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Magnetohydrostatic equilibrium in solar coronal arcades

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Abstract. We present a solution of magnetohydrostatic equations after including the effect of gravity. For low values of the parameter β, there exist two solutions for the common boundary condition at the base of the corona. These two solutions correspond to two different magnetic configurations with different energies. The high-energy configuration may become unstable due to physical conditions and relax to the low-energy configuration, releasing an amount of the energy that may be observed in the form of a solar activity, such as two-ribbon flares. When the value of β exceeds the critical limit, the magnetohydrostatic equations have no solution. If the system is forced to exceed beyond the critical limit, the magnetic field may show a violent behaviour.

Key words : magnetohydrodynamics—solar physics—solar flares

1. Introduction

Observations of the Sun from orbiting satellites established that the solar corona is highly structured by magnetic fields. These fields originate from eruption through the photosphere (Parker 1977; Vaiana & Rosner 1978) and play an important role in solar activities (see e.g., Pallavicini 1989, and references therein). Some magnetized plasma structures in the solar atmosphere (corona) appear to remain in a stable state on the time scales for days, weeks or more until the equilibrium becomes unstable due to the change in the plasma and/or magnetic pressures, and would initiate a flare. Therefore, the solution of magnetohydrostatic (MHS) equations is of considerable interest. When the magnetic pressure is much larger than the plasma pressure, the magnetic field can be assumed to be force-free. In order to explain the phenomena such as solar flares, scientists tried to find out two solutions of a set of basic equations under a common boundary condition at the base of the solar corona. For example, Low & Nakagawa (1975), Low (1977) and Jockers (1978) solved analytically the case of force-free magnetic fields, but could not succeed to get two solutions. Birn *et al.* (1978) included the plasma pressure and could succeed to get two solutions for the set of basic equations

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under a common boundary condition at the base of the corona. However, the effect of gravity was not taken into account. Melville *et al.* (1983) extended the work of Birn *et al.* (1978) and included the effect of gravity. In another paper, Melville *et al.* (1987) used the same idea to include the effect of gravity in the work of Low (1977). However, their expressions can only be applicable for the state of ideal gas. In the present investigation, we present a solution of MHS equations after including the effect of gravity where the plasma is not necessarily to be considered as an ideal gas. For low values of the parameter β, the MHS equations are found to have two solutions for the given common boundary condition at the base of the solar corona. These two solutions correspond to two different magnetic configurations with different energies. The high-energy configuration may become unstable and relax to the low-energy configuration releasing an amount of energy that may be observed in the form of a solar activity, such as two-ribbon flares. A large two-ribbon flare is a much more complex event and usually follows the eruption of active region filament (or prominence). When β exceeds the critical limit, the MHS equations have no solution and magnetic field may show a violent behaviour.

2. Approach of Melville, Hood and Priest

The basic MHS equations governing the equilibrium in solar atmosphere are

$$(\nabla \times \vec{B}) \times \vec{B}/\mu - \nabla p - \rho g \hat{e}_z = 0 \qquad \ldots (1)$$

$$(\nabla \cdot \vec{B}) = 0 \qquad \ldots (2)$$

$$p = \rho RT/m \qquad \ldots (3)$$

where $\vec{B}$, p, ρ, T, μ, R and m are the magnetic field, plasma pressure, mass density, kinetic temperature, permeability of the medium, gas constant and molecular weight of plasma, respectively. The acceleration due to gravity is directed opposite to the unit vector $\hat{e}_z$. In the earlier work, for example, by Birn *et al.* (1978), Low (1977), Low & Nakagawa (1975), Priest & Milne (1980), the effect of gravity was neglected and therefore, the term $\rho g \hat{e}_z$ was not considered in the basic MHS equations. Melville, Hood and Priest (MHP) included the effect of gravity by considering the term $\rho g \hat{e}_z$ in their basic equations.

