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Modelling sunspot equilibrium through a solution of the time dependent MHD equations

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Abstract. We numerically simulate the dynamical relaxation to equilibrium of a sunspot. Treating the sunspot as a thick axisymmetric flux tube in cylindrical geometry, we solve the time dependent MHD equations to examine the evolution of a sunspot towards equilibrium, starting from an arbitrary initial state.

Key words: Magnetohydrodynamics–Sun:magnetic fields–Sun:sunspots

1. Introduction

Solutions for thick flux have been obtained under various approximations, and can be classified into two types: current sheet models (e.g., Simon and Weiss 1970; Meyer, Schmidt and Weiss 1977; Simon, Weiss and Nye 1983; Jahn 1989; Pizzo 1990) and distributed current sheet models (e.g., Deinzer 1965; Yun 1970, 1971; Landman and Finn 1979; Low 1980; Pizzo 1986). In this paper, we consider a numerical technique to achieve equilibrium of a sunspot using dynamical relaxation.

2. MHD equations and the strong form of conservation laws

The non-relativistic MHD equations for a perfectly conducting fluid are

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{v} = 0\,, \qquad \rho\frac{D\vec{v}}{Dt} = -\nabla p - \rho\vec{g} + \frac{1}{c}\vec{j}\times\vec{B}\,, \tag{1}$$

$$\rho\frac{D}{Dt}\left(\frac{e}{\rho}\right) = -p\nabla\cdot\vec{v}\,, \qquad \frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B})\,, \tag{2}$$

where ρ, e and $\vec{v}$ are the fluid density, internal energy density and velocity, respectively; $\vec{B}$ is the magnetic flux density, and $\vec{g}$ is the acceleration due to gravity of the Sun. The current density $\vec{j}$ is related to $\vec{B}$ by $\vec{j} = (c/4\pi)\nabla \times \vec{B}$. The fluid equations are closed by an equation of state $p = p(\rho, e)$.

In order to minimize truncation errors associated with finite differencing, it is convenient to recast the above equations in strong conservation form, which may be derived by integrating over an arbitrary control volume and its surface.

We solve the finite-difference equations by expressing them in covariant form in a cylindrical coordinate system using ZEUS code. We ignore the azimuthal coordinate since we assume axial symmetry.

