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Asymptotic Values of Some Scattering Parameters from Mie Theory

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Abstract

The scattering parameters such as extinction efficiency, albedo, asymmetry parameter, etc. have been studied on the basis of the Mie theory of scattering of electromagnetic radiation by a sphere as well as the classical geometrical optics and diffraction (GOD). The spheres are assumed to be composed of water- or ice-like dielectric and absorbing materials with index of refraction, $m = m^1 - im^{11}; m^1 = 1.33$ or 1.333 and $m^{11} = 0$, 0.0033, 0.033, 0.1, 0.33 or 3.3. The minimum value of the size-to-wavelength parameter (the circumference of the sphere divided by the wavelength) has been estimated for each scattering parameter so that the asymptotic value obtained from Mie theory calculations agree, at moderate accuracy, with the corresponding result based on GOD.

Keywords: scattering, mie theory

Introduction

Mie Theory (see, for example, Mie 1908; van de Hulst 1957) is a rigorous solution to the problem of scattering of electromagnetic radiation by a spherical particle. The underlying assumptions are as follows: (1) The sphere is homogeneous and isotropic with a perfectly smooth surface, (ii) the sphere material is passive, i.e. it is composed of uncharged and unexcited matter whose temperature is such that its Planckian radiation is negligible at the frequency under consideration, (iii) the scattering process is coherent in the sense that the incident and the scattered radiations have the same wavelength, (iv) the scattering process is considered to be single and independent and (v) the size and the index of refraction of the sphere are arbitrary.

Define

$a =$ the radius of the sphere,

$\lambda =$ the wavelength of the incident radiation,

$m = m^{I} - im^{II} =$ the index of refraction of the material of the sphere,

$\theta =$ the scattering angle,

and $x = \frac{2\pi a}{\lambda} =$ the size-to-wavelength parameter.

Note that the size-to-wavelength parameter is denoted by x in the tables.

With (λ, a, m and θ) or (x, m and θ) as input parameters to a suitable computer code for the Mie theory (see, for example, Shah 1977) one can calculate all the scattering parameters including the scattering intensity functions.

In many applications, such as, for instance, rainbow, halo, glory, biological cells, scattering by interstellar and interplanetary grains as well as radar and lidar experiments, one encounters very large size-to-wavelength parameters, $x = \frac{2\pi a}{\lambda} >> 1$. Then the Mie calculations can become time consuming and expensive. Therefore, one may prefer some short-cut method. For very large particles with $x >> 1$ under the same assumptions mentioned above, one can use the concepts of Geometrical Optics and Diffraction (GOD). The exact calculations based on the Mie theory show that some of the scattering parameters in the limit of very large x, tend to approach asymptotically the values obtained on the basis of GOD. However, it is imperative to know the minimum value of the size-to-wavelength parameter, say x_{min}, such that, for $x \geq x_{min}$, GOD holds good for a specified accuracy. The purpose of this communication is to find approximate values of x_{min} for various scattering parameters.

Theoretical Consideration

The question is: What is GOD? Basically one uses the ray tracing method of geometrical optics to account for external and internal reflection as well as refraction at an interface between two media with different indices of refraction. Here the important physical quantities involved are the Fresnel reflection and transmission coefficients. To this is added the contribution due to Fraunhofer diffraction around the edge of the sphere. An appealing feature of GOD is that, conceptually in line with the familiar laws of physics at undergraduate level, one can keep track of various individual contributions due to reflection, refraction, transmission, absorption and diffraction phenomena.

The essential conditions for GOD to be valid are that the size-to-wavelength parameter $x >> 1$ and the central phase shift parameter (corresponding to the diameter of the sphere)

$2x \mid m-1 \mid >> 1$. For very large dielectric sphere (i.e.real index of refraction, $m^{11} = 0$) we have used the procedure described by van de Hulst (1957) and Irvine (1963). A modified version of the same expressions has been presented by Shah (1990) to facilitate computations. Bohren and Huffman (1983) have also given an elegant method for very large dielectric spheres. For very large absorbing sphere ($m^{11} > 0$) satisfying the conditions of GOD, we have used the expressions given by van de Hulst (1946) and Irvine (1965). Irvine has also included the procedure of Born and Wolf (1959) suitable for computing the reflectances. For very large and sufficiently absorbing sphere ($a\alpha >> 1, \alpha =$ the absorption coefficient), the refracted light will be completely absorbed within the sphere and so there will be no transmitted light. Therefore, one needs to consider only the contributions due to external reflection and diffraction.