Consider that the plane $z = 0$ represents the base of the solar corona and all physical parameters vary in a two-dimensional plane (x, z). (Since the length of two-ribbon flare configuration is much larger than its width, it is reasonable to consider two-dimensional solution.) The magnetic field can be expressed in terms of the generating function $A(x, z)$ (Low 1977; Birn *et al.* 1978; MHP)

$$\vec{B} = -\frac{\partial A}{\partial z}\hat{e}_x + B_y(A)\,\hat{e}_y + \frac{\partial A}{\partial x}\hat{e}_z. \qquad \ldots (4)$$

The magnetic field (4) satisfies the condition (2). Putting (4) into (1) we get a set of equations

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial x} + B_y \frac{\partial B_y}{\partial x}\right] + \frac{\partial p}{\partial x} = 0 \qquad \ldots (5)$$

$$\frac{\partial B_y}{\partial z}\frac{\partial A}{\partial x} - \frac{\partial B_y}{\partial x}\frac{\partial A}{\partial z} = 0 \qquad \dots (6)$$

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial z}\right] + \rho g + \frac{\partial p}{\partial z} = 0. \qquad \dots (7)$$

Since B_y is a function of A only, equation (6) is obviously satisfied. MHP expressed the plasma pressure p as

$$p = f(A)\, e^{-z/H} \qquad \dots (8)$$

where H is the pressure scale height (= RT/mg). Using equation (8) in equations (5) and (7), both equations reduces to the form

$$\frac{1}{\mu}\left[\nabla^2 A + B_y \frac{\partial B_y}{\partial A}\right] + e^{-z/H}\frac{\partial f(A)}{\partial A} = 0 \qquad \dots (9)$$

where the ideal gas equation (3) is used. In the following section 3 we have discussed another approach to include the effect of gravity.

3. Present approach

In order to include the effect of gravity, we considered (Prasad & Chandra 1992) the pressure p in the form

$$p = f(A) + G(z) \qquad \dots (10)$$

where f is a function of A only, and G is a function of z only. It separates the effects of the magnetic field and gravity which are independent of each other and it is not necessary to express the plasma as an ideal gas. Using (10) in (5), we get

$$\frac{1}{\mu}\left[\nabla^2 A + B_y \frac{\partial B_y}{\partial A}\right] + \frac{\partial f}{\partial A} = 0 \qquad \dots (11)$$

and using (10) in (7), we get

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial A}\frac{\partial A}{\partial z}\right] + \rho(z)\, g(z) + \frac{\partial f}{\partial A}\frac{\partial A}{\partial z} + \frac{\partial G(z)}{\partial z} = 0. \qquad \dots (12)$$

If we choose $G(z)$ such that

$$\frac{\partial G(z)}{\partial z} = -\rho(z)\, g(z) \qquad \dots (13)$$

then (12) reduces to (11). The function G is given by

$$G(z) = -\int_0^z \rho(z')\, g(z')\, dz'. \qquad \ldots (14)$$

Then the pressure at the base of the corona is given by

$$p_0 = f(A)\, |_{z=0}. \qquad \ldots (15)$$

Now we express the variables in terms of the dimensionless variables (denoted by putting bars on them)

$$\bar{x} = x/l, \quad \bar{z} = z/l, \quad \bar{B} = B/B_0, \quad \bar{A} = A/lB_0, \quad \bar{f}(\bar{A}) = f(A)/p_0$$

where B_0 and p_0 are characteristic values of field strength and pressure, respectively, at the origin ($z = 0$) and l is the characteristic length associated with the variations on the photospheric boundary. In terms of these, equation (11) reduces to

$$\bar{\nabla}^2 \bar{A} + \frac{1}{2} \frac{\partial}{\partial \bar{A}} [\bar{B}_y(\bar{A}) + \beta \bar{f}(\bar{A})] = 0 \qquad \ldots (16)$$

where $$\bar{\nabla}^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{z}^2},$$

and $$\beta = \frac{2\mu p_0}{B_0^2}$$

where β is a parameter representing the ratio of the characteristic plasma and magnetic pressures. For the sake of convenience, in the further discussion we would drop the bars from all dimensionless variables. For example, equation (16) would be written as

$$\nabla^2 A + \frac{1}{2} \frac{\partial}{\partial A} [B_y(A) + \beta f(A)] = 0. \qquad \ldots (17)$$

