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Star formation in molecular clouds and the initial mass function : A new model

H. C. Bhatt

Indian Institute of Astrophysics, Bangalore 560034, India

Abstract. The observed mass spectra for stars and interstellar clouds are discussed. Stars are born from clouds, but their mass spectra are fundamentally different. A new model is proposed in which the Salpeter function representing the stellar mass spectrum is shown to result from the parent cloud mass spectrum. Stars form in the cores of the clumps in a cloud and grow by accretion. The accretion energy is deposited into the clump and disrupts it when the star has grown to a limiting mass. The limiting mass is derived. Combined with the observed clump mass spectrum, this leads to the Salpeter function for the stellar mass spectrum. Several other observed features of clouds and stars also follow.

1. The stellar mass spectrum

The Milky Way Galaxy has ~ 10^{11} stars with masses in the range ~ $0.1M_\odot - 100M_\odot$. The field stars in the Galaxy show an *initial mass spectrum (i.e. at birth) :* $dn_*/dm_* \propto m_*^{-x}$ with x=2.35 (The Salpeter (1955) mass function) where dn_* is the number of stars in the mass range m_* to $m_* + dm_*$. Similar stellar mass spectra are exhibited by stars in star clusters in our Galaxy as well as by those in external galaxies (e.g. Miller & Scalo 1979; Sagar et al. 1986). A consequence of this stellar mass spectrum is that the total mass of a system of stars (e.g. in a star cluster) $M_* = \int m_* dn_* \propto \int m_*^{-x+1} dm_* \propto [m_{*max}^{-x+2} - m_{*max}^{-x+2}]$, is dominated by the low mass stars.

2. Origin of stars

Stars are born from interstellar clouds by gravitational collapse. According to the Jeans (1928) criterion, a cloud of mass M, radius R and temperature T collapses under gravity if its gravitational potential energy $3GM^2/5R$ is greater than its thermal energy $3MkT/2\mu$. The star-forming clouds show the following properties.

1. Cloud masses M_c ~ 1 $M_\odot$ – ~ 10^5 $M_\odot$, cloud radii R_c ~ 0.1pc – 30pc, cloud temperatures T ~ 10 – 20 K.

2. Cloud mass spectra : $dn_c/dm_c \propto m_c^{-\alpha}$; with α ~ 1.6 (Bhatt et al., 1984, Bhatt & Williams

1986; Solomon et al., 1987). For such a cloud mass spectrum the total mass $M \propto \int m_c^{-\alpha+1} dm \;\alpha\; [m_{cmax}^{2-\alpha} - m_{cmin}^{2-\alpha}]$ is dominated by the more massive clouds, in contrast with the case of the stars.

3. Cloud mass - size relation : $M_c \propto R_c^2$ (Myers et al., 1991).

4. Cloud luminosity - mass relation : $L_c \propto M_c^{-1}$ (Rengarajan 1984).

5. Mass of the most massive star formed in a cloud $m_{*max} \propto M_c^{\beta}$ with $\beta = 0.43$ (Larson 1982).

Why is the stellar mass spectrum much steeper (x = 2.35) than the spectrum of parent cloud masses (α = 1.6)? While the cloud mass spectrum (α = 1.6) can be understood in terms of the Oort (1954) model for the formation and destruction of interstellar clouds, there is no satisfactory theory for the origin of the stellar mass function that is also consistent with the properties of the clouds that give birth to them. The global star formation rate in the Galaxy, implied by the Jeans criterion applied to the interstellar clouds is $\sim 10^3\, M_{\odot} yr^{-1}$, whereas the observed rate is a factor of 100 - 1000 times smaller. How does one understand the $m_* - M_c$ and $L_c - M_c$ relations?

3. A physical model for the process of star formation in molecular clouds

Consider a cloud of total mass M_c, radius R_c, density ρ_c and temperature T_c. The cloud is made up of clumps (Blitz 1991). The clumps have a mass spectrum $dn/dm_c = am_c^{-\alpha}$ with $\sim \alpha$ 1.6 in the range of masses m_{cmin} to m_{cmax}; α is a constant. For such a mass spectrum the total mass $M_c = \int m_c dn = \frac{a}{2-\alpha} m_{cmax}^{2-\alpha}$; for $m_{cmax} \gg m_{cmin}$. The mass of the most massive clump m_{cmax} is given by the condition : Number of clumps = $1 = \int_{m_{cmax}}^{\infty} dn = \frac{a}{\alpha-1} m_{cmax}^{1-\alpha}$. This leads to : $a = \frac{(2-\alpha)^{(\alpha-1)}}{(\alpha-1)^{(2-\alpha)}} \times M_c^{(\alpha-1)}$, $m_{cmax} = \frac{(2-\alpha)}{(\alpha-1)} M_c$ and $\frac{dn}{dm_c} = [(2-\alpha)^{(\alpha-1)}(\alpha-1)^{(2-\alpha)} M_c^{(\alpha-1)}] m_c^{-\alpha}$.

