$

$$d_{UC} = \left[Q^2+19.18^2 - 2Q19.18\text{Cos}(i_c - 0.77)\right]^{1/2} \text{ and}$$

$$d_{NC} = \left[Q^2+30.06^2 - 2Q30.06\text{Cos}(i_c - 1.77)\right]^{1/2}.$$

where, 5.2 A.U. 9.55 A.U., 19.18 A.U. and 30.06 A.U. are the measures of the mean heliocentric orbital distances r_p and 1.3°, 2.49°, 0.77° and 1.77° are the inclinations i_p of the planets Jupiter, Saturn, Uranus and Neptune respectively (Appendix-II).

It has been found that 132 SP comet aphelia ranging from 4.09 A.U. to 7.24 A.U. are more close to the orbital path of the nearby planet Jupiter than any other planets (Table-11). Such a group of comets may be defined to constitute the Jupiter Family of Comets (JFC). Similar investigation reveals that 31 SP comets with aphelia from 7.43 to 14.20 A.U. constitute the Saturn Family of Comets and 4 SP comets with aphelia from 17.4 to 20.9 A.U. constitute Uranus Family of Comets. Out of the remaining 17 SP comets all with high orbital inclinations, 11 have aphelia ranging from 26.6 A.U. to 37.5 A.U. and 6 have aphelia beyond the planetary system at distances from 51.7 A.U. to 64.9 A.U. which are well within Kuiper Belt. Thus, those 11 SP comets can be considered as the Neptune Family of comets.

Table 11. Distribution of aphelia distances and family of SP comets.

Aphelion Q in A.U.	Log Q	Family	No. of comets
3.98 - 4.09	0.60 - 0.61		
4.09 - 5.01	0.61 - 0.70		37
5.01 - 6.31	0.70 - 0.80		81
6.31 - 7.24	0.80 - 0.86		14
4.09 - 7.24	0.61 - 0.86	Jupiter	132
7.24 - 7.43	0.86 - 0.87		
7.43 - 7.94	0.87 - 0.90		6
7.94 - 10.00	0.90 - 1.00		15
10.00 - 12.59	1.00 - 1.10		7
12.59 - 14.20	1.10 - 1.15		3
7.43 - 14.20	0.87 - 1.15	Saturn	31
14.20 - 15.85	1.15 - 1.20		
15.85 - 17.40	1.20 - 1.24		
17.40 - 19.95	1.24 - 1.30		2
19.45 - 20.40	1.30 - 1.32		2
17.40 - 20.40	1.24 - 1.32	Uranus	4
20.40 - 25.12	1.32 - 1.40		
25.12 - 26.60	1.40 - 1.42		
26.60 - 31.62	1.42 - 1.50		4
31.62 - 37.50	1.50 - 1.57		7
26.60 - 37.50	1.42 - 1.57	Neptune	11
37.50 - 39.82	1.57 - 1.60		
39.82 - 50.12	1.60 - 1.70		
50.12 - 51.70	1.70 - 1.71		
51.70 - 63.10	1.71 - 1.80		5
63.10 - 64.90	1.80 - 1.81		1

The distribution of cometary aphelia of 184 SP comet orbits shows that about 83% of the total aphelia are crowded within the domain of 4.09 A.U. to 10.00 A.U. (Table 12 and Fig. 6).

The studies on the aphelia of the 215 LP elliptic orbit comets have been made. The distribution of all the 399 periodic comet aphelia are shown in the Table 13 and Fig. 7. Some 50 of these orbits have aphelia ranging from 73 A.U. to 193 A.U. and the periods are less than 1000 years. The rest 165 orbits with periods greater 1000 years have aphelia even much more – extending upto a distance of 171825 A.U. which is perhaps the known farthest aphelion distance of any comet orbit. These orbits are thought to be elliptical, yet their eccentricities are so high that they

Table 12. Distribution of aphelion distance of SP comet orbits.

Aphelion Q in A.U.	Log Q	No. of comets	Aphelion Q in A.U.	Log Q	No. of comets
3.98 - 4.47	0.60 - 0.65	2	17.78 - 19.95	1.25 - 1.30	2
4.47 - 5.01	0.65 - 0.70	35	19.95 - 22.39	1.30 - 1.35	1
5.01 - 5.62	0.70 - 0.75	54	22.39 - 25.12	1.35 - 1.40	
5.62 - 6.31	0.75 - 0.80	28	25.12 - 28.18	1.40 - 1.45	1
6.31 - 7.08	0.80 - 0.85	12	28.18 - 31.62	1.45 - 1.50	3
7.08 - 7.94	0.85 - 0.90	7	31.62 - 35.48	1.50 - 1.55	5
7.94 - 8.91	0.90 - 0.95	8	35.48 - 39.82	1.55 - 1.60	2
8.91 - 10.00	0.95 - 1.00	7	39.82 - 44.67	1.60 - 1.65	
10.00 - 11.22	1.00 - 1.05	4	44.67 - 51.70	1.65 - 1.70	
11.22 - 12.59	1.05 - 1.10	3	51.70 - 56.23	1.70 - 1.75	4
12.59 - 14.13	1.10 - 1.15	2	56.23 - 63.10	1.75 - 1.80	1
14.13 - 15.85	1.15 - 1.20	1	63.10 - 70.79	1.80 - 1.85	1
15.85 - 17.78	1.20 - 1.25	1			

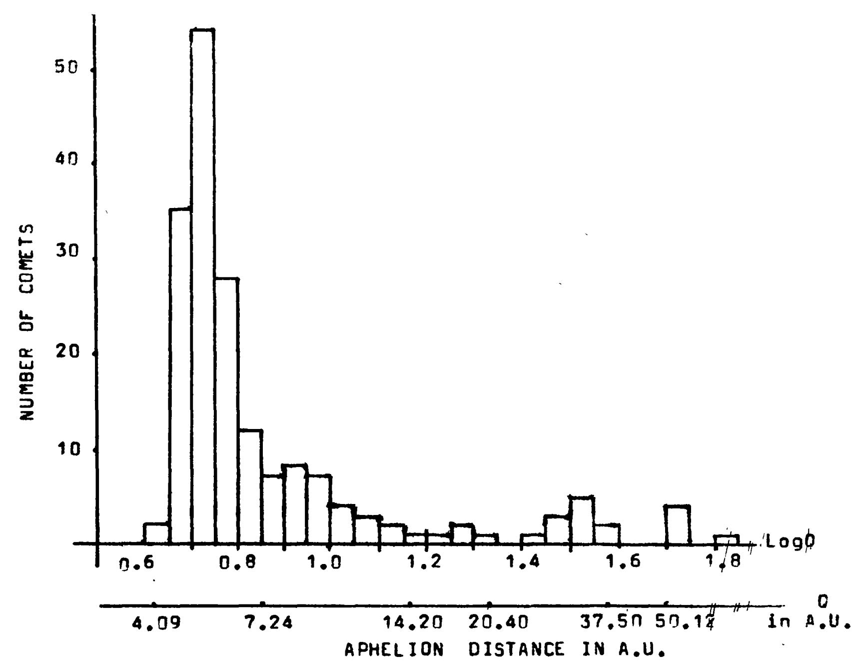

Figure 6. The distribution of Aphelion distances of SP comet orbits.

may be termed as near parabolic orbits. These elliptic orbits are the product of pertrubations exerted in varying degree by the giant planets, in particular, the massive planet Jupiter on the primitive comets which are considered as to orbit on the parabolic paths. This phenomenon clearly explains that the comets with elliptic orbit are mainly the product of planetary perturbation on the comets approaching the Sun. A certain fraction of all these comets have been captured and remained as the SP comets in the Solar System.

