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Collapsing void in an expanding universe

D.R. Mandal* and S. Banerji+

* *Department of Physics, Basirhat College, Basirhat, North 24 Parganas, W. Bengal, India*

+ *Department of Physics, The University fo Burdwan, Burdwan 713 104, India*

Abstract. We consider here the model of a spherical void in an expanding Robertson - Walker (RW) universe with flat space sections. The void is taken as a sphere of low density conducting perfect fluid (Region I) surrounded by a spherical shell of pure radiation (Region II). The metric in Region I is assumed to be special form of the solution of Maiti (1982) and that in Region II is that of Vaidya. The RW universe (Region III) surrounding the above combination is assumed to be filled with a perfect fluid having a linear equation of state so that the scale factor is given by t^n. The matching conditions are written down and solved. The arrow of time shows that the void appears to contract when seen by a comoving observer in the RW universe. It, however, the RW universe is filled up with dust ($p = 0$), then the void remains static and the Vaidya metric reduces to that of Scwarzchild. The co-ordinates of Region II are extended to Regions I and III.

1. Introduction

Astronomical observations in the last decade have indicated the existence of regions of the universe which appear to be empty, called voids. Later evidence indicated that the voids are deficient in luminous matter but are not completely empty. Voids of sizes upto $60h^{-1}$ Mpc ($H_0 = 100$ h km s^{-1} Mpc^{-1}) have been observed.

COBE satellite has shown that the microwave background is highly isotropic (anisotropy 1 part in 10^5). This means that the universe was highly isotropic at the time when radiation decoupled from matter. So many cosmologists have the impression that any small anisotropy or inhomogeneity present at the time of decoupling may have increased subsequently i.e. a small void at that time may have expanded to their present size. In the model of the void, we are going to consider here the reverse, i.e. voids initially present tend to disappear with the expansion of the universe. When the universe becomes matter dominated the void becomes static. Unless the result is very much model dependent, it will pose a problem to theorists.

In section 2 we present the metrics in the three regions, in section 3 we give the matching conditions and their solutions. In section 4 we extend r_2, t_2 coordinates to the interiors of Regions I and III and in section 5 we present the conclusions.

2. A model of the void

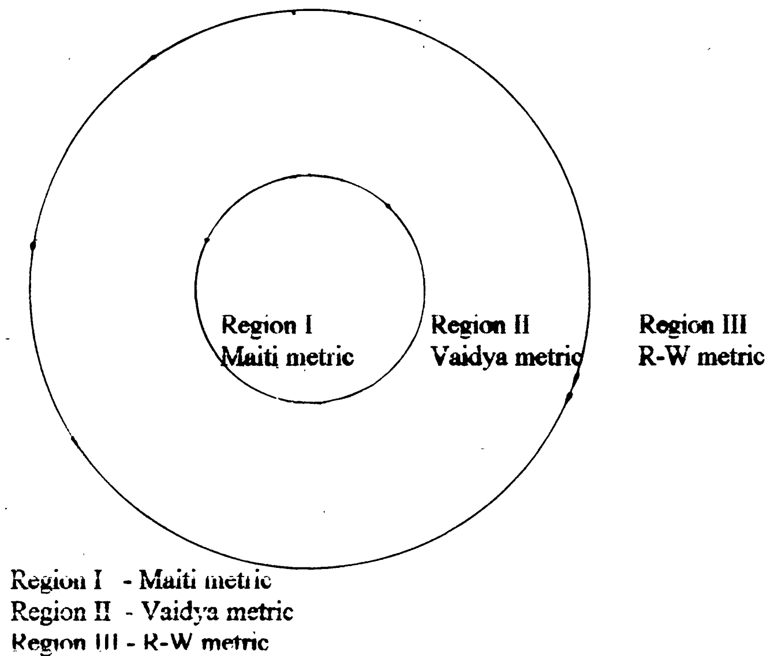

Figure 1.

The core of the void called Region I (fig. 1) has a metric of the form given by Maiti (1982) :

$$ds_I^2 = \left[1 + \frac{a}{1 + \xi r_1^2}\right]^2 dt_I^2 - \frac{R^2(t_1)}{(1 + \xi^2 r_1^2)^3} (dr_I^2 + r_I^2 d\theta^2 + \sin^2\theta \, d\phi^2) \tag{2.1}$$

where a and ξ are both constants. The energy momentum tensor is that of a fluid with heat flux expressed in the standard form as

$$T_\mu^\upsilon = (\rho + p) u_\mu u^\upsilon - p \delta_\mu^\upsilon - q_\mu u^\upsilon - u_\mu q^\upsilon \tag{2.2}$$

where q^μ represents the heat flux vector which is orthogonal to the velocity vector u^μ. In the present spherically symmetric case the radial comonent q' is nonvanishing. In regions II the metrices take Space for fig. 1 in the form given by Vaidya (1953) :

$$ds_{II}^2 = \left[1 - \frac{2m(v)}{r_2}\right] dv^2 + 2 \, dv \, dr_2 - r_2^2 (d\theta^2 + \sin^2\theta \, d\phi^2). \tag{2.3}$$

