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Non-conservative Rayleigh scattering in a finite atmosphere-II Polarization in telluric lines

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Abstract. We consider non-conservative Rayleigh scattering in finite atmospheres and study the polarization in the molecular lines both in the transmitted and reflected sunlight. We first investigate the parameterized models to estimate the effects of optical thickness, line strength, continuous absorption etc. on line polarization. This study helped us to analyse some typical O_2 and H_2O band lines in the earth's atmosphere. For the chosen parameters, we find that the polarization is more sensitive to optical thickness than the line strength. The continuous absorption affects the wing polarization selectively.

Key words: polarization—earth's atmosphere—telluric lines

1. Introduction

1.1 *Polarization by non-conservative Rayleigh scattering in continuum radiation*

Molecules and small particles can have various modes of absorption represented by a single scattering albedo less than unity at a particular wavelength. Techniques and results for polarization by non-conservative Rayleigh scattering are given by Sakera (1966), Dave (1965), Abhyankar & Fymat (1970) and Bond & Siewart (1971). All these studies pertain to semi-infinite atmospheres. Coulson *et al.* (1960) have brought out extensive tables of I, Q an U parameters for various ground reflection albedos. Kattawar *et al.* (1976) studied the effects of haze layers on polarization in finite media. Van de Hulst (1980) has shown how to construct solutions in semi-infinite atmospheres for non-conservative Rayleigh scattering by the use of simple formulas and tables. Rangarajan *et al.* (1994, paper I) studied the effect of non-conservative Rayleigh scattering in a finite atmosphere on polarization of sun-lit sky and showed that the shift of Babinet and Brewster points toward the sun can be explained by this process. Hansen & Hovenir (1974) were able to demonstrate the use of polarization to determine the effective radius, and index of reflection of the Venus cloud droplets.

The passive remote sensing technique leads to the global inference of atmospheric temperature, composition profiles and the radiative budget both from earth based measurements (Wang & Lenoble 1994) and from orbiting satellites and space shuttle polarimetric data (Egan *et al.* 1993). Santer *et al.* (1988) modelled the stratospheric aerosol observations from a balloon - borne polarimetric device and derived the particle mean dimension, aerosol size distribution and the particle refractive index. The analysis of the observational data to derive physical parameters requires a theoretical modeling based on radiative transfer calculations.

1.2 *Polarization across absorption lines*

The study of absorption lines or bands across which the absorption coefficient of the atmosphere will be rapidly changing function of frequency is of great importance to atmospheric and planetary physics. If the rapid variation of absorption is due primarily to atmospheric gases, we must consider the problem of solving the equation of transfer for a given phase function and for a sufficiently large number of values of ω_0 (single scattering albedos), and T (optional thickness) to define the spectral features of interest.

Absorption band studies show the possibility of vertical probing of a planetary atmosphere with spectropolarimetry (Coffen & Hansen 1974). A striking case is the 0.89 μ band of CH_4 in Jupiter and Saturn. Approximate calculations tell us that for a hypothetical two cloud layer model, measurments in the band could give this polarization phase curve for particles in a upper thin haze layer, while out of band measurments of 0.93 μ give a curve dominated by the lower particles.

These results show the importance of the theoretical study of polarization in absorption lines caused by the scattering particles.

1.3 *Earlier results on polarization in molecular lines*

Fymat (1974) made a systematic study of polarization in the reflected spectra of planets using a semi-infinite atmospheric model. His conclusions are as follows: 1) Polarization increases with the line strength but decreases afterwards as the line becomes strong and tends to saturate. Line strength is characterized by ω_{v_0}. A polarization reversal is predicted for latitudinal and longitudinal scans. The reversal happened at all phase angles considered. Forbes & Fymat (1974) observed Venus using a Fourier spectropolarimeter and confirmed some of their theoretical predictions. Bhatia & Abhyankar (1983 a,b,c) made a detailed study of intensity and polarization line profiles in a semi-infinite Rayleigh scattering planetary atmosphere. Buriez *et al.* (1979) showed that the equivalent widths of Q Stokes (parameter) profiles contain information regarding the gas to cloud particles ratio which is in agreement with the observation of 6900Å CH_4 band from Saturn.

1.4 *Aim of our study*

All the above mentioned calculations of line profiles are performed for the reflected sunlight and are useful for the study of semi-infinite planetary atmospheres. But for the earth's atmosphere, the transmitted light is of equal importance for *in-situ* measurements and also one has to consider finite atmospheres. Therefore we consider non-conservative Rayleigh scattering in finite atmospheres and study the polarization in molecular line both in the transmitted and the reflected sunlight. We describe the equation of transfer in the next section and the results in later sections of this paper. First we have studied parameterized models to get some idea on the effects of ω_0, T, μ (cosine of angle of observations), and ω_c continuous absorption on polarization. It seems reasonable to assume a small continuous absorption in the atmosphere. This may be due to the scattering particles or to any other absorbing matter. Depolarizing mechanisms like scattering due to anisotropic molecules and ground reflection are neglected in the present study. Later we use our code to study the polarization at some specific wavelengths in the telluric bands of O_2 and H_2O molecules.

Though small aerosols scatter the radiation according to Rayleigh's law (Van de Hulst 1980), larger particle scatter according to Mie theory. Therefore Mie scattering is an important mechanism and so we are planning to include it in our future work.

