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Effect of solar radiation pressure on a planet in the photogravitational restricted three-body problem

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Abstract. Here, we have deduced the equations of motion of the photogravitational restricted three-body problem considering the effect of solar radiation pressure on a planet and found that it is stabilizing for the triangular libration points.

Key words : photogravitational field – Lagrangian points – stability

1. Introduction

The photogravitational restricted three-body problem has been extensively studied (Viz. Radzievskii (1950, 1953), Chernikov (1970), Choudhry (1974), Perezhogin (1976), Bhatnagar & Hallan (1978), Simmons et al. (1985), Kumar & Choudhry (1986, 1987), Sharma (1987), Ahmad & Adhikary (1991, 1993), Ahmad & Islam (1994), Haque & Ishwar (1995) etc.) considering the bigger primary (both primaries) to be the source (sources) of radiation pressure (pressures) but neglecting its (their) effect (effects) on the other primary. It has been well established that for the triangular libration points the effect of solar radiation pressure on the infinitesimal mass is destabilizing.

Here we have deduced the equations of motion of the photogravitational restricted three-body problem considering the effect of solar radiation pressure on the planet and have studied the locations of the five Lagrangian points and their linear stabilities. We have also established the existence of the elliptic periodic orbits around the triangular libration points at critical mass.

As the cross-sectional areas of the planet and the asteroid as well as the sun-planet and sun-asteroid distances are different, we have used different parameters for the mass reduction factors due to solar radiation pressures on the planet and the asteroid.

2. Equations of motion

We have introduced the effect of solar radiation pressure on the planet in the photogravitational restricted three-body problem through the mean motion of the primaries.

Let r be the distance and U be the potential between the sun and the planet. Then

$$U = G\, q \frac{m_1 m_2}{r} \tag{1}$$

where

m_1 = the mass of the sun,
m_2 = the mass of the planet,
G = the gravitational constant,
$q = 1 - F_p / F_g = 1 - p$, $p \ll 1$, (Choudhry 1974),
F_g = the gravitational force between the sun and the planet,
F_p = the solar radiation pressure force.

Now proceeding, as for a two - body problem, we get the value of the mean motion of the planet which is given by:

$$n^2 = G\, q\, (m_1 + m_2) / r^3 \tag{2}$$

As usual, taking $m_1 + m_2 = 1$, $r = 1$ and $G = 1$, we get

$$n^2 = q = 1 - p < 1 \tag{3}$$

Therefore, the mean motion of the planet is retarded due to the radiation pressure of the sun on the planet.

Now, the equations of motion of the photogravitational restricted three-body problem in dimensionless synodic co-ordinates (x,y) may be written as

$$\ddot{x} - 2 n \dot{y} = \frac{\partial V}{\partial x}$$

$$\ddot{y} + 2 n \dot{x} = \frac{\partial V}{\partial y} \tag{4}$$

where

$$V=\frac{n^2}{2}(x^2+y^2)+q'(1-\mu)/r_1+\mu/r_2, \quad (5)$$

$$r_1^2=(x-\mu)^2+y^2, \quad (6)$$

$$r_2^2=(x-\mu+1)^2+y^2$$

$$q'=1-p',\ p'<<1.$$

3. Locations and linerar stability of the five Lagrangian points

To find the locations of the five Lagrangian points and to study the linear stability of these points one can proceed in the usual way (Bhatnagar & Hallan, 1978).

Here we have left the detail calculations and have pointed out important findings of our problem.

The co-ordinates of the triangular libration points upto first order in small parameters, are

$$[\mu-1/2+p'/3 \pm \frac{\sqrt{3}}{2}\{1+2(2p-p')/9\}] \quad (7)$$

The critical value of the mass parmeter is given by

$$\mu_C=\mu_o+2(2p-p')/27(69)^{1/2} \quad (8)$$

where

$$\mu_o=\frac{1}{2}\{1-(69)^{1/2}/9\}$$

If the solar radiation pressure on the planet be neglected, then $p=0$ and we get

$$\mu_c=\mu_o-2p'/27(69)^{1/2}<\mu_o \quad (9)$$

This confirms the well established result of the photogravitational restricted three-body problem that the effect of solar radiation pressure on the infinitesimal mass is destabilizing.

If the solar radiation pressure on the infinitesimal mass be neglected, then $p'=0$ and we get

$$\mu_c = \mu_o + 4p / 27(69)^{1/2} > \mu_o \qquad (10)$$

This shows that the effect of solar radiation pressure on the planet is stabilizing (New finding).

Proceeding, as Sharma & Subbarao (1979) have done, we can establish the existence of retrograde elliptic periodic orbits around the triangular libration points whose eccentricity is given by

$$\text{eccentricity} \quad = \quad 2^{1/2}(2^{1/2} - 1)^{1/2}. \qquad (11)$$

This shows that the solar radiation pressures on the planet and the infinitesimal mass have no effect on the eccentricity of the retrograde elliptic periodic orbits around the triangular libration points at critical mass.

Regarding the collinear libration points we found that L_1, L_2, L_3 exist in the intervals $(-\infty, \mu - 1)$, $(\mu - 1, 0)$ and (μ, ∞) respectively; and as usual they are found to be unstable.

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