In the present investigation, we considered

$$B_y(A) = \text{constant} \qquad \ldots (18a)$$

and $$f(A) = e^{-2\gamma A} \qquad \ldots (18b)$$

where γ is another free parameter. Using (18) in (17) we find that

$$\nabla^2 A - \beta e^{-2\gamma A} = 0. \qquad \ldots (19)$$

In order to transform it into a form that can be solved analytically, let us apply the transformation

$$F = \gamma A \qquad \text{... (20)}$$

so that equation (19) reduces to

$$\nabla^2 F - \beta\gamma^2 e^{-2F} = 0. \qquad \text{... (21)}$$

For the boundary condition at the base of the corona

$$F(x, 0) = \ln | x |$$

a pair of analytical solutions of equation (21) is (Birn *et al.* 1978)

$$F_{1,2} = \ln [r \cosh (2b_{1,2}\, \phi/\pi)/\cosh (b_{1,2})] \qquad \text{... (22)}$$

where $r = (x^2 + z^2)^{1/2}$, $\phi = \arctan (x/z)$ and b_1, $b_2(b_2 > b_1)$ are two solutions of the equation

$$\frac{b_{1,2}}{\cosh (b_{1,2})} = \frac{\pi\gamma\sqrt{\beta}}{2}. \qquad \text{... (23)}$$

The parameter β should always be a positive number. This situation can be understood on the basis of the physical grounds that both the plasma pressure as well as magnetic pressure are always positive quantities and therefore, β would always be a positive quantity. For a positive value of γ, both the roots of (23) are positive numbers, if they exist. When the value of γ is changed to $-\gamma$, both the roots of (23) would also change by their signs. For $\gamma = 0.1$, the variation of two solutions of (23) are shown in figure 1 as a function of β. Here, for $\beta \geq 17.0$ no solution of (23) exists. For other values of γ, say γ_c, the upper limit of β, say β_c, beyond which no solution of (23) exists, is given by $\gamma_c\sqrt{\beta_c} \approx \sqrt{17} \approx 0.41$. Similar figure can be obtained for the variation of the solution of (23).

Putting the reverse transformation, we get

$$A_{1,2} = \frac{F_{1,2}}{\gamma} = \frac{1}{\gamma} \ln [r \cosh (2b_{1,2}\, \phi/\pi)/\cosh (b_{1,2})]. \qquad \text{... (24)}$$

The value of the generating function A remains constant along a magnetic line of force. Therefore, for the given constant values of A and γ, the lines of force are the plots of the equation

$$r \cosh (2b_{1,2}\, \phi/\pi)/\cosh (b_{1,2}) = \text{constant} = C_1. \qquad \text{... (25)}$$

Figure 2 shows the lines of force corresponding to the solution b_1, $\gamma = 0.1$ and $\beta = 10.0$. The lines of force obviously end at the corona. In order to present the evolution of a line of force with β, we have plotted a line of force in figure 3 for $\gamma = 0.1$, $A = 10$ and different values