Table 13. Aphelion distribution of periodic comet orbits.

Aphelion distance Q in A.U.	Log Q	No. of comets	Aphelion distance Q in A.U.	Log Q	No. of comets
3.89 - 5.62	0.60 - 0.75	91	707.95 - 1000.00	2.85 - 3.00	15
5.62 - 7.94	0.75 - 0.90	47	1000.00 - 1412.54	3.00 - 3.15	17
7.94 - 11.22	0.90 - 1.05	19	1412.54 - 1995.26	3.15 - 3.30	8
11.22 - 15.85	1.05 - 1.20	6	1995.26 - 2818.38	3.30 - 3.45	13
15.85 - 22.39	1.20 - 1.35	4	2818.38 - 3981.07	3.45 - 3.60	11
22.39 - 31.62	1.35 - 1.50	4	3981.07 - 5623.41	3.60 - 3.75	5
31.62 - 44.67	1.50 - 1.65	7	5623.41 - 7943.28	3.75 - 3.90	8
44.67 - 63.10	1.65 - 1.80	5	7943.28 - 11220.18	3.90 - 4.05	6
63.10 - 89.13	1.80 - 1.95	12	11220.18 - 15848.93	4.05 - 4.20	5
89.13 - 125.89	1.95 - 2.10	14	15848.93 - 22387.21	4.20 - 4.35	6
125.89 - 177.83	2.10 - 2.25	17	22387.21 - 31622.78	4.35 - 4.50	3
177.83 - 251.19	2.25 - 2.40	18	31622.78 - 44668.36	4.50 - 4.65	1
251.19 - 354.81	2.40 - 2.55	19	44668.36 - 63095.73	4.65 - 4.80	2
354.81 - 501.19	2.55 - 2.70	21	63095.73 - 89125.09	4.80 - 4.95	
501.19 - 707.95	2.70 - 2.85	12	89125.09 - 125892.54	4.95 - 5.10	2
			125892.54 - 177827.94	5.10 - 5.25	1
				Total	**399**

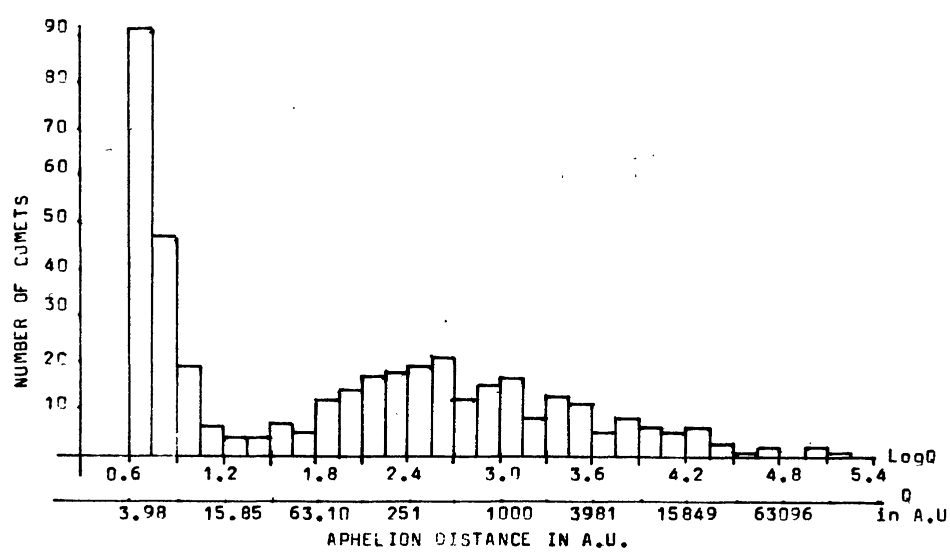

Figure 7. The Aphelion distribution of periodic comet orbits.

4.1 Jupiter family of comets (JFC)

Already 132 SP comets have been identified as the members of the JFC. The distribution of the closest aphelion distances from the mean orbital distance of the planet Jupiter reveals that some 22 aphelia of these JFC orbits are lying well within 0.5 A.U. (Table 14). For examining the proximity of these 22 aphelia more accurately from the actual orbital path of Jupiter,

Table 14. Distribution of the closest aphelion distances of JFC orbits from the orbit of Jupiter.

CLOSEST APHELION DISTANCES IN A.U.

LONGITUDE	0.0 – 0.5	0.5 – 1.0	1.0 – 1.5	1.5 – 2.0	2.0 – 2.5	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0	TOTAL
0° - 30°	2	2				2			6
30° - 60°	1	2	2	1	2				8
60° - 90°		5	4		1		1		11
90° - 120°	2	3	1	1					7
120° - 150°	1	2	2	1					6
150° - 180°	3	3	2	1	2				11
180° - 210°	2	8	3	1	2	1	1		18
210° - 240°	3	3	5	4	3			1	19
240° - 270°	4	5	2	2		2			15
270° - 300°	1	3	5	5		2			16
300° - 330°	2	2	2		1	1			8
330° - 360°	1	4	1						6
	22	42	29	16	11	8	2	1	131

[Excepting 96P (L = 281.9, d_{Jc} = 5.49 A.U.)

instead of its mean path, the radius vector of the planet along the line of respective cometary aphelion have been determined. The radius vector of the planet Jupiter r_J is determined by the relation

$$r_J = \frac{a(1-e^2)}{1+e\text{Cos}(l_a - l_p)} \tag{3}$$

where, a is the semimajor axis, e is the eccentricity and l_P is the longitude of perihelion of Jovian orbit and l_a is the longitude of aphelion line of the comet orbit.

By the application of Equations (2) and (3), it has been found that out of these 22 JFC aphelia, at least 5 are well within the distance of 0.322 A.U. from the actual orbital path of the planet Jupiter (Table 15). The distance 0.322 A.U. is the radius of the Sphere of Infuence (SOI) of Jupiter (Appendix-I). So, there are possibilities of comming any of these five comets into the space SOI of the planet Jupiter for being captured by the latter. The captured comet may

Table 15. Particulars of the five JFC aphelia within the SOI radius of Jupiter

Comet	Aphelion distance Q in A.U.	Inclination i	Longitude of		Jupiter's radius vector r_J	Closest aphelion distance in A.U.
			Comet's l_c	Jupiter's l_p		
70P	5.50	0.9	323.3	116.2	5.423	0.086
81P	5.30	3.2	357.8	116.2	5.312	0.176
57P	5.17	2.8	124.4	116.2	4.956	0.252
54P	5.21	3.6	170.4	116.2	5.050	0.260
88P	4.88	4.4	112.4	116.2	4.954	0.276

begin to revolve round the planet instead of the Sun and undergo any eventuality that may be caused by the perturbation of the planet Jupiter.

5. Distribution of nodes

The nodal distances of comets have been analysed without attributing any weightage for returned nodal distances of the SP comets (Table 16). It is found that out of 878 x 2 nodal distances 71% of them are crowded within the orbital distance of Jupiter at a heliocentric distance of about 5 A.U. and that or 83% are within the orbital distance of Saturn at about 10A.U. (Fig. 8). However, 93% of the nodal distances are within 49.96 A.U. which is the aphelion distance of the furthest planet of the planetary system.

The node of the comet C/1680 V1 at a distance of 0.006 A.U. is the closest one and that of C/1885 N1 at a distance of 14 827 A.U. is the furthest one from the Sun. The furthest antinode is of the comet C/1896 G1 (Swift) at a distance of 73 680 A.U. and the closest one is of the comet C/1989 N1 SMM 9 at a distance of 0.009 A.U. from the Sun.