2. Equation of transfer

Including absorbing particles in a Rayleigh scattering atmosphere, the equation of transfer for a polarized beam in a molecular band is,

$$\mu \frac{dI_\nu(\tau,\mu,\phi)}{d\tau} = I_\nu(\tau,\mu,\phi) - \frac{\omega_\nu}{4\pi}\int_0^{2\phi}\int_{-1}^{+1} P(\mu,\phi;\mu',\phi')\, I_\nu(\tau,\mu',\phi)\, d\mu'\, d\phi' - \frac{\omega_\nu}{4} P(\mu,\phi;-\mu_0,\phi_0)\, F e^{-\tau/\mu_0}. \quad (2)$$

If a_ν is the molecular absorption coefficient and σ and κ are the mass scattering and absorption coefficients of aerosols or other scattering particles in the atmosphere, then the albedo within the absorption line is

$$\omega_\nu = \frac{\sigma}{a_\nu + \sigma + k} = \left[\frac{1}{\omega'_\nu} + \frac{1}{\omega_c} - 1\right]^{-1}, \quad (3)$$

where

$$\omega_\nu' = \frac{\sigma}{a_\nu + \sigma} \text{ and } \omega_n = \frac{\sigma}{\sigma + \kappa'} \tag{4}$$

If we assume that the absorption profile is given by Lorentz profile, we have $a_\nu = a_0 / (1 + x^2)$, where $x = (\nu - \nu_0) / \alpha_L$, α_L being the Lorentz half-width of the line and ν_0 the frequency at the line centre. Now we can also express ω_ν as

$$\omega_\nu = \left[\frac{1 - \omega_0'}{\omega_0(1 + x^2)} + \frac{1}{\omega_c}\right]^{-1}, \tag{5}$$

where ω_0 is the albedo at the line centre.

We solve the system of equation (2) by using the discrete space theory technique (Grant & Hunt 1969; Peraiah 1978) as discussed in Paper I. Solutions are obtained for various combinations of μ, μ_0, $\phi - \phi_0$ in the parametric models with various values of T, ω_0 and ω_c.

3. Results of the parametric study

3.1 *General features*

We have plotted $I_{l,\nu} / I_{l,c}$, $I_{r,\nu} / I_{r,c}$ versus the frequency x in the top panel of all the figures. I_l and I_r are separated out to give the clarity and only one half of the frequency space has been shown because all these quantities are symmetric with respect to $x = 0$. In the middle panel, the equivalent width of Q against μ has been plotted. It is defined as (Buriez *et al.* 1979)

$$W(Q) = \int_{\infty}^{-\infty} \left(1 - \frac{Q_\nu}{Q_c}\right) d\nu.$$

Even though this quantity does not have a clear cut physical meaning and it can become negative, it is useful for deriving certain physical parameters like gas to cloud particles ratio. We define σ, the measure of polarization as $\sigma = \frac{\sqrt{Q^2 + U^2}}{I}$ where $Q = I_l - I_r$. σ is plotted in the lower panel of the figures.

We see that the polarization varies across the line profile and the line centre always shows more polarization than the wings because as the absorption increases, scattering decreases. Since the multiple scattering reduces the polarization, we obtain lesser polarization in the wings. The reflected light shows more polarization compared to the transmitted light. In the

Sun's direction the reflected light comes from the shallow layers of the atmosphere and hence undergoes less number of scatterings compared to the transmitted light. The Stokes parameter Q varies with respect to μ. This variation is not monotonic. Therefore we find $W(Q)$ to vary with μ. We can see from $W(Q)$ that the transmitted light at certain angles shows polarization reversal, whereas for the reflected light, there is no reversal for $\mu_0 = 0.5$.

3.2 *Optical thickness effects*

In Fig. 1, we have shown the effects of optical depth on polarization. We have considered the Sun's radiation to impinge in the direction $\mu_0 = 0.5$ and the observation is nearly at the zenith ($\mu = 0.98$ and $\phi - \phi_0 = 0$). We find that the higher optical thickness reduces the polarization throughout the line profile and changes it drastically from the line centre to the wings. The lower optical thickness gives a constant but high polarization throughout the profile. This can be explained from the fact that the number of scatterings is proportional to the optical depth. However $W(Q)$ increases with μ in the reflected light equally for all optical depths.

3.3 *Effects of line strengths on polarization*

In Fig. 2, we have plotted polarization related quantities for various albedos ($\omega_{v_o} = \omega_0$) at the line centre. The line strength is measured by this quantity. As ω_0 decreases, the line strength increases and the line becomes saturated for small $\omega_0's$. When the line gets saturated, it develops broad wings. We find that as ω_0 is reduced, the polarization at the line centre is increased for the particular chosen combination of sun's and the observational directions. It is explained as follows: As absorption in a line increases, the scattering decreases, that is, we are concerned with the lower scattering orders which induce relatively higher polarization. This behaviour is the same for both the transmitted and the reflected light even though there is a polarization reversal between them, i.e., $I_l > I_r$ for transmitted light and $I_l < I_r$ for relected light. But $W(Q)$ increases with μ in the reflected light in the case of strong lines, the increase being larger for the stronger lines.