Gravitational collapse of the clumps and protostar build up : The clumps (mass m_c, radius r_c, temperature T_c velocity dispersion $\sigma \sim (3kT_c/\mu)^{1/2}$) on the verge of gravitational collapse satisfy the Jeans criterion $Gm_c/r_c = 5kT_c/2\mu$, so $m_c = (5kT_c/2\mu G)r_c$. The clump collapses, central density rises rapidly and soon the core becomes optically thick. This *protostellar core* accretes the infalling matter and grows in mass.

The core mass - radius relation : The optically thick central core is in hydrostatic equilibrium. Let m_*, r_*, ρ_*, p_* be the mass, radius, density and gas pressure at the surface of the protostellar core. For pressure equilibrium at the protostellar surface : gas pressure = gravitational pressure + ram pressure of accretion flow, so that $p_* = \frac{Gm_*^2}{r_*^4} + \frac{\dot{m}_*}{4\pi r_*^2}(2Gm_*/r_*)^{1/2}$, where the accretion rate $\dot{m}_* = \pi(2Gm_*/\sigma^2)^2\sigma\rho$.

Upon simplification $p_* = \frac{Gm_*^2}{r_*^4}\left\{1 + \left(\frac{m_* r_*^3}{m_c r_c^3}\right)^{1/2}\right\} = \frac{Gm_*^2}{r_*^4}$ as $m_* \ll m_c$ and $r_* \ll r_c$. Since the core is optically thick, radiative energy transport is unimportant and the core compression is adiabatic. Hence $p_* = S\rho_*^{\gamma}$, where $\gamma = c_p/c_v = 5/3$ for a monatomic gas and S is a constant. With $\rho_* \sim m_*/r_*^3$, $p_* = S(\frac{m_*}{r_*^3})^{5/3} = \frac{Gm_*^2}{r_*^4}$. This lead to : $r_* = \frac{S}{G} m_*^{-1/3}$.

Clump dispersal and limit to the stellar mass : The central adiabatic protostellar hydrostatic core halts the accretion flow at r_* where a shock front is formed. The gravitational potential energy of the accreted matter is deposited into the parent clump gas in the form of radiative and mechanical (shocks and outflows, e.g. Bachiller & Gomez-Gonzalez 1992) energy. When the total energy so deposited into the clump equals and exceeds the binding energy of the original clump, the clump becomes gravitationally unbound. The clump gets disrupted and merges with the parent cloud (M_c) gas. Further growth of the protostar is arrested. The protostar has reached its limiting mass m_{*max}. m_{*max} is given by the condition :

Accretion energy liberated = Grav. binding energy of the clump. That is, $Gm^2_{*max}/r_* = Gm_c^2/r_c$. With $r_* = \frac{S}{G} m_*^{-1/3}$ and $m_c = (5kT_c/2\mu G)r_c$ this leads to : $m_{*max} = G^{-6/7} S^{3/7}(5kT_c/2\mu)^{6/7} m_c^{3/7}$, i.e. $m_{*max} \propto m_c^{0.43}$ as 3/7 = 0.43. Compare this with Larson's (1982) observations : $m_{*max}/m_\odot = 0.33(m_c/m_\odot)^{0.43}$.

Star formation efficiency : For a clump of mass m_c, assuming that all the mass accreted by the protostellar core is converted into stellar mass, the star formation efficiency $SFE = m_*/m_c$ is given by $SFE(m_c) = (S\sigma^2 G^{-2})^{3/7} . m_c^{-4/7}$. The global star formation efficiency in the cloud of mass M_c is $SFE(M_c) = \int m_* dn/M_c \propto M_c^{\alpha-2}[m_{cmin}^{-\alpha+10/7}] \sim M_c^{-0.4}$ for $\alpha \sim 1.6$.

The stellar mass function : As stars form in the cores of the clumps of different masses a spectrum of stellar masses is produced. For a clump mass spectrum $\frac{dn}{dm_c} = [(2-\alpha)^{(\alpha-1)} (\alpha-1)^{(2-\alpha)} M_c^{(\alpha-1)}] m_c^{-\alpha}$ the resulting stellar mass spectrum will be : $\frac{dn_*}{dm_*} = \frac{dn}{dm_c}\frac{dm_c}{dm_*} \propto m_*^{(-\alpha+4/7)(7/3)}$. Thus a stellar mass function $\frac{dn_*}{dm_*} \propto m_*^{-x}$ is produced with $x = (-\alpha+4/7)(7/3)$. For $\alpha = 1.6$ (the observed value for molecular cloud clumps; Blitz 1993) $x = 2.4$, compared with the Salpeter value of 2.35.