The following scattering parameters for the Mie theory as well as GOD have been considered in the present study:

Q_{ext} = extinction efficiency (QEXT),

Q_{sca} = scattering efficiency (QSCA),

Q_{abs} = absorption efficiency (QABS),

$< cos\theta >$ = asymmetry parameter (ASYM),

Q_{pr} = efficiency for radiation pressure (QPR),

Q_{back} = backscattering efficiency (QBACK).

Albedo = QSCA/QEXT, also denoted by ALBEDO and

NN = the number of Mie coefficients considered from 1 to NN.

The equivalent symbols used in the tables are indicated within the brackets.

The cross sections corresponding to each efficiency can be defined by the product of particular efficiency and the geometrical cross section of the target sphere. For example, extinction cross section, $C_{ext} = \pi a^2 Q_{ext}$. The functional dependence of the above quantities has the form

$$Q = Q(a, \lambda, m) \quad or \quad Q(x, m)$$

where Q represents any of the above quantities including asymmetry parameter or albedo. The derivation of all the relevant equations can be found in the appropriate references cited earlier.

TABLE I

INDEX OF REFRACTION: REAL PART=1.333, IMAGINARY PART=0.0

X	QEXT	QABS	ALBEDO	ASYM	QPR	QBACK	NN
.10000E-04	.11283E-20	-.37616E-36	.10000E+01	.18352E-10	.11283E-20	.16924E-20	2
.10000E-03	.11283E-16	-.15407E-32	.10000E+01	.18353E-08	.11283E-16	.16924E-16	2
.10000E-02	.11283E-12	.00000E+00	.10000E+01	.18353E-06	.11283E-12	.16924E-12	2
.10000E-01	.11283E-08	.20680E-24	.10000E+01	.18352E-04	.11283E-08	.16923E-08	3
.10000E+00	.11275E-04	.00000E+00	.10000E+01	.18344E-02	.11254E-04	.16837E-04	3
.10000E+01	.95652E-01	.00000E+00	.10000E+01	.18473E+00	.77982E-01	.86135E-01	5
.10000E+02	.21577E+01	.00000E+00	.10000E+01	.70590E+00	.63457E+00	.54242E+00	17
.20000E+02	.20999E+01	.00000E+00	.10000E+01	.77222E+00	.47832E+00	.18695E+01	29
.40000E+02	.19583E+01	.00000E+00	.10000E+01	.84638E+00	.30083E+00	.81009E+00	51
.60000E+02	.20007E+01	.00000E+00	.10000E+01	.86310E+00	.27390E+00	.65156E-02	74
.80000E+02	.20708E+01	.00000E+00	.10000E+01	.87209E+00	.26489E+00	.52031E+00	94
.10000E+03	.21200E+01	.00000E+00	.10000E+01	.87578E+00	.26334E+00	.24723E+00	114
.20000E+03	.20222E+01	.00000E+00	.10000E+01	.86696E+00	.26902E+00	.16687E+01	218
.30000E+03	.20705E+01	.00000E+00	.10000E+01	.87846E+00	.25165E+00	.23618E-01	318
.40000E+03	.20241E+01	.00000E+00	.10000E+01	.87831E+00	.24631E+00	.38854E+01	423
.50000E+03	.20303E+01	.00000E+00	.10000E+01	.87960E+00	.24444E+00	.39818E+00	523
.60000E+03	.20342E+01	-.44409E-15	.10000E+01	.88056E+00	.24297E+00	.16000E+01	623
.70000E+03	.20147E+01	.00000E+00	.10000E+01	.87984E+00	.24209E+00	.25727E+01	723
.80000E+03	.20334E+01	.00000E+00	.10000E+01	.88206E+00	.23981E+00	.27687E+01	828