3.4 *Effect of continuous absorption* ω_c

Since the continuous absorption ω_c is supposed to be very small in the atmosphere, we have selected only a narrow range of ω_c for our study (i.e. $\omega_c = 0.9$, 0.95 and 1.0). When there is no continuous absorption, $\omega_c = 1.0$. As the continuous absorption is increased, polarization increases. This may be the only mechanism which has a considerable influence over the wing polarization. Even though at the line centre, all the above values for ω_c give the same polarization, we find a marked difference in the wings as shown in Fig. 3. But ω_c does not affect the behaviour of $W(Q)$.

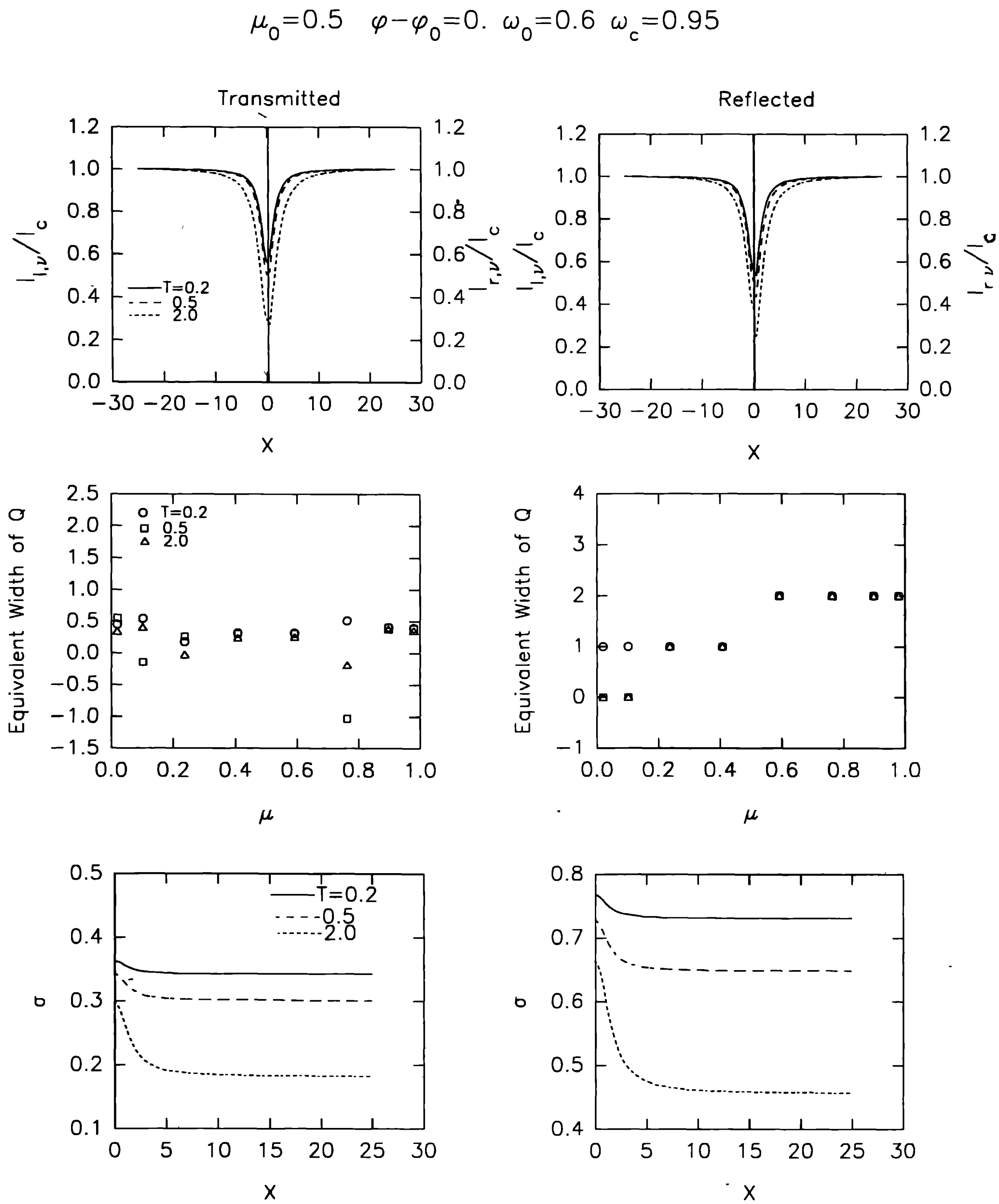

Figure 1. Top panel : The abscissa represents the frequency $x = (\nu - \nu_0)\, \alpha_L$ where α_L is the Lorentz width. In the left side of the graph, $I_{l,\nu}/I_c$ and in the right side $I_{r,\nu}/I_c$ plotted. In the left side of the page, the transmitted light and in the right side the reflected light are shown. In the middle panel the equivalent width of the Stokes parameter Q is plotted against the cosine of the viewing zenith angle μ. In the bottom panel, the amount of polarization σ is plotted against frequency x. In this figure various curves represent the results for different optical thickness. The common parameters for all the curves are indicated at the top of the page.

