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Generalised photogravitational restricted three body problem

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Abstract. The problem is generalised in the sense that the rigid spherical shell, filled with homogeneous, incompressible fluid is taken as an oblate spheroid and the second body is radiating outside the shell. The equations of motion of the problem has been found, which is different from Robe's. The locations and stability of collinear points of the problem have further been studied.

Keys Words : restricted problem, collinear points

1. Introduction

Robe (1977) has considered a new kind of restricted three body problem, in which one body m_1 is a rigid spherical shell, filled with homogeneous, incompressible fluid of density ρ_1 The second one, m_2 is a mass point outside the shell ; and m_3 is a small sphere of density ρ_3 moving inside the shell and is subject to the attraction of m_2 and the buoyancy force due to the fluid ρ_1. He obtained an equilibrium point and studied the linear stablity of the point corresponding to the particular solutioin of the rotating system.

Shrivastava & Gorain (1991) considered the effect of perturbation due to Coriolis and centrifugal forces on the location of liberation points in Robe- circular RTBP.

Here in the present paper we have considered the rigid shell m_1 of density ρ_1 as an oblate spheroid and m_2 as a radiating mass point in Robe-circular RTBP. It is of interest now to study the effect of perturbation due to oblateness and radiation of the respective primaries on the location of liberation points in Robe's RTBP.

The following forces are acting on m_3:

(i) Attraction of m_2.
(ii) Oblateness effect of m_1.
(iii) Effect of Radiation of m_2.
(iv) The Gravitational Force, exerted by the Fluid of density ρ_1 i.e $F_A = -(^4/_3)\, \pi\, \rho_1\, G\, m_3 M_1 M_3$;
(v) The Buoyancy Force, $F_B = (^4/_3)\, \pi\, G\, \rho_1^2\, m_3 M_1 M_3/\rho_3$,

Where $r_1 = M_1M_3$; $r_2 = M_2M_3$;

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M_1M_2 and M_3 being the centres of m_1,m_2 and m_3 respectively and G is the Gravitational Constant. (x,y,z) are the co-ordinates of the infinitesimal mass, m_3 and the line joining m_1 and m_2 is the x axis.

The total potential acting at m_3 is

$$-\frac{G\,m_2 q}{r_2} + {}^4/_3\pi\, G\, \rho_1\left(1-\frac{\rho_1}{\rho_3}\right) r_1^2 - \frac{G\,m_1}{r_1} - \frac{G\,m_1 A_1}{2\,r_1^3} \qquad (1)$$

Where q is the mass reduction factor due to radiation effect and A_1 stands for oblateness effect,

2. Equation of motion

$$\ddot{x} - 2n\dot{y} = \frac{\partial \Omega}{\partial x}$$

$$\ddot{y} + 2n\dot{x} = \frac{\partial \Omega}{\partial x}$$

$$\ddot{z} = \frac{\partial \Omega}{\partial x} \qquad (1)$$

Where $$\Omega = \frac{n^2(x^2+y^2)}{2} - Kr_1^2 + \frac{\mu q}{r_2} + \frac{1-\mu}{r_1} + \frac{(1-\mu)A_1}{2r_1^3} \qquad (2)$$

Here $$K = \frac{4}{3}\pi\rho_1\left(1-\frac{\rho_1}{\rho_3}\right);\ \mu = \frac{m_2}{m_1+m_2},\quad 0<\mu<1$$

$$n^2 = 1+\frac{3A_1}{2};\quad q = 1-\frac{F_p}{F_g};\quad r_1^2 = (x^1-x_1)^2 + y^2 + z^2;$$

or $r_2^2 = (x-x_2)^2 + y^2 + z^2;\ \ x_1 = -\mu;\ x_2 = 1-\mu,\ 0<\mu<1$

Equilibrium exists when $\Omega_x = \Omega_y = \Omega_z = 0$

When $\rho_1 = \rho_3$ i.e $k = 0$ the equation (2) reduces to

$$\Omega = \frac{n^2}{2}(x^2+y^2) + \frac{\mu q}{r_2} + \frac{1-\mu}{r_1} + \frac{(1-\mu)A_1}{2r_1^3} \qquad (3)$$

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Therefore $$\Omega_x = n^2 x - \frac{\mu q\,(x-1+\mu)}{r_2^3} - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{3}{2}\frac{(1-\mu)A_1(x+\mu)}{r_1^5} = 0 \tag{4.1}$$

$$\Omega_y = y\,[\,n^2 - \frac{\mu q}{r_2^3} - \frac{(1-\mu)}{r_1^3} - \frac{3A_1(1-\mu)}{2\,r_1^5}\,] = 0 \tag{4.2}$$

$$\Omega_z = z\,[\,-\frac{\mu q}{r_2^3} - \frac{(1-\mu)}{r_1^3} - \frac{3A_1(1-\mu)}{2\,r_1^5}\,] = 0 \tag{4.3}$$

From (4.2) and (4.3) we see that $y = 0$ and $z = 0$, and the collinear liberation points exist.