.10000E+04	.20228E+01	.00000E+00	.10000E+01	.88010E+00	.24254E+00	.34795E+01	1028
.11000E+04	.20203E+01	.00000E+00	.10000E+01	.88254E+00	.23732E+00	.65095E+00	1129
.12000E+04	.20131E+01	-.44409E-15	.10000E+01	.88157E+00	.23841E+00	.29374E+01	1229
.13000E+04	.20253E+01	.00000E+00	.10000E+01	.88169E+00	.23962E+00	.38065E+00	1329
.14000E+04	.20117E+01	.44409E-15	.10000E+01	.88168E+00	.23803E+00	.16180E+01	1429
.15000E+04	.20140E+01	.00000E+00	.10000E+01	.88258E+00	.23649E+00	.48273E+00	1534
.16000E+04	.20157E+01	.44409E-15	.10000E+01	.88318E+00	.23548E+00	.27003E+00	1634
.17000E+04	.20113E+01	.00000E+00	.10000E+01	.88258E+00	.23616E+00	.24852E+01	1730
.18000E+04	.20179E+01	.00000E+00	.10000E+01	.88252E+00	.23706E+00	.56602E+00	1830
.19000E+04	.20103E+01	.00000E+00	.10000E+01	.88184E+00	.23753E+00	.11700E+01	1935
.20000E+04	.20116E+01	.44409E-15	.10000E+01	.88292E+00	.23551E+00	.28005E-01	2035
.25000E+04	.20097E+01	.00000E+00	.10000E+01	.88357E+00	.23400E+00	.37563E+00	2536
.30000E+04	.20090E+01	.00000E+00	.10000E+01	.88379E+00	.23347E+00	.26212E+00	3038
.35000E+04	.20088E+01	.00000E+00	.10000E+01	.88376E+00	.23351E+00	.49323E+00	3539
.40000E+04	.20087E+01	.00000E+00	.10000E+01	.88358E+00	.23385E+00	.94766E+00	4040
.45000E+04	.20084E+01	.00000E+00	.10000E+01	.88343E+00	.23413E+00	.23413E+01	4546
.50000E+04	.20082E+01	.00000E+00	.10000E+01	.88312E+00	.23472E+00	.41188E+01	5047
.60000E+04	.20057E+01	.44409E-15	.10000E+01	.88359E+00	.23348E+00	.22486E+01	6045
.70000E+04	.20052E+01	.00000E+00	.10000E+01	.88433E+00	.23196E+00	.22470E+00	7048
.80000E+04	.20052E+01	.00000E+00	.10000E+01	.88459E+00	.23141E+00	.85630E+00	8051
.90000E+04	.20045E+01	.00000E+00	.10000E+01	.88415E+00	.23221E+00	.20990E+00	9053
.10000E+05	.20049E+01	.00000E+00	.10000E+01	.88354E+00	.23350E+00	.14421E+01	10051
.15000E+05	.20035E+01	.00000E+00	.10000E+01	.88383E+00	.23275E+00	.27825E+01	15060
.20000E+05	.20032E+01	.00000E+00	.10000E+01	.88377E+00	.23284E+00	.31227E+01	20064
.25000E+05	.20024E+01	.00000E+00	.10000E+01	.88405E+00	.23218E+00	.61941E+01	25073
.30000E+05	.20023E+01	.00000E+00	.10000E+01	.88419E+00	.23188E+00	.61611E+01	30077
.40000E+05	.20018E+01	.00000E+00	.10000E+01	.88442E+00	.23136E+00	.89830E+01	40086
.50000E+05	.20014E+01	.00000E+00	.10000E+01	.88438E+00	.23141E+00	.30267E+01	50090
.60000E+05	.20014E+01	.00000E+00	.10000E+01	.88413E+00	.23191E+00	.11715E+02	60090
.70000E+05	.20013E+01	.00000E+00	.10000E+01	.88416E+00	.23184E+00	.27564E+01	70081
.80000E+05	.20012E+01	.44409E-15	.10000E+01	.88432E+00	.23150E+00	.21134E+00	80094
.90000E+05	.20010E+01	.00000E+00	.10000E+01	.88438E+00	.23137E+00	.35018E+01	90094
.10000E+06	.20009E+01	.00000E+00	.10000E+01	.88430E+00	.23150E+00	.17454E+01	100107
x >> 1	.20000E+01	.00000E+00	.10000E+01	.88427E+00	.23147E+00		