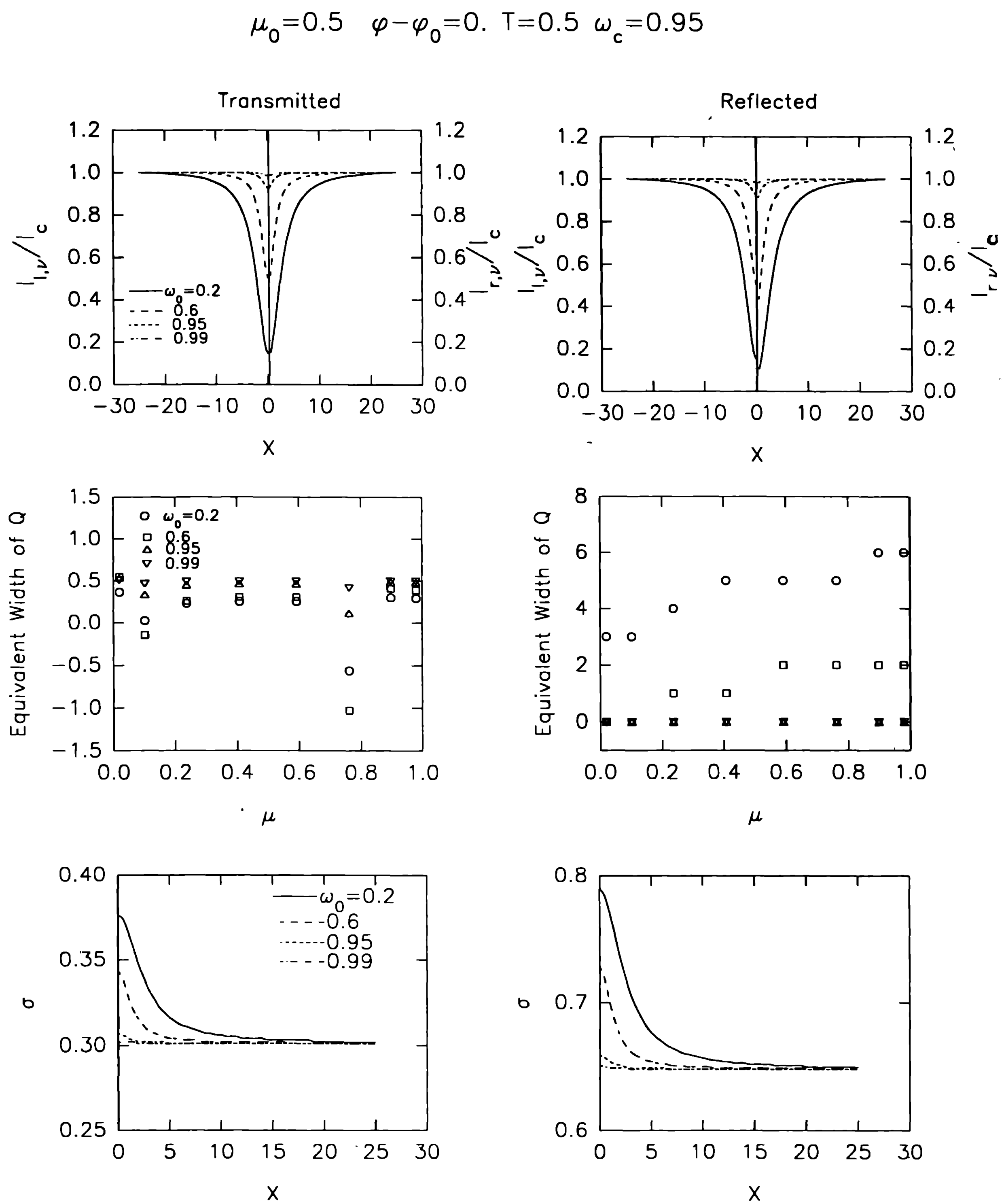

Figure 2. Same as Fig. 1 but for different line strengths (ω_c's)