We now investigate the proximity of the orbital paths of comets to those of planets at the cometary nodes. In the process we define the annular space swept by the SOI of a particular planet while, it circumscribes on its orbit, as the corresponding Planetary Annular Space of Influence (PASI) of the said planet. The Table 17 shows that the number of cometary nodes lie within the distances from the inner radius to the outer radius of the PASI.

Table 16(a). Distribution of nodes

Distance in A.U.

ECLIPTIC LONGITUDE	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	Total
0° - 20°	26	12	8	2	1													1								3	53
20° - 40°	21	15	3	1	1	2		1	2				1													3	50
40° - 60°	27	11	2	2	1	3	1			1			1	1												3	53
60° - 80°	20	10	10	1	2	1			1					1	1				1								48
80° - 100°	34	11	3	1	1			1		2	1															4	58
100° - 120°	25	11	5	2	1		1				1			1												4	51
120° - 140°	18	8	8		1	1	2	2							1											4	45
140° - 160°	17	11	3	4		1	2							2							1		1		1	5	48
160° - 180°	20	9	7	3				1			1					1										2	44
180° - 200°	13	10	8	4	3	1			1					1	1									1		4	47
200° - 220°	19	14	9	1	3		2		1	1			1													2	53
220° - 240°	17	5	5	2	1				1	1				1					1						1	4	39
240° - 260°	16	11	7		3	2	1	2		1	1			1		1						1				1	48
260° - 280°	29	7	4	2	1		1				1	1										1	1		1	1	50
280° - 300°	23	4	5	1	2		5	1				1									1					4	47
300° - 320°	24	9	8	3		2												1								3	50
320° - 340°	24	5	9	2						1		1														2	44
340° - 360°	28	9	1	2	1	3							1													5	50
TOTAL =	401	172	105	33	22	16	15	8	6	7	5	3	4	8	3	2		2	2		2	2	2	1	3	54	878

Table 16(b). Distribution of anti-nodes

Distance in A.U.

ECLIPTIC LONGITUDE	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-26	26-28	28-30	30-32	32-34	34-36	36-38	38-40	40-42	42-44	44-46	46-48	48-50		Total
0^o - 20^o	24	12	4		1	1	1	1		1																2	47
20^o - 40^o	27	7	6	1	2			2		1								1					1			5	53
40^o - 60^o	15	10	2	2	2	1				1	1							1								4	39
60^o - 80^o	24	10	4	2						1		1														6	48
80^o - 100^o	17	7	3	1		2	3	2					1		1			1			1					11	50
100^o - 120^o	21	8	4	2		3	2	1					1	1					1							3	47
120^o - 140^o	20	11	3	2	3	1								3			1									6	50
140^o - 160^o	23	6	3	5			1	1	1																	4	44
160^o - 180^o	31	6	7.		3	1						1														1	50
180^o - 200^o	33	7	7	1	1	1			1						1											1	53
200^o - 220^o	25	10	6	4	2			1																		2	50
220^o - 240^o	21	6	9	3	1	2	1	2	1	1						1						1				4	53
240^o - 260^o	23	9	7	2			1		2		2	2															48
260^o - 280^o	21	15	11	1		3		1	1					1			1									3	58
280^o - 300^o	25	13	4	1		1												1	2				1			3	51
300^o - 320^o	19	6	8	2	2	2		1	1			1				1										2	45
320^o - 340^o	17	14	5	2	3		2								1					1						3	48
340^o - 360^o	16	9	3	1	1			1	1	2				2	1						1	1				5	44
TOTAL=	402	166	96	32	21	18	11	13	8	7	3	5	2	7	4	2	2	4	3	1	2	2	2			65	878
16a + 16b =	803	338	201	65	43	34	26	21	14	14	8	8	6	15	7	4	2	6	5	1	4	4	4	1	3	119	1756

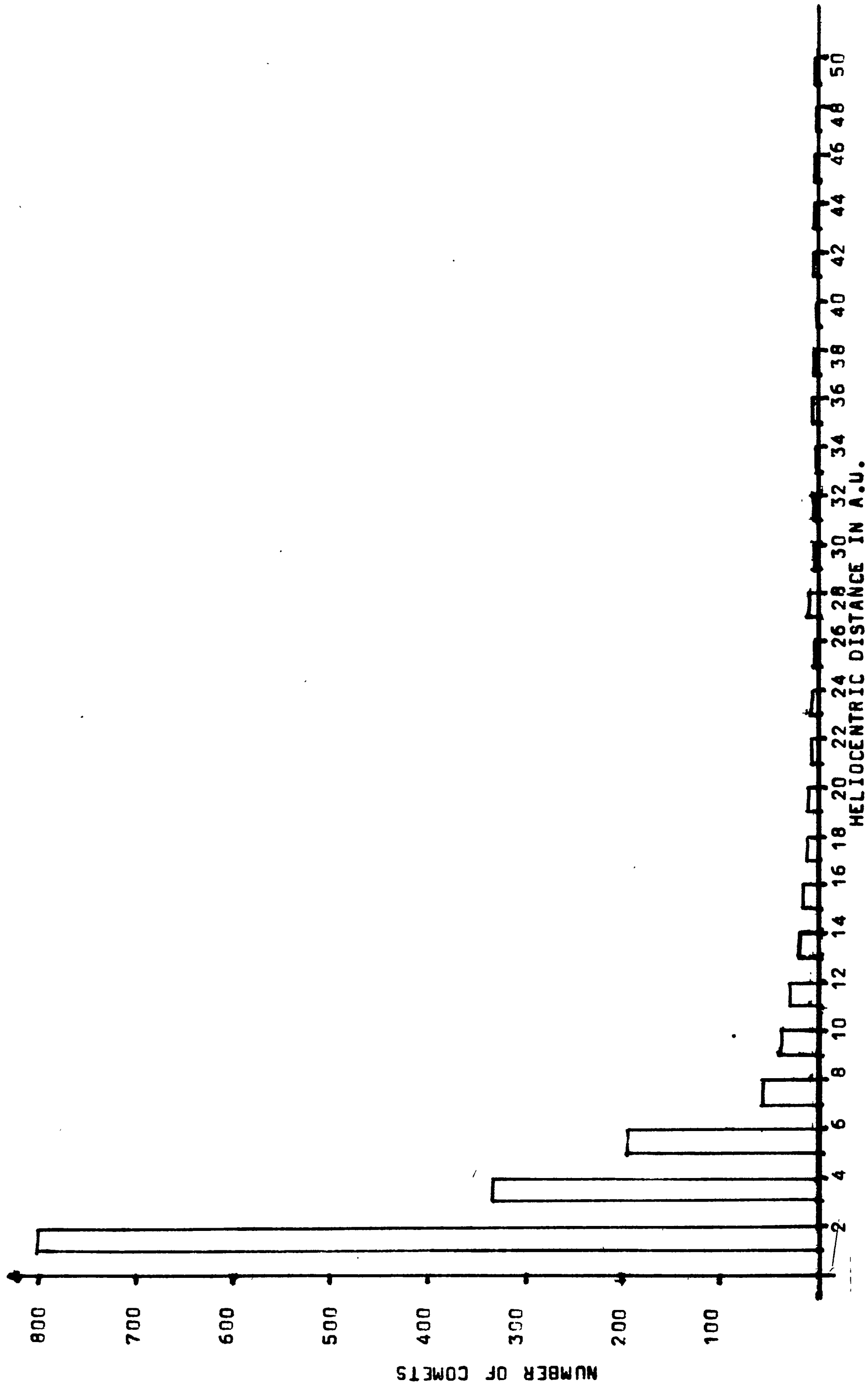

Figure 8. Distribution of nodes and antinodes.

Table 17. Number of nodes within PASI.