In Region III we have the Robertson – Walker metric with flat space sections filled with a perfect fluid with $p = \gamma\rho$ $0 \leq \gamma \leq 1/3$. The corresponding scale factor involves a power in t.

$$ds^2_{III} = dt_3^2 - t_3^{2n} (dr_3^2 + r_3 \, d\theta^2 + r_3^2 \sin^2\theta \, d\phi^2) \tag{2.4}$$

$\gamma = 0$ (dust), n = 2/3 and for $\gamma = 1/3$ (pure radiation), n = 1/2.

3. Boundary conditions

The matching conditions on the boundary between Regions I and II obtained by equating the first and the second fundamental form on the two sides are given below :

$$A^2 \dot{t}_1^2 - B^2 \dot{r}_1^2 = 1 \tag{3.1}$$

$$r_2 = B \, r_1 \tag{3.2}$$

$$\left\{1 - \frac{2m(v)}{r_2}\right\} \dot{v}^2 + 2\dot{r}_2 \, \dot{v} = 1 \tag{3.3}$$

$$r_1^2 A \, \dot{t}_1 \frac{\partial B}{\partial r_1} + r_1 A \, B\dot{t}_1 + \frac{B^2 \dot{r}_1^2 \, r_1}{A} \frac{\partial B}{\delta t} = \left\{1 - \frac{2m \cdot \dot{v}}{r_2}\right\} \dot{v} \, r_2 + r_2 \, \dot{r}_2 \tag{3.4}$$

$$\dot{t}_1 A B \ddot{r}_1 - \ddot{r}_1 A B \dot{t}_1 - A \, \dot{t}_1 \, \dot{r}_1^2 \frac{\partial B}{\partial r_1} - \frac{A^2}{B} \dot{t}_1^3 \frac{\partial A}{\partial r_1} + 2 B \, \dot{r}_1^2 \, \dot{t}_1 \frac{\partial A}{\partial r_1}$$
$$- 2 A \, \dot{r}_1 \, \dot{t}_1^2 \frac{\partial B}{\partial t_1} + \frac{B^2 \dot{r}_1^3}{A} \frac{\partial B}{\partial t_1} = \frac{\ddot{v}}{\dot{v}} - \frac{m \dot{v}}{r_2^2} \tag{3.5}$$

where $A = 1 + \frac{a}{1 + \xi r_1^2}$, $B = \frac{R(t_1)}{1 + \xi r_1^2}$

A dot denotes derivative with respect to τ, the time corresponding to the comoving coordinates in the metric intrinsic to the bounding 3 – space (Santos 1985). Banerjee *et al.* (1989) showed that the boundary is static ($r_1 = r_0$) when

$$R = b \, t_1 \tag{3.6}$$

$$m = c \, R \tag{3.7}$$

where b and c both are constants.

The matching of the Vaidya metric of Region II with that of R.W in Region III has been done by a number of workers (Aguirregabiria *et al.* 1991; Fayos *et al.* 1992). Equating the first and second fundamental forms on the two sides we obtain at $r_3 = u(t_3)$:

$$r_2 = u\, t_3^n \tag{3.8}$$

$$\dot{t}_3^2 - t_3^{2n}\, \dot{u}^2 = 1 \tag{3.9}$$

$$\left(1 - \frac{2\,m}{r_2}\right) \dot{v}^2 + 2\, \dot{r}_2\, \dot{v} = 1 \tag{3.10}$$

$$\left(1 - \frac{2\,m}{r_2}\right) \dot{v}\, r_2 + \dot{r}_2\, r_2 - u\, t_3^n\, \dot{t}_3 - n\, u^2\, \dot{u}\, t_3^{3n-1} = 0 \tag{3.11}$$

$$\frac{m\ \dot{v}}{r^2} - \frac{\ddot{v}}{\dot{v}}\, \ddot{u}\, \dot{t}_3\, t_3^n - 2n\, \dot{u}\, \dot{t}_3^2\, t_3^{n-1} + n\, \dot{u}^3\, t_3^{3n-1} + \dot{u}\, t_3^n\, \ddot{t}_3 = 0. \tag{3.12}$$