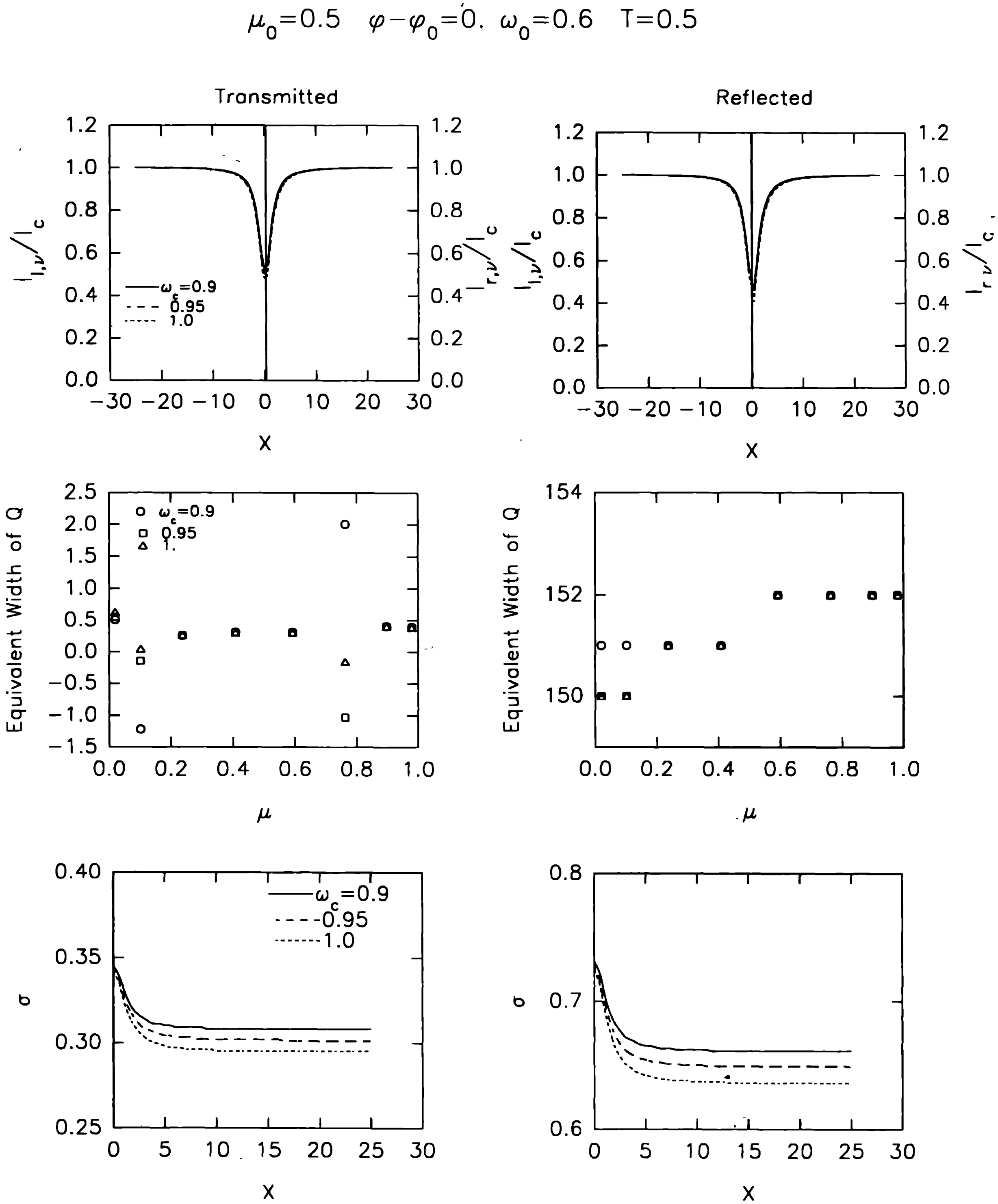

Figure 3. Same as Fig. 1 but for different continuous absorption coefficient ω_c's

3.5 *Effect of the Sun's direction on polarization*

It is well known that the polarization is very sensitive to Sun's position in the sky. We find from our calculations that the polarization across the profile is affected in a uniform way. These facts are borne by Figs 4 and 5.

4. Study of telluric line polarization

We applied the method developed so far to study the polarization in the strongest H_2O and O_2 band lines. We have calculated I_l, I_r, and σ for a particular combination of $\mu_0 = 0.5$, $\phi - \phi_0 = 0$ and at various zenith distances in the O_2 and H_2O lines. The wavelengths of the bands, the scattering coefficeints ω_0, ω_c and the optical depths are shown in tables 1 and 2. These values are obtained from Allen (1973). All the quantities are plotted against the frequency x in the line. Each figure contains a title which gives the molecular band, μ_0, and $\phi - \phi_0$ values. For $\phi - \phi_0 = 0$, π or $\mu_0 = 1.0$, Rayleigh phase matrix gives $U = 0$ for an unpolarized incident light. Therefore we have plotted only I_l, I_r and σ for these cases. Lorentz half width is chosen as 1.

The H_2O line at 13500Å is a very strong line ($\omega_0 = 0.009$) and therefore shows saturation and broad wings. From our parametric study (which we have described so far), we can expect it to have higher polarization compared to the lines at 9000Å and 11000Å wavelengths. But the optical thickness of this line is higher than the others which reduces the polarization (see Fig.6). The effect of an order of magnitude variation in line strength parameter on polarization is compensated by a factor of ~ 3 in optical depth. The diagnostic can be as follows. The broad line indicates higher optical thickness and hence there should be less polarization.

Table 1. Bands of O_2.

	B band	A band	IR band
λ	6900Å	7600Å	12000Å
τ_c	0.012	0.006	0.000
τ_σ	0.088	0.072	0.029
τ_0 (line)	0.430	1.670	0.430
$\tau = \tau_c + \tau_\sigma + \tau_0$	0.530	1.746	0.459
ω_c	0.880	0.950	1.000
ω_0	0.200	0.040	0.060

Table 2. Bands of H_2O.

λ	9000Å	1100Å	13500Å
τ_c	0.001	0.000	0.000
τ_σ	0.048	0.034	0.022
τ_0 (line)	0.690	0.690	2.460
$\tau = \tau_c + \tau_\sigma + \tau_0$	0.739	0.724	2.480
ω_c	1.000	1.000	1.000
ω_0	0.065	0.046	0.009