Planet	SOI radius A.U.	Semimajor axis A.U.	PASI limits from the sun A.U.	No. of nodes		No. of antinodes		Total
				SP Comet	LP Comet	SP Comet	LP Comet	
Mercury	0.00075	0.387	0.386 - 0.388					
Venus	0.00409	0.723	0.719 - 0.727		2		3	5
Earth	0.00618	1.000	0.994 - 1.006	1	2		3	6
Mars	0.00386	1.524	1.520 - 1.528	1	1	1	2	5
Jupiter	0.32200	5.203	4.881 - 5.525	18	19	25	13	75
Saturn	0.36500	9.630	9.165 - 9.895		6	3	6	15
Uranus	0.34700	19.24	18.893 - 19.587		1		3	4
Neptune	0.58100	30.14	29.559 - 30.727				3	3
Pluto								
			Total =	**20**	**31**	**29**	**33**	**113**

Table 17 shows that 20 SP cometary nodes and 29 antinodes have locations within the different PASI and out of 64 LP comets 31 crossed their nodes and 33 crossed antinodes through these PASI. It is remarkable that 43 SP and 32 LP nodes and antinodes are located within the PASI of the planet Jupiter alone. The orbits of the planet Mercury which is the closest to the Sun and the planet Pluto which orbits at the furthest distance for 231 years out of its total period of 251.2 years, are highly elliptic having eccentricities 0.206 and 0.225 respectively. These two are the most elliptic orbits in the planetary system. None of the nodes or antinodes have been found to be located with the PASI of the planet Mercury. Since the mass of the planet Pluto is very small, even smaller than the seven planet-size satellites viz., Moon (Earth), Io, Europa, Ganymedes, Callisto (Jupiter), Titan (Saturn) and Triton (Neptune), and also the population densities of the nodes or antinodes at the heliocentric distances between 29 and 49 A.U. is very small, so its influence on the cometary masses have been considered as negligible.

6. Multiple nuclei

Basically, cometary nuclei are extremely fragile and weakly bounded by light and freezed volatile materials. Also these constituents of the cometary nuclei are held together by mutual gravity. According to Weidenschilling (1994), the cometary nuclei are formed of a layer of dust particles at the outer solar system by collisional coagulation accompanied by gravitational collapse. In an attempt to determine the mass of the comet 1P/1982 U1 (Halley), H. Rickman in 1986 obtained it to be in between 0.5×10^{17} to 2×10^{17} gm and in 1988 R.Z. Sagdeev et al found the mass as 1×10^{17} to 8×10^{17} gm. Considering some uncertainty, Moroz (1998), suggested that "the average density is most likely lower than 1 and higher than 0.1 gm. cm^{-3}." From the space studies of the Halley's comet, Moroz probably considered the 8 x 8 x 16 km sized and irregularly shaped nucleus to be 500 km^3 in volume. The rigid mass like the cometary nuclei may suffer disruption under the influence of differential gravitational force, if it happens to approach the Sun within a certain limiting distance which is known as the Roche's Limit. Thus the Roche's Limit of the Sun r for a comet may be obtained from the relation

$$r = 1.44 \left(\frac{d_o}{d_c} \right)^{\frac{1}{3}} R_o \qquad (4)$$

where, d_o is the density of the Sun,

d_c is the density of the comet, and

R_o is the radius of the Sun.

Since R_o = 696,000 km = 0.00465247 A.U. and d_o = 1.41 gm cm^{-3}, then the maximum value of the Roche's Limit of the Sun can be obtained as

$$r_{max} = 0.01618 \text{ A.U.}$$

The 'Catalogue 1995' contains no fewer than 25 comet orbits, listed in Table 4, having perihelion within the Roche's Limit of the Sun. All of them are long period comets of which five have elongated elliptic and the rest 20 have parabolic orbits.

Sometimes cometary nuclei suffer disruption during their orbital motion while they happen to cross their nodes or to pass their aphelia through the SOI of the giant planets, in particular the planet Jupiter, and also when they happen to make perihelion passage within the Roche's Limit of the Sun. A few comets have been observed even to crash either into the globe of the planet Jupiter or the Sun !

The comet C/1975 V1-A (West) with nodal distance of 0.197 A.U., is an example for displaying split of a cometary nucleus. This comet was observed to break into four components of nuclei in March, 1976, just after its passage of perihelion on 25 February, 1976 (Hartmann, 1985). This incident was observed through the 1.5m telescope of Lunar Planetary Laboratory, University of Arizona. The 'Catalogue 1995' shows that nine comets listed in Table 18 have been observed to have such multiple nuclei.

Table 18. Particulars of multiple nuclei comets.

Sl. No.	Comet	No. of nucleus	Perihelion distance q A.U.	Incli- nation i	Nodal distance Ascen- ding r_A A.U.	Nodal distance Descen- ding r_D A.U.	Aphelion distance Q A.U.
1.	3D/1846 W1-A	2	0.856440	12.58	3.37	0.974	6.19
	3D/1846 W1-B		0.856460	12.58			6.19
	3D/1852 Q1-A	2	0.860594	12.55			6.19
	3D/1852 Q1-B (Biela)		0.960625	12.55			6.19
2.	C/1860 D1-A	2	1.1989	79.68			
	C/1860 D1-B (Liasi)		1.1982	79.62	18.2	1.28	
3.	C/1882 R1-A	4	0.007750	142.0	0.01	0.024	
	C/1882 R1-B		0.007751	142.0			
	C/1882 R1-C		0.007751	142.0			
	C/1882 R1-D (Great September Comet)		0.007749	142.0			
4.	C/1947 X1-A	2	0.110032	138.5	5.51	0.112	
	C/1947 X1-B (Southern Comet)		0.110023	138.5			
5.	C/1956 F1-A (Writanen)	1	4.447262	33.2	4.51	371.0	
6.	C/1965 S1-A	2	0.007786	141.9	0.01	0.024	
	C/1965 S1-B (Ikeya-Seki)		0.007778	141.9			
7.	C/1975 V1-A (West)	1	0.196626	43.07	0.197	969.0	
8.	D/1993 F2-A, B,.. W. Omitting I, J, M&O (Shoemaker -Levy 9)	19	5.38	5.75	5.38	8.24	8.25
9.	D/1994 P1-A	3	0.752548	12.79	3.71	0.801	5.27
	D/1994 P1-B		0.752556	12.79			5.27
	D/1994 P1-C (Mechholz 2)		0.752558	12.79			5.27

There is another list of split comets prepared by Festou et al (1993) shown in Table 19. This list contains seven split comets of Jupiter Family which includes the parent of the two SP comets 42P/Neujmin 3 and 53P/Biesbroeck. The parent comet of these two might have suffered split preceeded by outbrust activity. Recently Chen and Jewitt (1994) used time resolved Charge-Couple Device (CDD) image to assess the frequency of splitting of comets. They noticed that the three comets listed in Table 20, are split in a sample of 49 comets. They made observations on these 49 comets from 1986 to 1993. From their observations they were able to estimate that the cometary splitting rate S~0.01 per year per comet or even larger. They also suggested that splitting may be an important destructive process for the cometary nuclei. But H. Rickman in 1990 argued otherwise. A Carusi and others in 1985 reported that a comet