We obtained the following solutions of the above

$$u = \alpha_0 - \frac{2 - 3n}{3n\,(1 - n)}\, t_3^{1-n} \qquad (\alpha_0 > 0) \tag{3.13}$$

$$r_2 = u\, t_3^n = \alpha\, t_3^n - \frac{2 - 3n}{3n\,(1 - n)}\, t_3 \tag{3.14}$$

$$t_3 = \frac{3n}{2\,(3n - 1)^{1/2}} \tag{3.15}$$

$$m = \frac{n^2\, u^2}{2}\, t_3^{3n-2} \tag{3.16}$$

$$v = \frac{2\,(1 - n)}{n} \int \frac{dt_3}{3n\,(1 - n)\, \alpha_0\, t_3^{n-1} + 1}\,. \tag{3.17}$$

It is evident from (3.13) and (3.16) that

$$\frac{dm}{dt_3} = -\frac{n\,(2 - 3n)u^2}{2}\, t_3^{2(n-1)}\, (1 + n\, u\, t_3^{n-1}). \tag{3.18}$$

This is negative for $\frac{1}{2} \leq n < \frac{2}{3}$ and zero for $n = \frac{2}{3}$. The void, therefore, contracts till $u \to 0$. If, however, n changes with time and the void survives till the present epoch when $n = 2/3$ and the pressure vanishes, the void ceases to contract. In this case Vaidya metric reduces to that of Schwarzschild with constant m and the time coordinate t_2 is given by

$$v = t_2 - \int \frac{dr_2}{1 - \frac{2\,m}{r_2}} \tag{3.19}$$

using (3.14) and (3.16) we obtain,

$$t_2 = t_3 + 4/3\ \alpha_0^2\ t_3^{1/3} + 4/9\ \alpha_0^3\ \text{in} \left| \frac{3t_3^{1/3} - 2\alpha_0}{3t_3^{1/3} + 2\alpha_0} \right| . \tag{3.20}$$

The relation was found earlier by Dey and Banergi (1991). Here t_2 and t_3 are both future directed. Since both Newton's law of gravitation and Einstein's field equations are symmetric with respect to time reversal, we always obtain a solution with past directed times corresponding to each of the above solutions. Details will be published elsewhere.

In the general case we have

$$t_2 = \{3\alpha_0\,(1-n)\}^{1/1-n}\,(n)^{n/1-n} \left[(3n-1) \int \frac{x^{(2-n)/(n-1)}\,dx}{x-(5-6n)} - \int \frac{x^{(2-n)/(n-1)}\,dx}{1+x} \right] \tag{3.21}$$

where $x = (3n\,(1-n)\,\alpha_0\,t_3^{n-1}$

4. Extension of r_2, t_2 coordinates to the interior of Regions I and III

Within Region I

$$r_2 = \frac{b\,r_1\,t_1}{1 + \xi\,r_1^2} \tag{4.1a}$$

$$t_2 = e^{b^2 z_1}\ x\ \frac{(1 + a + \xi\,r_0^2)\,(1 - \xi\,r_0^2)}{(1 + \xi\,r_0^2)^2 \left\{1 - \frac{2c\,(1 + \xi\,r_0^2)}{r_0}\right\}} \tag{4.1b}$$

where

$$z_1 = b^{-2}\ \ln t_1 + [(a+2)^{-2}\,\xi\,]^{-1}\ \ln \frac{(1 + a + \xi\,r_1^2)\,(1 - \xi\,r_0^2)}{(1 - \xi\,r_1^2)\,(1 + a + \xi\,r_0^2)} + \frac{a\,(r_0^2 - r_1^2)}{2\,(a+2)\,(1 + a + \xi\,r_1^2)\,(1 + a + \xi\,r_0^2)} \tag{4.1c}$$

Within Region III

$$r_2 = r_3 t_3^n \tag{4.2a}$$

We may obtain t_2 within Region III replacing x in (3.20) by

$$x = 3n(1-n)\alpha_0 \left[z_2 + \frac{3n(1-n)(2-3n)}{4-3n}\alpha_0 \right]^{-1} \tag{4.2b}$$

where

$$z_2 = \left[\left\{ t_3^{1-n} - \frac{3n(1-n)(2-3n)}{4-3n}\alpha_0 \right\}^2 + \frac{9n^2(1-n)^2}{4-3n}(r_3^2 - u^2) \right]^{1/2} \tag{4.2c}$$

5. Conclusion

In this model of the spherical void, the radius of the void formed in the early universe goes on contracting as the universe expands. If, however, the void persists till the present epoch of zero pressure, then it becomes static with a constant radius. The coordinates r_2 and t_2 have been extended to the interiors of Regions I and III. It will be interesting to examine if Region II vanishes before Region I or vice versa. Work is proceeding along these lines.

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