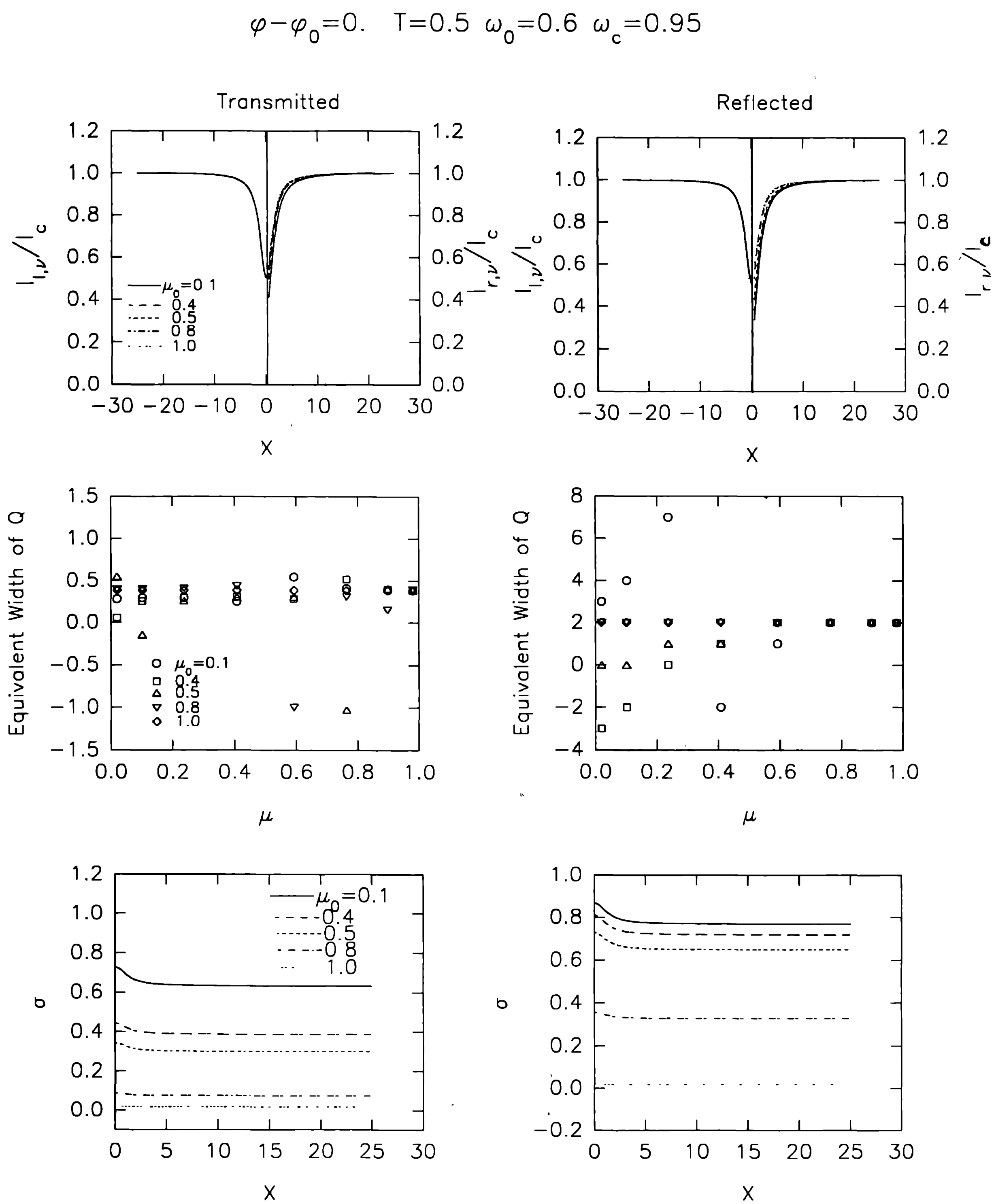

Figure 4. Same as Fig. 1 but for different viewing zenith directions μ.