(GEOMETRICAL OPTICS PLUS DIFFRACTION)

Results and Discussion

In what follows the results of calculations based on the GOD and the Mie theory are given for representative real and complex indices of refraction corresponding to pure and impure water, ice or frost in the visual wavelength range. They are relevant to the study of atmospheric optics (terrestrial clouds) and icy planets and satellites. In particular, the real part $m^1 = 1.33$ *or* 1.333 and the imaginary part m'' takes on various values like $m^{11} = 0, 0.0033, 0.033, 0.1, 0.33$ and 3.3. The Mie theory calculations have been performed with the help of author's FORTRAN program MIEHSS (Shah, 1977) operated on Mighty Frame II computer. The size-to-wavelength parameter is varied between $x = 10^{-5}$ to 10^5 at selected intervals. The primary objective here is to find x_{min}, the minimum value of the size-to-wavelength parameter from the results calcualted on the basis of the Mie theory. This x_{min} should be such that, for $x \gtrsim x_{min}$, the Mie theory and GOD both give nearly the same results for a given scattering parameter. Thus x_{min} for each scattering parameter can be regarded as approximate lower bound on the size-to-wavelength parameter for GOD to be valid within a specified accuracy.

The exact values of the scattering parameters according to the Mie theory have been listed in Table I for very large dielectric spheres with index of refraction $m = 1.333 - i0.0$. The bottom line corresponds to the results based on GOD (i.e. the classical theory of geometrical optics and diffraction). It is seen that various scattering parameters such as extinction efficiency, asymmetry parameter, etc. reach asymptotically the results given by GOD at different values of x_{min}. Suppose we restrict the tolerance of accuracy to three significant digits. Then Q_{ext} (GOD) = 2.00. The same value of Q_{ext} in Table I for Mie theory begins to occur at $x_{min} \simeq 7000$. This means that the approximations based on GOD are valid for $x \gtrsim 7000$ in the case of extinction efficiency provided the accuracy is limited to 3 significant digits. Similarly, x_{min} for the asymmetry parameter ($< cos\theta > =$ ASYM) at the same level of accuracy turns out to be $x_{min} \simeq 9000$. However, x_{min} in the cases of the efficiency for radiation pressure (Q_{pr} = QPR) or the backscattering efficiency (Q_{back} = QBACK) in Table I is difficult to specify because of the well known oscillations in the values of these quantitites as function of x.

In particular, the backscattering efficiency Q_{back}, for very large dielectric sphere, may oscillate drastically with large amplitudes depending on not only the size-to-wavelength parameter ($x >> 1$) but the index of refraction also (Shah, 1991). The functional dependence of Q_{back} on m is such that initially Q_{back} increases as m increases from $m = 1$. The maximum of Q_{back} is attained at the index of refraction $m \simeq 1.8$ (Kerker, 1969). As m goes on increasing still further, the trend is expected to reverse, finally, to conform to the case of very large perfectly reflecting/conducting sphere ($m = \infty, x >> 1$). It may also be mentioned that the oscillations in the backscattering efficiency and other scattering parameters are damped out considerably if the sphere is composed of absorbing material with complex index of refraction. Consequently, as x goes on increasing, the Mie theory results approach asymptotically to the corresponding results given by GOD in case of each scattering parameter. Besides, this asymptotic limit in many cases may approach at much smaller values of the size-to-wavelength parameter. But x_{min}, for a given scattering parameter does not show any clear cut systematic monotonic trends as the absorptivity (m^{11}) varies (see, for example, Table III).