Table 19. Split Jupiter Family Comets

Sl. No.	Comet	Date of Split	Inclination i	Perihelion q A.U.	Aphelion Q A.U.	Nodal distance: Ascending r_A A.U.	Nodal distance: Descenling r_D A.U.
1.	3D/Biela	1840	12.58	0.86	6.19	3.37	0.974
2.	Parent comet of						
	42P/Neujmin 3		4.0	2.001	7.67	6.24	2.13
	53P/Van Biesbroeck		6.6	2.400	8.33	6.06	2.69
3.	16P/Brooks 2	1886-88	5.5	1.843	5.40	5.15	1.87
4.	D/1896 R1, (Giacobini)	April. 1896	11.4	1.455	5.62	4.23	1.59
5.	69P/Taylor	Dec. 1915	20.5	1.950	5.35	1.95	5.33
6.	79P/du Toit-Hartley	Dec. 1976	2.9	1.199	5.91	2.37	1.61
7.	101P/Chernykh	April.1991	5.7	2.356	9.24	4.04	3.51

Table 20. Particulars of three split comets (Chen and Jewitt, 1994)

Sl. No.	Comet	Perihelion q A.U.	Aphelion Q A.U.	Nodal distance: Ascending A.U.	Nodal distance: Descending A.U.	Observed magnitude: Primary	Observed magnitude: Secondary
1.	101P/1977 Q1, Chernykh	2.36	9.24	4.04	3.51	16.37	19.25
2.	108P/1985 V1, Ciffrco	1.71	5.77	1.71	5.77	19.06	18.27
3.	C/1986 P1, Wilson	1.20	-	5.06	1.57	13.68	22.68

split around 1850, resulting in two SP comets of Jupiter family. These two are the 42P and 53P comets which have been existing for more than a century. This investigation inspired Rickman to conclude that the splitting of cometary nuclei may also be a process of creating comets.

On a few occasions space telescopes have detected comets to collide into the Sun. The comet C/1979 Q1 is such an example. An orbiting telescope mounted on the SOLWIND satellite succeeded to photograph the comet while it was about to crash into the globe of the Sun on 30 August, 1979 (Hartmann, 1985). Very recently a comet which was originally orbiting around the Sun, happened to experience strong gravitational pull of the giant planet Jupiter. The pull was so severe that the comet was displaced from its original orbit round the Sun to a transformed orbit round the captor planet Jupiter. Subsequently, the comet also suffered disruption under the influence of differential gravitational force of the planet in July, 1992, much earlier to its discovery on 25 March, 1993, by the couple Caroyln and Eugene Shoemaker (1928-97) and David H. Levy. All the 19 compoments of the comet D/1993 F2-A, B,.........W (Shoemaker-Levy 9) (omitting I, J, M & 0), instead of making perihelion passage round the planet Jupiter, were forced to crash into its globe in July, 1994.

There are a few more evidences of collisions between comets and major members of the planetary system like the collision of Shoemaker-Levy 9. Observing from the images returned by the spacecraft Voyager 1 in January, 1979, of several prominent chains of impact craters on the surface of Callisto and Ganymede which are the two largest satellites of Jupiter, H.J. Melosh and P. Schenk (1993) suggested that the impact of split comets may be responsible for the creation of these straight chains or catenae.

The stastistical investigations of 1472 cometary orbits contained in the 'Catalogue 1995' reveal that :

i) the sun-grazing comets whose perihelion distances are less than the Roche's Limit of the Sun, have been observed recently to make frequent visit to the latter (Table 4);

ii) quite a good number of SP comet aphelia are close to the planetary orbits and at least five of them have such aphelia that these may come within the SOI radius of the planet Jupiter (Table 15), and

iii) already some 20 nodes and 29 anti-nodes of SP comets have been found to lie well within the PASI (Table 12) besides the possible passage of many other LP comets which may keep their respective nodes within the same.

In view of these considerations, incidents like the splitting and followed by crashing of comets into the globe of the Sun, planets or even the planet-size satellites occurred in the past and may recur in future.

7. Comet groups

Porter (1963) listed 39 cometary orbits and divided these orbits into 15 groups on the basis of closeness in the values of their orbital coordinate elements within certain tolerable limits. These 15 groups were assigned by the English capital letters from A to Q, omitting I and O, according

Table 21. Up-dated F and M comet groups.

Gr.	Sl. No.	q A.U.	e	ω °	Ω °	i °	L °	B °	Nodes r_A A.U.	Nodes r_D A.U.	Comets	
F	1	0.504	1.0	344.7	179.9	126.4	189.2	-12.3	0.514	28.4	C/1822 J1	
	2	0.626	1.0	346.1	176.9	135.0	186.8	- 9.8	0.636	42.7	C/1864 R1	
	3	0.812	0.996	347.5	176.4	129.8	184.5	- 9.6	0.822	59.3	C/1893 U1	
	4	2.511	1.0005	346.2	175.9	138.6	186.3	- 9.0	2.55	178.0	C/1973 A1	
M	1	0.006	0.999914	82.6	3.5	144.4	282.6	+35.3	0.010	0.013	C/1843 D1	
	2	0.005	1.0	86.2	7.8	144.7	282.4	+35.2	0.010	0.012	C/1880 C1	
	3	0.008	0.999907	69.9	347.7	142.0	282.9	+35.9	0.011	0.024	C/1882 R1-B	
	4	0.005	1.0	83.5	4.6	144.4	282.5	+35.4	0.009	0.011	C/1887 B1	
	5	0.008	1.0	72.1	351.2	141.9	283.6	+36.0	0.011	0.022	C/1945 X1	
	6	0.005	0.999946	86.2	7.9	144.6	282.6	+35.3	0.009	0.011	C/1963 R1	
	7	0.008	0.999915	69.1	347.0	141.9	283.0	+35.2	0.011	0.024	C/1965 S1-A	
	8	0.009	1.0	61.3	337.0	139.1	283.0	+35.1	0.012	0.034	C/1970 K1	
	9	0.005	1.0	67.7	345.0	141.5	282.7	+35.2	0.007	0.015	C/1979 Q1	SOLWIND 1
	10	0.008	1.0	65.4	342.1	140.7	282.7	+35.2	0.011	0.027	C/1981 B1	SOLWIND 2
	11	0.006	1.0	68.4	346.0	141.7	282.7	+35.2	0.009	0.019	C/1981 O1	SOLWIND 3
	12	0.004	1.0	77.7	357.6	143.8	282.7	+35.2	0.007	0.011	C/1981 V1	SOLWIND 4
	13	0.008	1.0	78.6	358.7	144.0	282.7	+35.2	0.013	0.019	C/1983 S2	SOLWIND 6
	14	0.005	1.0	80.6	1.2	144.3	282.7	+35.2	0.009	0.013	C/1987 T2	SMM 1
	15	0.006	1.0	82.6	3.7	144.5	282.7	+35.2	0.011	0.014	C/1987 U4	SMM 2
	16	0.005	1.0	85.9	7.7	144.7	282.7	+35.2	0.010	0.011	C/1988 M1	SMM 3
	17	0.006	1.0	82.3	3.2	144.4	282.7	+35.2	0.010	0.014	C/1988 Q1	SMM 4
	18	0.005	1.0	88.1	10.4	144.8	282.7	+35.2	0.010	0.011	C/1988 T1	SMM 5
	19	0.006	1.0	86.1	8.0	144.7	282.7	+35.2	0.011	0.012	C/1988 U1	SMM 7
	20	0.006	1.0	91.1	14.0	144.8	282.7	+35.2	0.012	0.012	C/1988 W1	SMM 6
	21	0.006	1.0	87.7	6.2	144.6	282.7	+35.2	0.010	0.012	C/1989 L1	SMM 8
	22	0.005	1.0	91.8	14.9	144.8	282.7	+35.2	0.010	0.009	C/1989 N3	SMM 9
	23	0.005	1.0	87.5	9.6	144.8	282.7	+35.2	0.009	0.010	C/1989 S1	SMM 10

to the ascending order in the values of their ecliptic Longitude L. Each of these groups contains at least two sets of orbital elements q, e, ω and i, including the ecliptic Longitude L and latitude B. The group M is conspicuous for having the largest number of component orbits of those comets whose perihelions are closest to the surface of the Sun and hence they are known as the sun-grazing comets (Table 21).