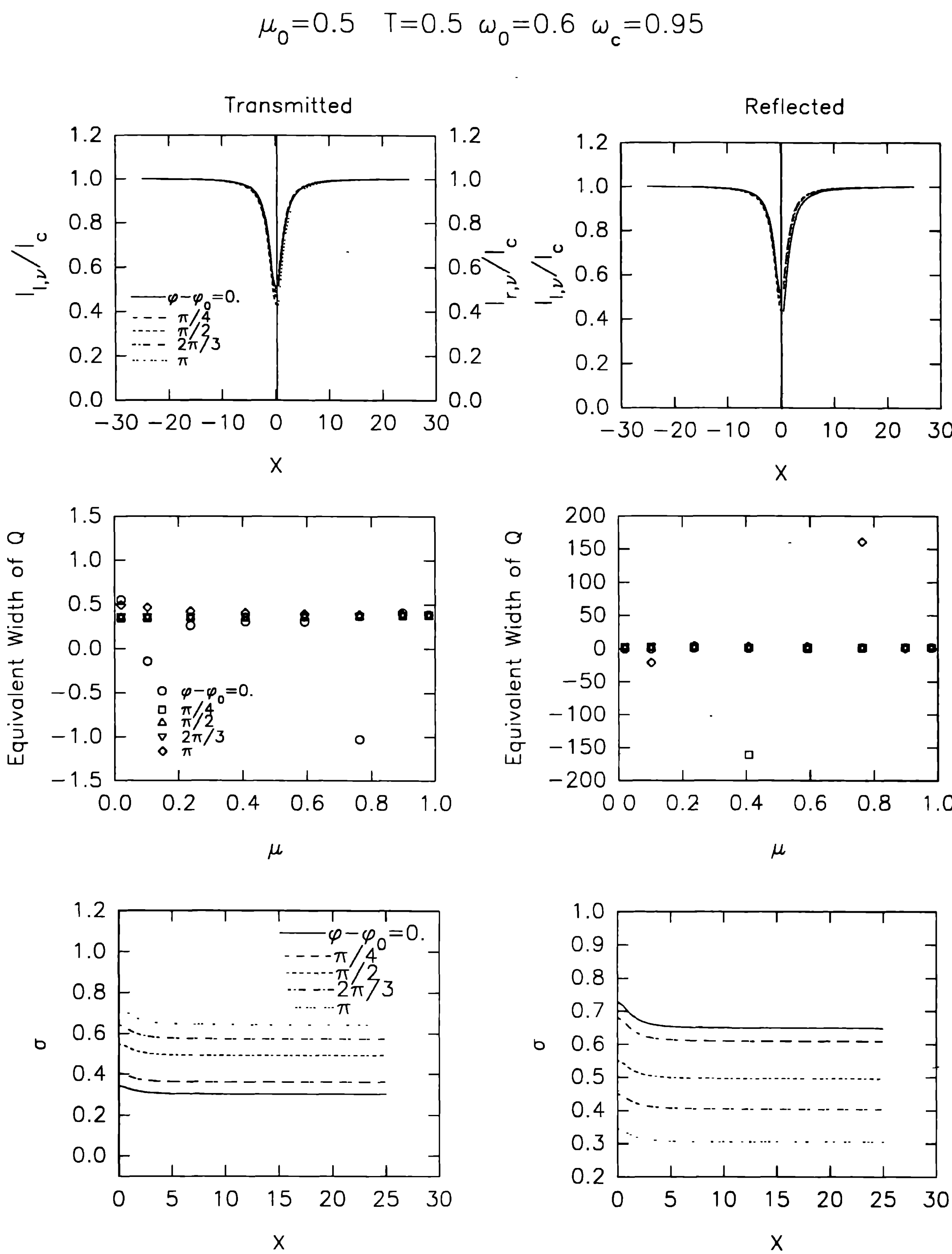

Figure 5. Same as Fig. 1 but for different viewing azimuth directions $\phi - \phi_0$.

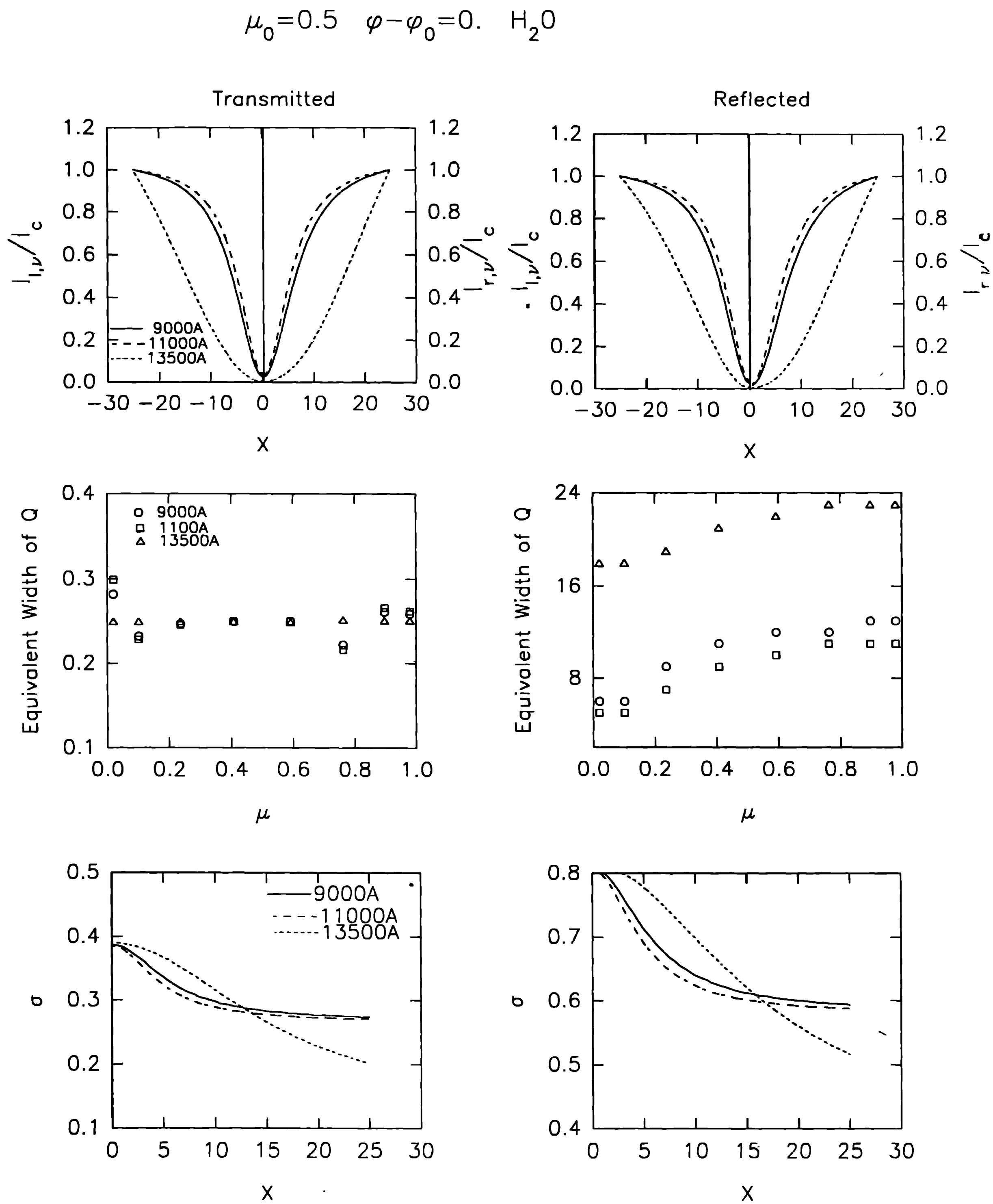

Figure 6. Same as Fig. 1 but for different lines of H_2O.