TABLE II

INDEX OF REFRACTION: REAL PART=1.333, IMAGINARY PART=-0.1

X	QEXT	QABS	ALBEDO	ASYM	QPR	QBACK	NN
.10000E-04	.22434E-05	.22434E-05	.54946E-15	.18316E-10	.22434E-05	.18490E-20	2
.10000E-03	.22434E-04	.22434E-04	.54947E-12	.18316E-08	.22434E-04	.18490E-16	3
.10000E-02	.22434E-03	.22434E-03	.54947E-09	.18316E-06	.22434E-03	.18490E-12	3
.10000E-01	.22435E-02	.22435E-02	.54944E-06	.18316E-04	.22435E-02	.18489E-08	4
.10000E+00	.22530E-01	.22518E-01	.54670E-03	.18309E-02	.22530E-01	.18394E-04	5
.10000E+01	.36731E+00	.27012E+00	.26461E+00	.19073E+00	.34877E+00	.85763E-01	8
.10000E+02	.23921E+01	.12122E+01	.49324E+00	.91853E+00	.13084E+01	.21568E-01	21
.20000E+02	.22506E+01	.11341E+01	.49611E+00	.96112E+00	.11775E+01	.19298E-01	34
.40000E+02	.21596E+01	.10464E+01	.51545E+00	.96631E+00	.10839E+01	.22244E-01	57
.60000E+02	.21223E+01	.10131E+01	.52262E+00	.96771E+00	.10489E+01	.22169E-01	80
.80000E+02	.21013E+01	.99528E+00	.52636E+00	.96835E+00	.10303E+01	.22171E-01	102
.10000E+03	.20876E+01	.98404E+00	.52863E+00	.96871E+00	.10186E+01	.22170E-01	123
.20000E+03	.20558E+01	.95980E+00	.53312E+00	.96932E+00	.99342E+00	.22170E-01	229
.30000E+03	.20428E+01	.95094E+00	.53449E+00	.96947E+00	.98427E+00	.22170E-01	333
.40000E+03	.20355E+01	.94627E+00	.53510E+00	.96953E+00	.97946E+00	.22170E-01	436
.50000E+03	.20306E+01	.94337E+00	.53543E+00	.96955E+00	.97648E+00	.22170E-01	538
.60000E+03	.20272E+01	.94139E+00	.53562E+00	.96956E+00	.97444E+00	.22170E-01	640
.70000E+03	.20246E+01	.93994E+00	.53573E+00	.96957E+00	.97295E+00	.22170E-01	742
.80000E+03	.20225E+01	.93884E+00	.53580E+00	.96957E+00	.97182E+00	.22170E-01	844
.10000E+04	.20194E+01	.93726E+00	.53588E+00	.96956E+00	.97020E+00	.22170E-01	1046
.11000E+04	.20183E+01	.93667E+00	.53590E+00	.96956E+00	.96960E+00	.22170E-01	1148
.12000E+04	.20172E+01	.93618E+00	.53591E+00	.96955E+00	.96910E+00	.22170E-01	1249
.13000E+04	.20164E+01	.93576E+00	.53591E+00	.96955E+00	.96867E+00	.22170E-01	1350
.14000E+04	.20156E+01	.93540E+00	.53591E+00	.96955E+00	.96830E+00	.22170E-01	1451
.15000E+04	.20149E+01	.93508E+00	.53591E+00	.96954E+00	.96797E+00	.22170E-01	1552
.16000E+04	.20143E+01	.93480E+00	.53591E+00	.96954E+00	.96769E+00	.22170E-01	1653