The investigation of the 'Catalogue 1995' has resulted in the increase of the number of component comet orbits of the group F and M. Only one component orbit has been added to the group F at the serial number 4 and eighteen to the group M in the serial numbers from 6 to 23. As a result, with 23 component orbits the group M remains as the largest one. Of these 23 comets the space telescope SOLWIND have discovered 5 from 1979 to 1983 and the SMM (Solar Maximum Mission) succeeded to discover 10 comets within a short period of three years from 1987 to 1989. With the inclusion of these 19 component orbits, one in the group F and 18 others in the group M, the total number of the constituent orbits of all the groups have increased to 58. It is very interesting to note that these 58 comets listed in 15 groups are all long periodic.

The component orbits of a single group may be the fragments of a primitive cometary nucleus which might have suffered tidal disruption at a great distance from the Sun where the cometary nucleus moves at a very slow velocity. The slightest variations in the velocity of the component nuclei at that long distance from the Sun may cause very large change of their semi-major axis and consequently in the perihelion distances and also in the period of revolution, if there be any. But can we assign the apparitions of some 15 comets of the group M having almost identical orbital elements as the shower of comets from a single source ?

8. Discovery and recovery of comets

The first discovery of a comet with the help of a telescope was done in 1680 by Gottfried Kirch (Festou et al, 1993). The comet C/1680 V1 was later found to have double nuclei. The discovery of comets and naming them after their respective discoverers began since the year 1760. The first such comet recorded in the 'Catalogue 1995' is C/1760B1 which was one of the twelve comets discovered by Charles Messier (1730-1817). Thus he initiated the process of dedicated comet search with the help of telescopes and binoculars. But the first recovery of a periodic comet was done by a well-to-do German farmer named Johann G. Palitzsch on 25 December, 1758, as already stated.

During the period of 18th century, only 63 discovered and recovered comets were observed to pass their respective perihelion. Comets were detected visually through the optical telescopes until the last decade of the 19th century, during which only 309 perihelion passages of discovered and recovered comets were observed. With the advent of photographic technique around 1900 and the application of CCD detector in more recent years, the rate of cometary detection and their perihelion passage as well have enhanced considerably. Upto 1989 of the 20th century the number of comets observed to pass their perihelion have been recorded to be 806.

The growth rate of discovery and perihelion passage of comets per decade have been studied for the period from 1700 to 1989 (Table 22). The rate of discovery and perihelion passage of comets per decade which includes the returned perhilion passage of SP comets, have been shown in the Fig 9.

As recorded in the 'Catalogue 1995' as many as 878 comets have been discovered upto the year 1994. the monthwise discoveries, excluding the recoveries of SP comets, have been analysed (Table 23). It has been found that the maximum number of 83 comets were discovered in the months of November and minimum number of 55 comets were discovered in the months of May (Fig. 10).

The analysis of the 'Catalogue 1995' reveals that in all 466 observers including the three instruments SMM, SOLWIND and IRAS succeeded so far to discover or recover or both.

Tabel 22. Rate of discovery and perihelion passage of comets per ten years.

Period Decade	Rate of discovery	Rate of perihelion passage	Period Decade	Rate of discovery	Rate of perihelion passage
1700-09	4	4	1850-59	28	39
1710-19	1	1	1860-69	27	35
1720-29	2	2	1870-79	24	38
1730-39	3	3	1880-89	42	51
1740-49	7	7	1890-99	36	55
1750-59	3	6	1900-09	30	42
1760-69	8	6	1910-19	28	47
1770-79	7	7	1920-29	29	51
1780-89	13	13	1930-39	29	54
1790-99	13	14	1940-49	47	75
1800-09	7	10	1950-59	39	84
1810-19	14	15	1960-69	42	95
1820-29	16	20	1970-79	71	144
1830-39	7	11	1980-89	115	214
1840-49	30	35			

Table 23. Monthwise discovery of comets

Comets	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Total
SP	13	10	8	6	6	8	6	14	13	10	11	11	116
LP	65	51	71	62	49	55	69	65	67	69	72	67	762
Total	78	61	79	68	55	63	75	79	80	79	83	78	878

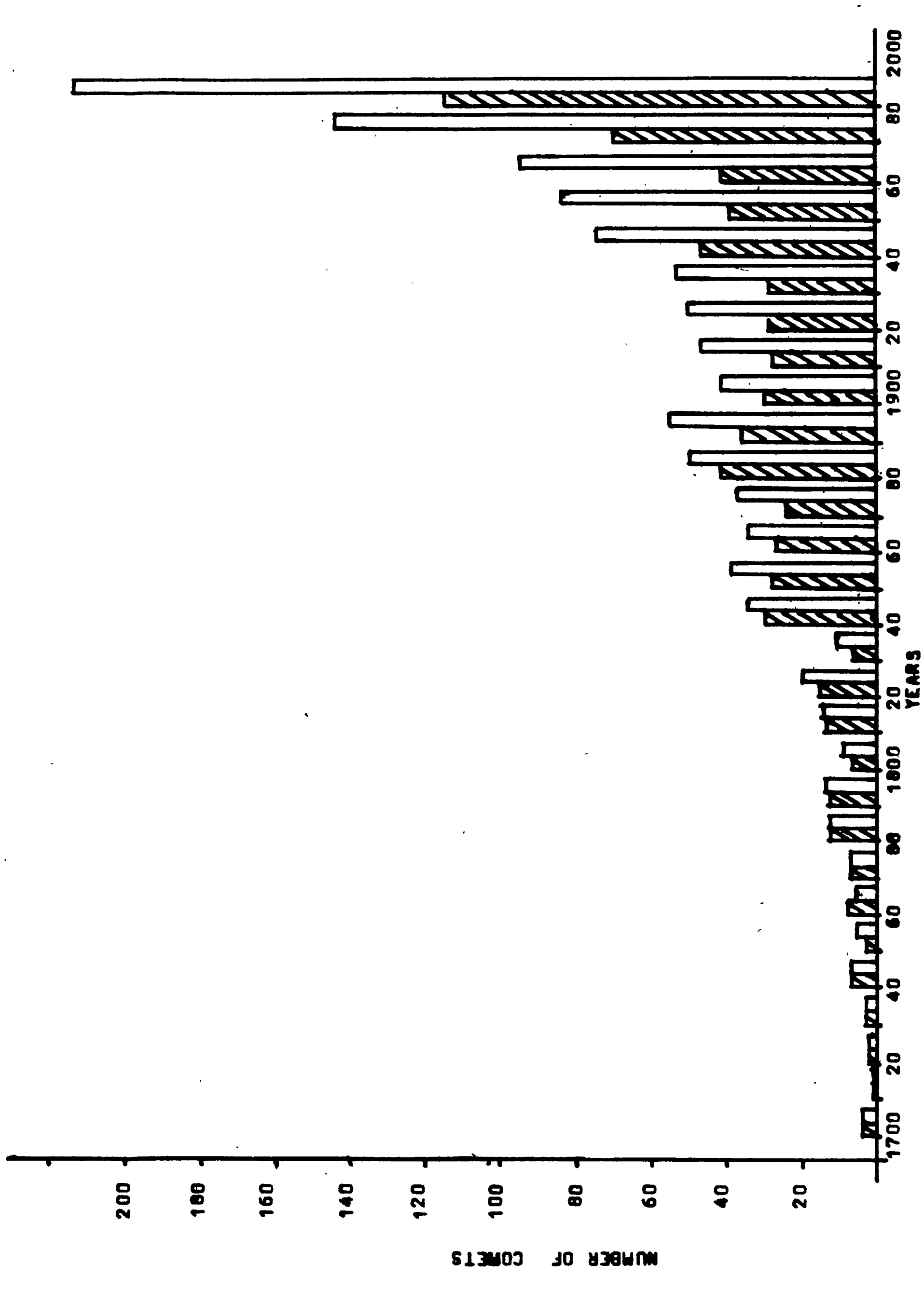

Figure 9. The rate discovery [hatched] and the rate of Perihelion Passage [open] of comets.