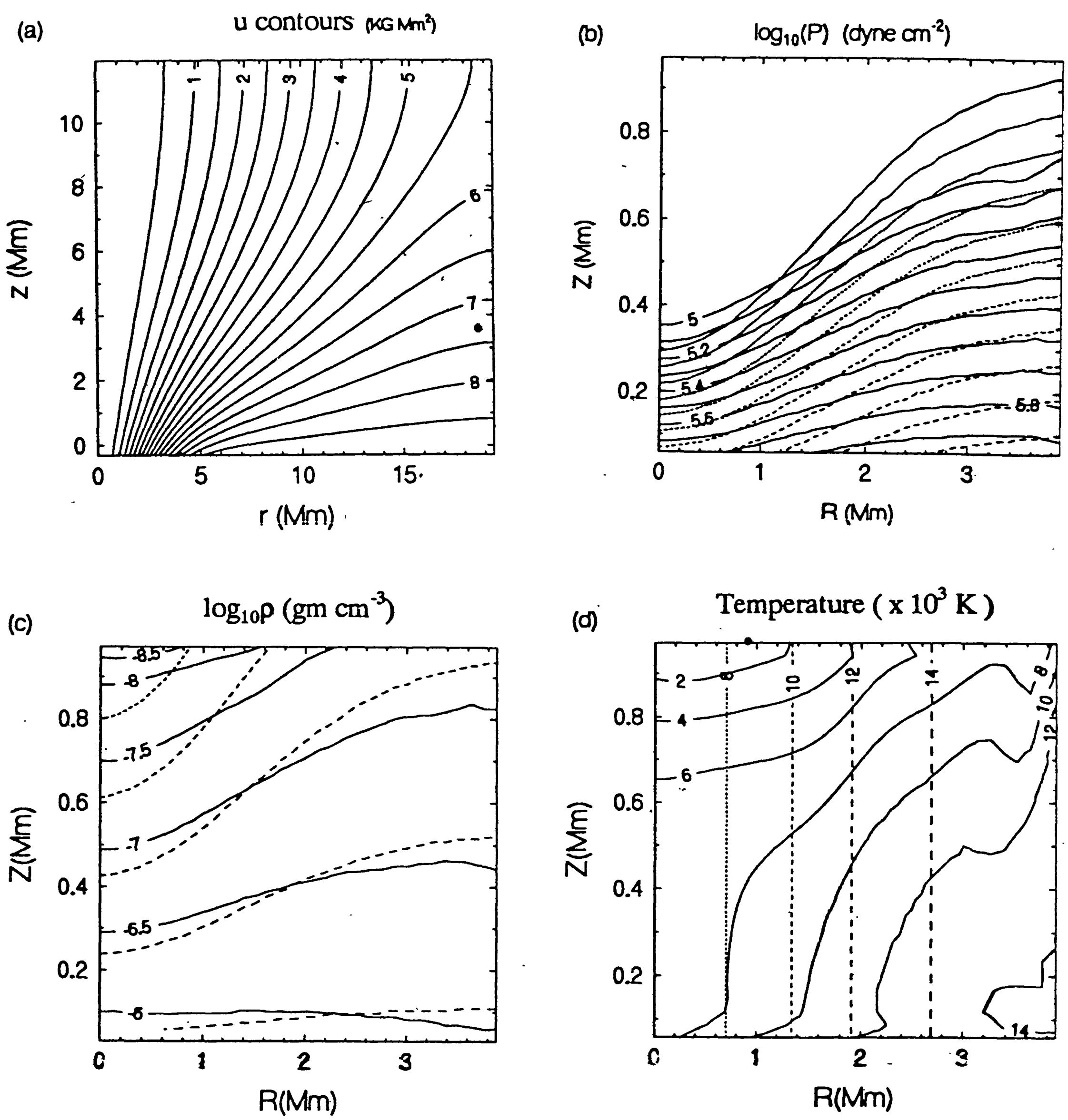

Figure 1. (a) Field line constant u, (b) pressure p, (c) density ρ and (d) temperature T contours at the initial (broken) and equilibrium (continuous) stages.

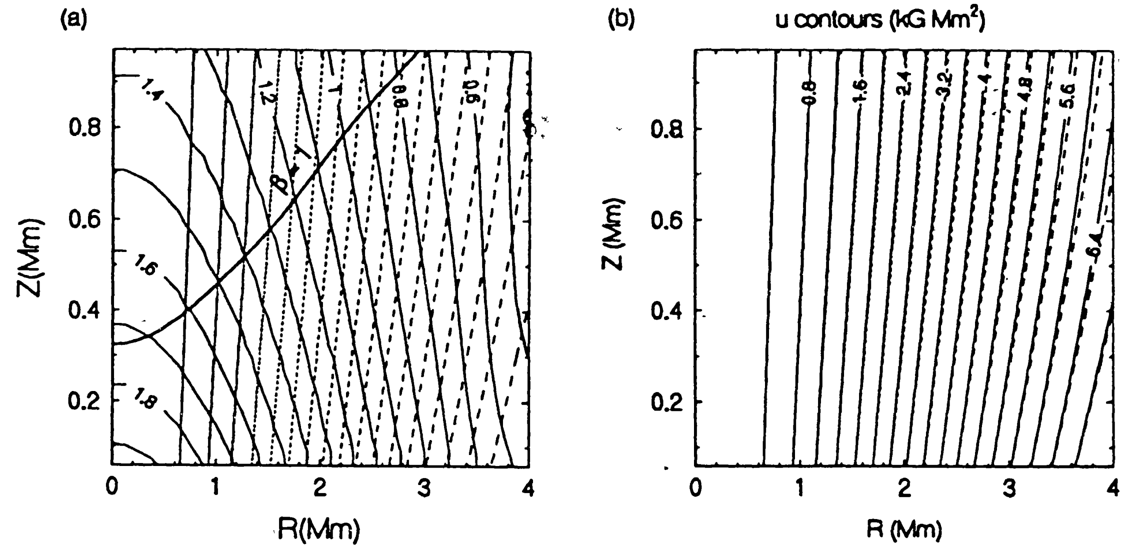

Figure 2. (a) Contours of $\mid B \mid$ for the prototype; labels indicate field strength in kG. The dashed lines are the contours of u and heavy solid line denotes the surface where the plasma beta, $p/(B^2/8\pi)$, equals unity. (b) The contours of u in the equilibrium (continuous) and initial (broken) stages.

conditions. The assumption of axial symmetry precludes gas flow across the axis of the spot. On the right boundary, we allow for an inflow boundary condition. We allow for the escape of matter along field lines from the upper boundary.

Having initialized all the magnetohydrodynamic variables, we numerically solve the MHD equations as a time dependent problem with the goal of achieving an equilibrium configuration for a sunspot.

At $t = 0$ the fluid velocity over the entire mesh is set to zero. However, the initial state is clearly not one which is in equilibrium. Thus, as soon as we begin the simulation, we expect that the negative horizontal pressure gradient will lead to a radial inflow of matter from the right boundary. The vector field of fluid velocity oscillates and dampens.

In Fig. 2a, the continuous lines indicate contours (continuous) of magnetic field at the equilibrium. The labels on contours indicate the magnitude of magnetic field in units of kG. The broken line contours indicate the field line constant. The heavy line indicate a contour of $\beta = P/(B^2/8\pi) = 1$ with $\beta > 1$ below the line and < 1 above it.

Fig. 2b shows contours of field line constant u at the equilibrium (continuous) and initial (broken) stages. The labels on the contours are in units of kG Mm^2. We find that the field away from the axis is compressed relative to the potential field and tends to become more vertical owing to them being squeezed inwards towards the axis by the radial inflow. This flow is due to the uncompensated radial pressure gradient at the initial instant. Momentum balance in the horizontal direction is eventually achieved primarily through an buildup of the Lorentz forces, which counteract the tendency of the flow to sweep in field lines.

3. Discussion

The main focus of the present study has been been to see whether dynamic relaxation of the time dependent equations results in a equilibrium state which is magnetostatic. Our simulation indeed shows, that a quasi-equilibrium state can be achieved by this method, which is roughly similar to the configuration computed by solving the magnetostatic equations. An interesting feature of our simulation is the development of transient flows, which have a peak value of about 1.5 km s^{-1}. It takes about 5000 s for the flow to become very small.

This study presents the beginning of a comprehensive investigation of time dependent processes in sunspots. In forthcoming papers, we hope to enlarge the scope of this work by examining various dynamical phenomena associated wth the excitation of oscillations, nonlinear wave propagation, heating and transport of energy in sunspots.

6. Acknowldegments

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