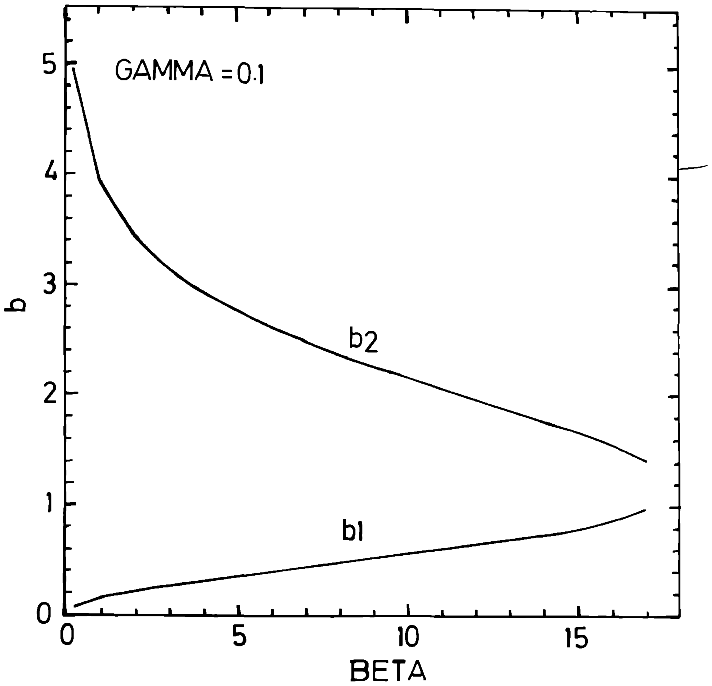

Figure 1. The variations of two results (b_1, b_2) as a function of β for $\gamma = 0.1$ are shown. Here, beyond $\beta \approx 17.0$, no solutions of equation (23) exists. When the value of γ is changed to $-\gamma$, the values of the roots of equation (23) would also change by their signs. For a given value of γ, say γ_c, the upper limit of β, say β_c, up to which the solution of equation (23) exists, is given by $\gamma_c\sqrt{\beta_c} \approx 0.41$.

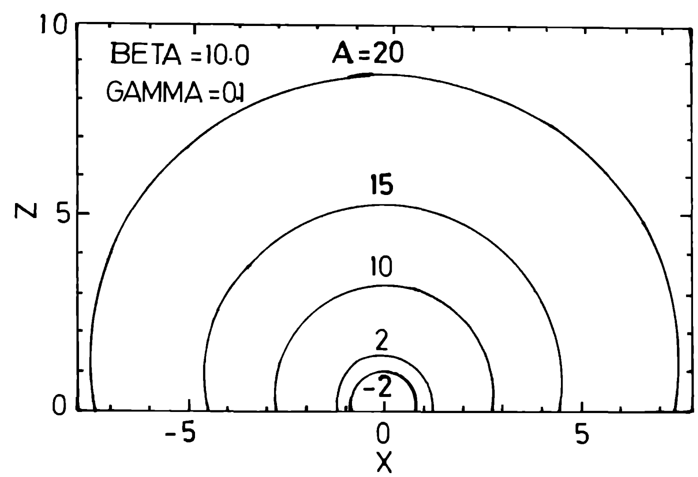

Figure 2. The magnetic field configurations corresponding to the solution b_1 of equation (23), $\gamma = 0.1$ and $\beta = 10.0$ are shown. The values of A are written near the field lines. (The parameters in the figure are dimensionless quantities).

of β. Figure 3 shows that the foot-points of the line of force are stationary and at the base of the corona, and the height of the line of force increases with β. The lines of force corresponding to the solution b_2, $\gamma = 0.1$ and $\beta = 10.0$ are shown in figure 4. In this configuration also, the lines of force end at the base of the corona. We found that in this case also, the foot-points of a line of force are stationary and at the base of the corona. The height of the line of force however, decreases with the increase of β. Using (10), (13), (18b) and (24) the plasma pressure is given by the equation

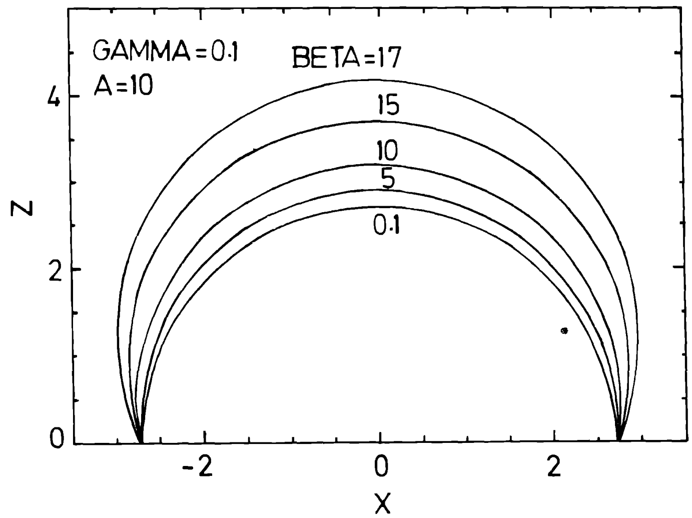

Figure 3. The magnetic field configurations corresponding to the solution b_1 of equation (23), $\gamma = 0.1$ and $A = 10$ are shown. It can be noticed that with the increase of β, the foot-points of the line of force are stationary and at the base of the corona, and the height of the line of force increases. (The parameters in the figure are dimensionless quantities).