.17000E+04	.20137E+01	.93456E+00	.53590E+00	.96954E+00	.96743E+00	.22170E-01	1754
.18000E+04	.20132E+01	.93434E+00	.53589E+00	.96953E+00	.96721E+00	.22170E-01	1855
.19000E+04	.20127E+01	.93414E+00	.53589E+00	.96953E+00	.96700E+00	.22170E-01	1956
.20000E+04	.20123E+01	.93396E+00	.53588E+00	.96952E+00	.96682E+00	.22170E-01	2057
.25000E+04	.20106E+01	.93326E+00	.53583E+00	.96951E+00	.96611E+00	.22170E-01	2560
.30000E+04	.20094E+01	.93279E+00	.53579E+00	.96950E+00	.96563E+00	.22170E-01	3064
.35000E+04	.20085E+01	.93245E+00	.53575E+00	.96949E+00	.96529E+00	.22170E-01	3566
.40000E+04	.20078E+01	.93220E+00	.53571E+00	.96948E+00	.96502E+00	.22170E-01	4069
.45000E+04	.20072E+01	.93199E+00	.53568E+00	.96947E+00	.96482E+00	.22170E-01	4571
.50000E+04	.20067E+01	.93183E+00	.53565E+00	.96947E+00	.96465E+00	.22170E-01	5073
.60000E+04	.20060E+01	.93158E+00	.53559E+00	.96945E+00	.96440E+00	.22170E-01	6077
.70000E+04	.20054E+01	.93140E+00	.53555E+00	.96945E+00	.96422E+00	.22170E-01	7081
.80000E+04	.20049E+01	.93126E+00	.53551E+00	.96944E+00	.96408E+00	.22170E-01	8084
.90000E+04	.20046E+01	.93116E+00	.53548E+00	.96943E+00	.96397E+00	.22170E-01	9087
.10000E+05	.20042E+01	.93107E+00	.53545E+00	.96943E+00	.96388E+00	.22170E-01	10089
.15000E+05	.20032E+01	.93081E+00	.53535E+00	.96941E+00	.96362E+00	.22170E-01	15100
.20000E+05	.20027E+01	.93068E+00	.53528E+00	.96940E+00	.96348E+00	.22170E-01	20108
.25000E+05	.20023E+01	.93060E+00	.53524E+00	.96939E+00	.96340E+00	.22170E-01	25115
.30000E+05	.20020E+01	.93054E+00	.53520E+00	.96939E+00	.96335E+00	.22170E-01	30120
.40000E+05	.20017E+01	.93047E+00	.53516E+00	.96938E+00	.96328E+00	.22170E-01	40130
.50000E+05	.20015E+01	.93043E+00	.53512E+00	.96937E+00	.96323E+00	.22170E-01	50138
.60000E+05	.20013E+01	.93040E+00	.53510E+00	.96937E+00	.96321E+00	.22170E-01	60145
.70000E+05	.20012E+01	.93038E+00	.53508E+00	.96937E+00	.96319E+00	.22170E-01	70151
.80000E+05	.20011E+01	.93037E+00	.53506E+00	.96936E+00	.96317E+00	.22170E-01	80156
.90000E+05	.20010E+01	.93036E+00	.53505E+00	.96936E+00	.96316E+00	.22170E-01	90161
.10000E+06	.20009E+01	.93035E+00	.53504E+00	.96936E+00	.96315E+00	.22170E-01	100166
x >> 1	.20000E+01	.93024E+00	.53488E+00	.96934E+00	.96304E+00	.22170E-01	