Thus the observed stellar mass spectrum for the field stars and stars in a cluster is naturally explained as a consequence of the cloud mass spectrum with $\alpha \sim 1.6$ (produced by competing coagulation and fragmentation processes as in the Oort model) that give them birth, the protostellar mass - radius relation $r_* \propto m_*^{-1/3}$ (due to the adiabatic compression of the growing protostellar core) and dispersal of the gaseous mother clump as the accretion energy is deposited into its body and it reaches a limiting value (also the mass of the star $m_* \propto m_c^{3/7}$) equal to its binding energy.

The luminosity - mass relation for the star forming clouds : After the dispersal of the clump gas the new born stars find themselves in the general environment of the cloud, of which the clump was a part. The young star may accrete matter (now at a much reduced rate) from this cloud medium and radiate the accretion energy. Assuming that all the accretion energy is emitted as radiation, the accretion luminosity l_* for a young star of mass m_* will be : $l_* = Gm_*\dot{m}_*/r_*$; where $\dot{m}_*$ is the accretion rate $\pi(2Gm_*/\sigma^2)^2\sigma\rho_c = 4\pi G^{8/7} S^{3/7} \sigma^{1/7} m_c^{10/7} \rho_c$. Now $\rho_c \propto R_c^{-1} = C.M_c^{-1/2}$, where C = $3.10^{-19}\ m_\odot^{1/2}$ is a constant (Solomon et al., 1987), so that $l_* = [4\pi C G^{8/7} S^{3/7} \sigma^{1/7}] M_c^{-1/2}\ m_c^{10/7}$.

The total luminosity of a cloud mass M_c will be $L_c = \int l_* dn$

$$= [const.] M_c^{-1/2} \int m_c^{10/7}\, dn$$

$$= [const.][const.](1/(\tfrac{17}{7} - \alpha)) M_c^{\alpha - 3/2}\, m_{cmax}^{17/7-\alpha}.$$

With $m_{cmax} = \frac{(2-\alpha)}{(\alpha-1)} M_c$ this gives : $L_c \propto M_c^{13/14}$. We thus have $L_c \propto M_c^{0.93}$ to be compared with the observed $L_c \propto M_c^{1\pm0.15}$ relation.

4. Summary

1. Clouds form from the interstellar medium with a mass spectrum $dN/dM_c \propto M_c^{-\alpha}$ with $\alpha \sim 1.6$ as in the Oort's coagulation - fragmentation model.

2. Substructure within the clouds arises in the form of clumps with the same mass spectrum, $\alpha \sim 1.6$ (*self similar structure*).

3. When clumps satisfy the *Jeans criterion*, they undergo gravitational collapse. At the verge of collapse $m_c \propto r_c$.

4. A central core is rapidly built up that soon becomes optically thick and whose further evolution is adiabatic. This leads to a *mass - radius relation for the protostellar core* $r_* \propto m_*^{-1/3}$.

5. The protostellar core accretes matter from the clump. The accretion energy is deposited into the clump. When this energy equals the original binding energy of the clump, the mother clump is dispersed. The protostar has reached its limiting mass. $m_* \propto m_c^{3/7}$. This is observed.

6. This *star mass - clump mass* relation together with the clump mass spectrum naturally leads to the *Salpeter mass function* $x = (\alpha - 4/7)(7/3)$ for the population of stars so produced.

7. Further accretion of matter by the young stars from the cloud leads to the observed $L_c \propto M_c^{-1}$ relation.

References

Bachiller R., Gomez-Gonzalez J., 1992, A&A, Review, 3, 257.

Bhatt H. C., Rowse D. P., Williams I. P., 1984, MNRAS, 209, 69.

Bhatt H. C., Williams I. P., 1986, MNRAS, 219, 217.

Blitz L., 1993, in Protostars and Planets III, ed. by E. H. Levy and J. I. Lunine, University of Arizona Press, Tucson, 125.

Jeans J. H., 1928, Astron. Cosmogony, Cambridge University Press, London.

Larson R. B., 1982, MNRAS, 200, 159.

Miller G. E., Scalo J. M., 1979, ApJ Supp, 41, 513.

Myers P. C., 1991, in Fragmentation of Molecular Clouds and Star Formation, ed. by E. Falgarone, F. Boulanger and G. Duvert, Dordrecht, 221.

Oort J. H., 1954, BAIN, 12, 177.

Rengarajan T. N., 1984, ApJ, 287, 671.

Sagar R., Piskunov A. E., Myakutin V. I., Joshi U. C., 1986, MNRAS, 220, 383.

Salpeter E. E., 1955, ApJ, 121, 161.

Solomon P. M., Rivolo A. R., Barrett J., Yahil A., 1987, ApJ, 319, 370.