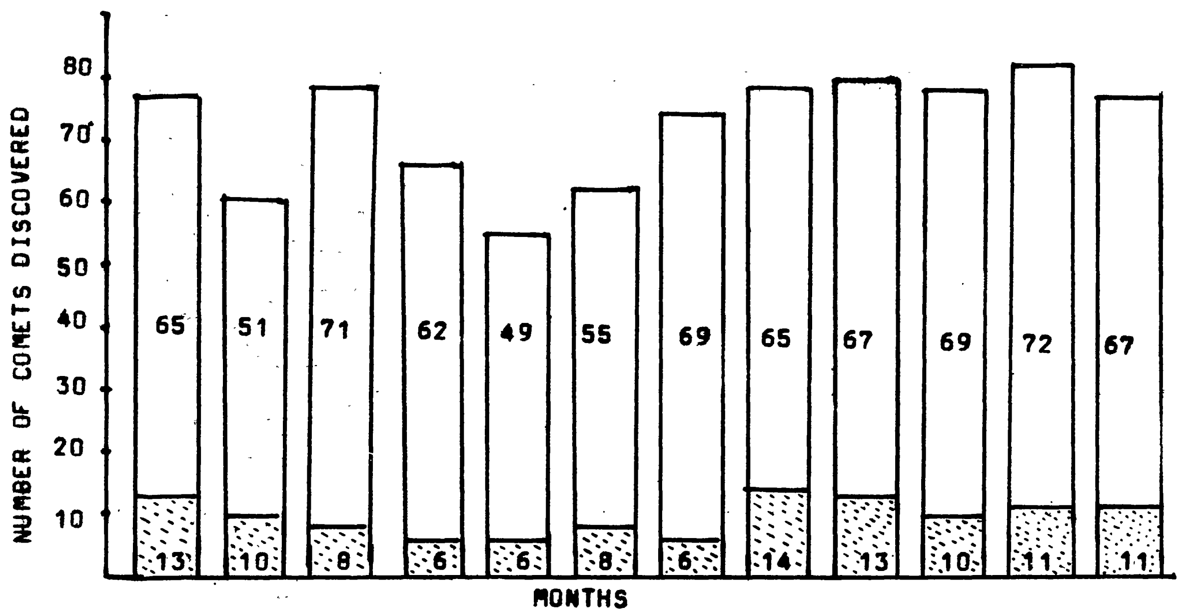

Figure 10. Monthwise discovery of SP comets and LP comets

Of these 275 observers discovered only, 120 observers recovered only and 71 observers had the credit of both discovering and recovering the comets. The most successful observer, so far, is Eugene Shoemaker (1928-97) who individually discovered 14 comets and jointly discovered 18 comets (Table 24). The other four leading discoverers are Pons, Brooks, Levy and Barnard. Incidentally David H. Levy and Shoemaker jointly discovered 13 comets, including the comet C/1993 F2 (Shoemaker-Levy 9) which ultimately crashed into the globe of Jupiter in 1994.

Table 24. List of highest number of comet discoverers.

Sl. No.	Discoverer	Period	No. of comets discovered		Total
			Alone	Jointly	
1.	Shoemaker	1983-84	14	18	32
2.	Pons	1801-27	22	4	26
3.	Levy	1984-94	5	16	21
4.	Brooks	1885-1912	18	2	20
5.	Barnard	1881-92	14	2	16

Among the recoverers, Elizabeth Roemer tops the list, recovering as many as 79 periodic comets of which 32 were recovered by her alone. It is a matter of great surprise that Roemer could not have the chance to discover even a single comet !

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Appendix - I

The sphere of influence radius

The Sphere of Influence (SOI) is defined by the space around a planet within which a body can describe a planeto-centric orbit more conveniently inspite of the gravitational pull of the Sun than to orbit on a heliofocal path. By solving many-body problem, SOI radius r_{SOI} of a planet has been obtained as

$$r_{SOI} = \left[\frac{M_p}{M_o} \right]^{\frac{2}{5}} r_p \ldots \qquad (6)$$

where, M_p is the mass of the planet,

r_p is the radius vector of the planetary orbit, and

M_o is the mass of the Sun.

Considering the mass of the Sun $M_o = 1.99 \times 10^{30}$ kg and the corresponding value of M_p and r_p of all nine planets of the solar system, their respective SOI radius r_{SOI} have been determined with the application of the relation (6) and are listed in Table 25.

Table 25. Sphere of Influence radius of planets.

Planet	Planet's			Sphere of influence radius		
	Mass in 10^{24} kg	Distance A.U.	Radius km	in A.U.	in 10^4 Km	in radius of planet
Mercury	0.3302	0.387	2487	7.518×10^{-4}	11.24	45.18
Venus	4.871	0.723	6187	4.097×10^{-3}	61.29	99.06
Earth	5.975	1.000	6378	6.180×10^{-3}	92.95	144.96
Mars	0.642	1.524	3380	3.859×10^{-3}	57.73	170.80
Jupiter	1900	5.203	71370	0.322	4821.00	675.50
Saturn	568.8	9.555	60330	0.365	5465.00	905.87
Uranus	87.0	19.22	25400	0.347	5187.0	2042.26
Neptune	103.0	30.11	22320	0.581	8694.0	3895.25
Pluto	0.0131	39.44	1151	0.021	3149.0	2736.24

If any comet happens to traverse on a path passing through any of the SOI radius of a planet or planet size satellite, then it may be captured to on an orbit round the planet or satellite instead of orbiting round the Sun and finally may crash into the body of the latter.

Appendix - II

The closest distances of the SP comet aphelia from the nearby planetary orbits, d_{PC}.
i) Closest aphelia distances

$$d_{JC} = \left[Q^2 + 5.2^2 - 2Q5.2\mathrm{Cos}\,(i_c - 1.3^o) \right]^{1/2}$$

to the orbit of Jupiter whose $r_p = 5.2$ A.U. and $i_p = 1.3^o$.

Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{JC} A.U.	Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{JC} A.U.
2P	4.09	11.9	1.40	P/1991 F1	4.93	31.5	2.65
107P	4.29	2.8	0.92	P/1990 R1	4.93	7.1	0.58
39P	4.53	4.0	0.71	P/1992 G2	4.93	6.1	0.50
111P	4.60	4.2	0.64	P/1991 C2	4.95	8.5	0.68
82P	4.64	1.1	0.56	86P	4.96	15.5	1.28
71P	4.68	9.5	0.88	48P	4.98	13.7	1.12
P/1990 R2	4.68	14.9	1.28	P/1991 S1	4.99	7.3	0.57
P/1991 C1	4.68	5.0	0.52	D/1819 W1	5.03	9.1	0.72
9P	4.73	10.6	0.93	83P	5.06	22.7	1.91
10P	4.73	12.0	1.04	D/1783 W1	5.06	45.1	3.83
116P	4.73	3.7	0.51	41P	5.14	9.2	0.71
P/1991 R2	4.74	10.0	0.88	46P	5.15	11.7	0.94
65P	4.74	10.4	0.91	91P	5.15	14.1	1.33
110P	4.75	11.7	1.00	57P	5.17	2.8	0.14
94P	4.79	6.2	0.59	44P	5.17	7.0	0.52
100P	4.80	25.7	2.15	73P	5.18	11.4	0.91
87P	4.80	2.6	0.42	D/1952 B1	5.20	16.3	1.36
74P	4.81	6.6	0.60	54P	5.21	3.6	0.21
79P	4.81	2.9	0.41	17P	5.21	19.2	1.62
31P	4.82	3.8	0.44	D/1918 W1	5.21	5.6	0.39
25D	4.84	10.6	0.89	11D	5.22	5.4	0.37
D/1977 C1	4.84	3.2	0.40	37P	5.25	7.2	0.54
P/1993 X1	4.85	2.4	0.36	113P	5.25	5.8	0.41
D/1886 K1	4.86	12.7	1.05	36P	5.25	9.9	0.79
D/1884 O1	4.86	5.5	0.50	P/1994 P1-A	5.27	12.8	1.05
88P	4.88	4.4	0.42	P/1989 T2	5.27	9.8	0.78
105P	4.89	9.2	0.76	81P	5.30	3.2	0.20
77P	4.90	24.4	2.04	75P	5.30	5.9	0.43
D/1766 G1	4.92	7.9	0.65	89P	5.31	12.0	0.99
26P	4.93	21.1	1.76	61P	5.31	6.1	0.45

Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{JC} A.U.	Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{JC} A.U.
76P	5.33	30.5	2.66	14P	5.74	27.5	2.53
47P	5.34	12.5	1.04	49P	5.75	17.9	1.67
22P	5.35	4.7	0.35	108P	5.77	13.1	1.26
69P	5.35	20.6	1.77	103P	5.84	9.3	1.00
P/1889 E2	5.36	17.7	1.51	18P	5.85	17.8	1.71
43P	5.37	18.5	1.59	P/1993 K2	5.85	9.9	1.05
104P	5.39	15.8	1.35	19P	5.87	30.3	2.85
16P	5.40	5.5	0.44	98P	5.88	9.5	1.04
60P	5.41	6.7	0.54	96P	5.91	60.1	5.49
D/1978 C2	5.42	7.0	0.57	52P	5.95	10.2	1.14
P/1990 S1	5.43	9.4	0.79	P/1987 U1	5.95	4.4	0.81
84P	5.44	7.3	0.61	106P	5.96	20.1	1.97
114P	5.47	18.2	1.59	4P	5.96	9.1	1.07
D/1978 R1	5.48	5.9	0.51	P/1992 G3	6.00	29.8	2.86
70P	5.50	0.9	0.30	21P	6.01	31.8	3.05
P/1991 V2	5.50	10.3	0.89	50P	6.14	19.9	2.05
102P	5.51	26.2	2.33	D/1895 Q1	6.16	3.0	0.97
P/1989 E1	5.53	5.2	0.49	3D-A	6.19	12.5	1.49
45P	5.54	4.2	0.44	15P	6.19	3.7	1.02
D/1892 T1	5.55	31.3	2.80	P/1994 A1	6.20	4.2	1.04
P/1927 U2	5.55	8.8	0.79	P/1989 U1	6.20	7.4	1.77
62P	5.57	10.5	0.94	97P	6.25	13.0	1.57
P/1989 E3	5.58	15.4	1.38	29P	6.31	9.4	1.37
6P	5.59	19.4	1.74	59P	6.42	9.0	1.45
51P	5.59	8.7	0.80	D/1896 F1	6.45	5.5	1.32
D/1984 H1	5.59	3.0	0.42	80P	6.46	29.8	3.12
5D	5.61	29.4	1.65	115P	6.46	11.7	1.64
112P	5.61	24.2	2.18	P/1991 T1	6.46	11.8	1.64
78P	5.61	6.7	0.65	P/1991 V1	6.56	16.9	2.02
7P	5.62	22.3	2.01	D/1984 W1	6.57	21.6	2.41
D/1896 R2	5.62	11.4	1.08	32P	6.68	13.0	1.91
D/1770 L1	5.63	1.6	0.43	58P	6.84	14.1	2.11
30P	5.66	8.1	0.79	24P	6.94	11.8	2.06
P/1986 W1	5.69	1.5	0.49	P/1990 UL3	6.99	4.6	1.82
33P	5.71	20.1	1.85	93P	7.03	12.2	2.16
67P	5.73	7.1	0.77	P/1993 W1	7.24	16.5	2.60

ii) Closest aphelia distances

$$d_{SC} = \left[Q^2 + 9.54^2 - 2Q9.54\mathrm{Cos}(i_c - 2.49^o)\right]^{1/2}$$

to the orbit of Saturn whose r_p = 9.54 A.U. and i_p = 2.49°.

Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{SC} A.U.	Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{SC} A.U.
64P	7.43	9.3	2.33	90P	9.21	9.6	1.21
99P	7.50	4.4	2.13	63P	9.24	19.9	2.86
P/1988 V1	7.58	11.7	2.39	101P	9.24	5.1	0.52
42P	7.67	4.0	1.88	P/1987 Q3	9.30	4.7	0.44
56P	7.70	8.2	2.03	P/1983 M1	9.45	46.2	7.07
72P	7.88	8.6	1.90	P/1983 J3	10.1	4.3	0.64
40P	7.98	11.6	1.71	8P	10.3	54.7	8.76
68P	8.09	10.9	1.94	P/1983 C1	10.6	3.8	1.08
P/1992 Q1	8.12	18.1	2.78	66P	10.9	18.7	3.18
D/1993 F3-K	8.25	5.9	1.39	P/1986 A1	11.4	6.4	1.99
53P	8.33	6.6	1.37	P/1990 V1	11.8	24.3	4.61
D/1960 S1	8.54	6.7	1.20	28P	12.3	14.2	3.54
34P	8.70	11.7	1.67	P/1994 N2	12.6	17.6	4.20
85P	8.91	5.8	0.82	P/1994 X1	12.7	29.1	5.90
92P	8.96	18.7	2.67	P/1983 V1	14.2	95.7	16.66
P/1994 J3	8.97	24.8	3.62				

iii) Closest aphelia distance

$$d_{UC} = \left[Q^2 + 19.18^2 - 2Q19.18\mathrm{Cos}(i_c - 0.77^o)\right]^{1/2}$$

to the orbit of Uranus whose r_p = 19.18 A.U. and i_p = 0.77°.

Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{UC} A.U.	Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{UC} A.U.
27P	17.4	29.1	9.12	55P	19.6	162.7	6.34
95P	19.0	6.9	2.05	38P	20.9	18.0	6.24

iv) Closest aphelia distances

$$d_{NC} = \left[Q^2 + 30.06^2 - 2Q30.06\text{Cos}(i_c - 1.77^0)\right]^{1/2}$$

to the orbit of Nepture whose r_p = 30.06 A.U. and i_p = 1.77^0.

Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{NC} A.U.	Comet	Aphelion Q A.U.	Inclination i_c o	Proximity d_{NC} A.U.
P/1991 L3	26.6	19.2	9.93	23P	33.7	19.3	10.36
D/1827 M1	29.0	136.5	22.75	1P	35.3	162.2	11.96
20D	30.0	40.9	20.11	D/1846 D1	35.3	85.1	43.62
D/1921 H1	30.3	22.3	10.76	D/1989 A3	37.3	83.1	44.24
13P	32.6	44.6	23.0	D/1942 EA1	37.5	38.0	22.16
12P	33.5	74.2	37.65				
109P	51.7	113.4	33.61	D/1984 A1	55.5	51.8	32.16
D/1889 M1	54.2	31.2	18.59	35P	56.9	64.2	41.65
D/917 F1	55.1	32.7	20.11	D/1937 D1	64.9	26.0	26.63