(GEOMETRICAL OPTICS PLUS DIFFRACTION)

TABLE III

The variation of x_{min} with absorptivity (m'') and the scattering parameters.

Index of Refraction $m=m'-im''$		Extinction efficiency Q_{ext}	Absorption efficiency Q_{abs}	Albedo	Asymmetry Factor $\langle\cos\theta\rangle$	Radiation pressure efficiency Q_{pr}	Back scattering efficiency Q_{back}
1.33-i0.0	Mie	2.0047	0.00000	1.0000	0.88474	0.22943	Oscillates
	x_{min}	6000	Arbitrary	Arbitrary	6000	80000	heavily upto $x \simeq 10^5$
	GOD	2.0000	0.00000	1.0000	0.88532	0.22936	--------
1.33-i0.0033	Mie	2.0054	0.93450	0.53344	0.97169	0.96452	0.020055
	x_{min}	7000	20000	15000	600	25000	700
	GOD	2.0000	0.93405	0.53298	0.97176	0.96415	0.020061
1.33-i0.033	Mie	2.0054	0.93454	0.53344	0.97146	0.96449	0.020294
	x_{min}	7000	10000	40000	200	20000	100
	GOD	2.0000	0.93368	0.53316	0.97153	0.96403	0.020256
1.33-i0.33	Mie	2.0049	0.90340	0.54941	0.95083	0.95741	0.039341
	x_{min}	8000	10000	6000	100	9000	40
	GOD	2.0000	0.90269	0.54866	0.95086	0.95661	0.039330
1.33-i3.3	Mie	2.0053	0.32944	0.83555	0.59938	0.99939	0.67400
	x_{min}	90000	7000	2500	25000	20000	200
	GOD	2.0000	0.32850	0.83575	0.59884	0.99904	0.67400

Note: Mie – Exact calculations from Mie theory,
GOD – Results based on GOD,
x_{min}– Mie theory and GOD agree to 3 *significant places for* $x \gtrsim x_{min}$.

The asymmetry parameter and the efficiency for radiation pressure, for scattering by a very large dielectric sphere, do not follow monotonic trends as function of the index of refraction. Rather a new feature of shallow resonance has been found in $< cos\theta >$ as well as Q_{pr} (Shah 1991). The asymmetry parameter has the minimum value, $< cos\theta >_{min} = 0.476792$ and the corresponding efficiency for radiation pressure has the maximum value $(Q_{pr})_{max} = 1.04642$ in the interval of index of refraction defined by $11.201 \lesssim m \lesssim 11.203$. The question is: can these results be derived analytically?

As a representative case of a very large absorbing sphere, we have chosen the index of refraction, $m = 1.333 - i\ 0.1$. The resulting Mie calculations are listed in Table II. The corresponding results based on GOD are entered in the bottom line in Table II. The minimum values of the size-to-wavelength parameter, for the Mie theory and GOD to agree at the level of 3 significant digits, are given approximately by x_{min} = 7000, 30000, 10000, 6000, 25000 and 40 corresponding to Q_{ext}, Q_{abs}, albedo, $< cos\theta >$, Q_{pr} and Q_{back}, respectively. Note that the oscillations, which are prominent in the case of real $m = 1.333$, are now highly damped out with the addition of absorptivity (m^{11}).

Regarding the variation of x_{min} with different scattering parameters as function of the imaginary part of the index of refraction, the results for both GOD and the Mie theory are summarized in Table III. Note that , for $x \gtrsim x_{min}$, GOD holds good provided the tolerance of accuracy is restricted to three significant digits for all the scattering parameters listed in Table III. Mie and GOD indicate the exact values obtained according to the Mie theory and Geometrical Optics plus Diffraction, respectively. Some interesting features may be noted. According to Table III, $< cos\theta >$ and Q_{pr} are almost independent of absorptivity provided absolute value of the imaginary part (m'') of the index or refraction is sufficiently small (but not zero) compared to the real part (m'). The same is true for Q_{abs} and albedo too. Such a feature of near constancy of $< cos\theta >$ or Q_{pr} has been described by Irvine (1965) for index of refraction satisfying the condition $0 < | \frac{m''}{m'-1} | \lesssim 1$. The asymptotic value of the backscattering efficiency (Q_{back}), for very large absorbing sphere according to the Mie theory, conforms to the results given by GOD remarkably well at much smaller value of x_{min} compared to the cases of other scattering parameters.

Conclusion

The theoretical formulation, for scattering of electromagnetic radiation based on the classical theories of geometrical optics and diffraction in the case of very large homogeneous, isotropic and smooth spheres, can be used provided the size-to-wavelength parameter is larger than certain minimum value, say, x_{min}. It is not possible to define a unique value of x_{min} for all the scattering parameters. However, for routine application in the case of particular scattering parameter, index of refraction and tolerance of moderate accuracy, x_{min} can be established with reference to sample calculations based on the Mie theory.

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