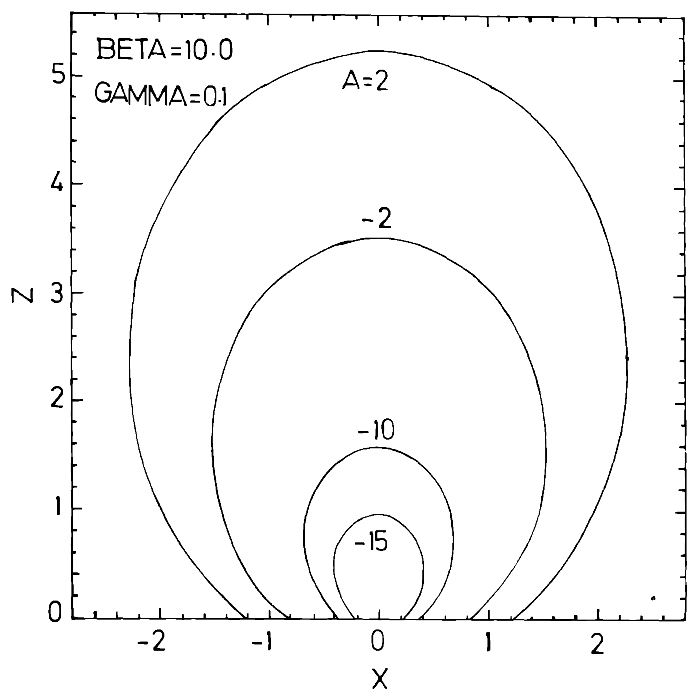

Figure 4. The magnetic field configurations corresponding to the solution b_2 of equation (23), $\gamma = 0.1$ and $\beta = 10.0$ are shown. The values of A are written near the field lines. (The parameters in the figure are dimensionless quantities).

$$p_{1,2} = [r \cosh (2b_{1,2}\, \phi/\pi)/\cosh (b_{1,2})]^{-2} - \int_0^z \rho(z')\, g(z')\, dz' \qquad \ldots (26)$$

Thus, the pressure isobars given by the equation

$$[r \cosh (2b_{1,2}\ \phi/\pi)/\cosh (b_{1,2})]^{-2} - \int_0^z \rho(z')\ g(z')\ dz' = \text{constant} = C_2 \qquad \dots (27)$$

are no longer along the magnetic lines of force. If the gravitational effect is neglected, the pressure isobars become a long magnetic lines of force.

4. Discussion

Figure 1 shows that for the given value of γ, and the common boundary condition at the base of the corona, there are two possible magnetic configurations. The total energy of these configurations defined by

$$W_{1,2} = \int_v \left(\frac{B_{1,2}^2}{2\mu} - G(z) + \frac{p_{1,2}}{\gamma' - 1} \right) dV \qquad \dots (28)$$

would be different. Here γ' is the adiabatic index. It can be easily shown that $W_2 \neq W_1$. Therefore, the configuration corresponding to larger energy may become unstable due to physical reasons and relax to the low energy configuration, releasing an amount of energy that may be observed in the form of a solar activity. The amount of energy released would depend on the values of the parameters β and γ. When this energy is small, the solar activity may not be noticeable, but when the energy is large, the activity such as a two-ribbon flare may be observable.

The increase in the value of parameter β means to increase the value of plasma pressure and/or to decrease the value of magnetic pressure. For a given value of γ, when the value of β is increased slowly, it reaches to a critical value beyond which no magnetic configuration exists. At this critical point the magnetic field may erupt violently.

For isothermal case, soft x-rays observations would show up the structures of constant density (Melville *et al.* 1983). For constant temperature and density, the pressure becomes constant. Since the pressure isobars are no longer along the magnetic lines of force, therefore, one should be cautious to interpret X-ray structures as magnetic loops.

5. Conclusions

We found that the MHS equations have a pair of solutions, if they exist, for the common boundary condition at the base of the corona. After including the effect of gravity, the pressure isobars are no longer along the magnetic lines of force, as it was the case when the effect of gravity was neglected. The two solutions correspond to two different magnetic configurations with different energies. The magnetic configuration with larger energy may become unstable and relax to the low energy configuration releasing an amount of energy that may cause solar activity, such as two ribbon-flares. For a given value of γ, when β is increased slowly a critical limit arrives beyond which no solutions of MHS equations exist and the magnetic field may show a violent behaviour.

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