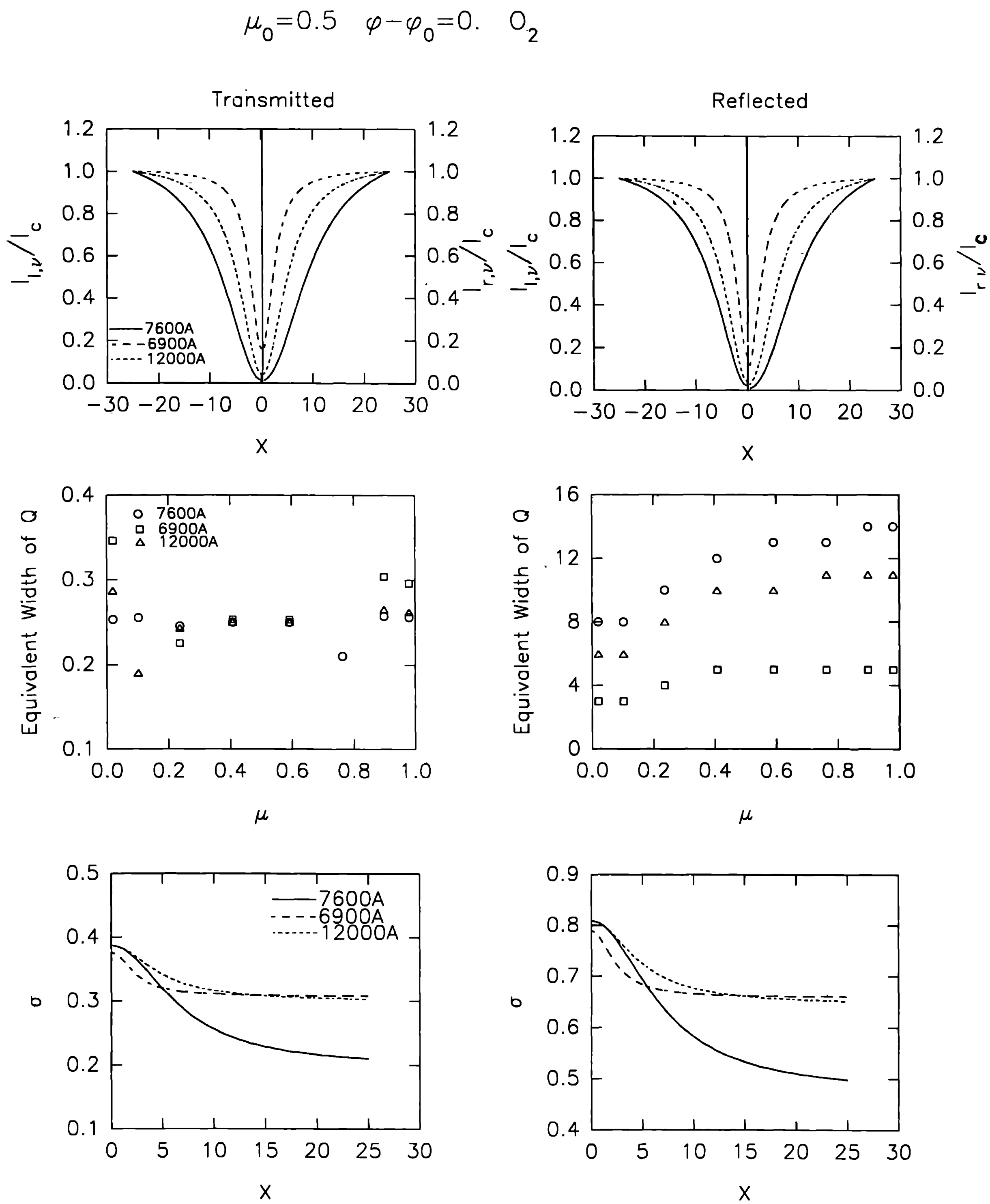

Figure 7. Same as Fig. 1 but for different lines of O_2.

In Fig. 7, we have plotted our calculated results for the O_2 lines at 7600Å, 6900Å and 12000Å. From the figures one can clearly see that 7600Å line is quite broad due to a large optical thickness and this gives the variation of polarization across the line profile. The lines at 6900Å and 12000Å have nearly the same optical thickness, even though they have different ω_0's and so we obtain nearly the same polarization. The effect of line strength on the behaviour of $W(Q)$ in the reflected light can be seen in both H_2O and O_2 lines.

5. Conclusions

From our parametric models we find that 1) higher optical thickness reduces the polarization throughout the line profile and changes it from the line core to the wings in a drastic way. 2) The line strength ω_0 affects the polarization at the line centre and its effect on the line wing polarization is negligible. 3) The continuous absorption ω_c affects the wing polarization only. 4) The position of the sun affects the polarization uniformly throughout the profile. With this parametric study, we could analyze the profiles of O_2 and H_2O lines at certain specific wavelengths. From this analysis, we find that the optical thickness effects are more pronounced on polarization compared to the line strengths. It may be interesting to do spectro polarimetric observations in the molecular lines and compare them with the theoretical calculations.

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