3. Location of the collinear points

When $y = 0$ & $z = 0$, Equation (4.1) determines the location of the

Collinear points $L_1\,(x_1,o,o)$, $L_2\,(x_2,o,o)$, $L_3(x_3,o,o)$

Where $x_1 = \xi_1 + 1-\mu$, $x_2 = \xi_2 - 1+\mu$, $x_3 = \xi_3+\mu$,; ξ_1,ξ_2,ξ_3 satisfying the quintics.

$$(2+3A_1)\,\xi_1^7 + (2+3A_1)(5-\mu)\,\xi_1^6 + (2+3A_1)(10-4\mu)\,\xi_1^5$$
$$+[(2+3A_1)(10-6\mu) - 2\mu q + 2\mu - 2]\,\xi_1^4, + [\,(2+3A_1)$$
$$(5-4\mu) - 8\mu q + 4\mu - 4]\,\xi_1^3 - 12\mu q\,\xi_1^2 - 8\mu q\xi_1 - 2\mu q = 0,$$

$$(2+3A_1)\,\xi_2^7 - (2+3A_1)(5-\mu)\,\xi_2^6 + (2+3A_1)(10-4\mu)\,\xi_1^5$$
$$-[(2+3A_1)(10-6\mu) + 2\mu q + 2 - 2\mu]\,\xi_2^4 + [(2+3A_1)(5-4\mu) + 8\mu q$$
$$+ 4(1-\mu)]\,\xi_2^3$$
$$-[2(2+3A_1)(1-\mu) + 12\mu q]\,\xi_2^2 + 8\mu q\xi_2 - 2\mu q = 0,$$

$$(2+3A_1)\,\xi_3^7 + (2+3A_1)(\mu+2)\,\xi_3^6 + (2+3A_1)(2\mu+1)\,\xi_3^5$$
$$+[(2+3A_1)\,\mu - 2\mu q + 2\mu - 2]\,\xi_3^{4C} - 4(1-\mu)]\,\xi_3^3$$
$$-(2+3A_1)(1-\mu)\,\xi_3^2 - 6(1-\mu)A_1\xi_3 - 3(1-\mu)A_1 = 0 \tag{5}$$

For $y \neq 0$, Equations (4.1) and (4.2) disclose that $r_2^3 = q/n^2$, $r_1 = 1$
Equations (5) locate the other two points L_4 and L_5. These points forming isoceles triangles with the primaries are known as Triangular points. It may be noted that $r_2 < 1$

4. Stability of the liberation points

Putting $x = x_0 + \xi$, $y = y_0 + \eta$, in equation (1)

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for studying the motion near any of the equilibrium points L (x_0,y_0), we get the first variational equations as :

$$\ddot{\xi} - 2n\dot{\eta} = \Omega_{xx}(x_0,y_0)\,\xi + \Omega_{xy}(x_0,y_0)\,\eta$$
$$\ddot{\eta} + 2n\dot{\xi} = \Omega_{xy}(x_0,y_0)\,\xi + \Omega_{yy}(x_0,y_0)\,\eta \qquad (6)$$

The characteristic equation of equations (6) is

$$\lambda^4 + (4n^2 - \Omega^0_{xx} - \Omega^0_{yy})\,\lambda^2 + \Omega^0_{xx}\,\Omega^0_{yy} - \Omega^{0\,2}_{xy} = 0 \qquad (7)$$

5. Stability of the collinear points

At the Collinear points, we get

$$\Omega^0_{xx} = n^2 + \frac{2\mu q}{r_2^3} + \frac{2(1-\mu)}{r_1^3} + \frac{6(1-\mu)\,A_1}{r_2^5} > 0$$

$$\Omega^0_{xy} = 0$$

$$\Omega^0_{yy} = n^2 - \frac{\mu q}{r_2^3} - \frac{(1-\mu)}{r_1^3} - \frac{3A_1(1-\mu)}{2r_1^5} < 0$$

Consequently $\quad \Omega^0_{xx}\,\Omega^0_{yy} - \Omega^{0\,2}_{xy} < 0$

It can easily be found that the roots $\lambda_i (i = 1,2,3,4)$ of the characteristics equation (7) are

$$\lambda_{1,2} = \pm\,[-C_1 + (C_1^2 + C_2^2)^{1/2}]^{1/2} = \pm\,\lambda$$
$$\lambda_{3,4} = \pm\,[-C_1 - (C_1^2 + C_2^2)^{1/2}]^{1/2} = \pm\, is$$

Where $\quad C^1 = 2n^2 - (\Omega^0_{xx} - \Omega^0_{yy})/2$

$$C_2^2 = -\,\Omega^0_{xx}\,\Omega^0_{yy} > 0$$

The general solution of Equation (6) can be written as :

$$\xi = \sum_{i=1}^{4} A_{ie}\,\lambda it, \quad \eta = \sum_{i=1}^{4} B_{ie}\lambda it,$$

and $\quad (\lambda_i^2 - \Omega^0_{xx})\,A_i = (2n\lambda_i + \Omega^0_{xy})\,B_i \qquad (8)$

It may be noted that $\lambda_{1,2}$ are real whereas $\lambda_{3,4}$ are purely imaginary. Hence the Collinear equilibria are unstable in the general case : However, it is possible to choose the initial conditions (ξ_0, η_0) as in Szebehely (1967 a) and Sharma & Subba Rao (1976) such that $A_{1,2} = 0$ and the equation (8) represent an ellipse, whose eccentricity and semi- major axis are $(1 - C_3^{-2})^{1/2}$ and $(C_3^2\xi_0^2 + \eta_0^2)^{1/2}$ respectively,

where $\quad C_3 = (s^2 + \Omega^0_{xx})/2ns$

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Conclusion

(i) We find that the position of liberation points are affected by oblateness and radiation of the primaries and this can be compared with the classical Robe Circular Restricted Problem of Three Bodies by putting $q = 1$ & $A_1 = 0$ in case when $\rho_1 = \rho_3$ i.e. $K = 0$.

(ii) Collinear equilibria are unstable in general case but through special choice of initial condition elliptical periodic orbits exist.

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