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On heating of stellar (solar) chromospheres and coronae by magnetoacoustic waves : a review

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Abstract. Stellar chromospheres and coronae may be heated by waves of various types. Magnetoacoustic waves are one of them. Subphotospheric turbulence generates various types of waves. Here the basic theory of generation, propagation and dissipation of waves is briefly described. Heating of stellar atmosphere by magnetoacoustic waves is reviewed. Because of our poor knowledge of subphotospheric turbulent motions the estimated wave fluxes seem to be erroneous consequently the estimates of heating are also in error. The fast mode waves suffer total internal reflection during propagation in the chromosphere. Unless mode conversion to other waves occurs they cannot heat stellar atmosphere. Thus mode conversion could play an important role in understanding the heating of stellar chromospheres and coronae.

Key words : MHD waves—stellar atmosphere—heating

1. Introduction

It is now well known that the solar chromosphere and corona lie in between two cooler regions, namely photosphere and the interplanetary space (see, e.g. figure 1). Probably all stars with the possible exception of A-stars exhibit such a behaviour. Chromospheres lose energy predominantly by radiation and coronae by conduction, radiation and stellar winds. To replenish these losses some source of energy must be present. The source cannot be thermal because the thermal conductivity of the medium is very high and so within no time ($1\text{-}10^3$ s; see e.g. Krieger 1978) the chromosphere and corona would attain photospheric temperature. The source cannot be radiative because the matter density of the solar atmosphere is too low to be opaque to solar radiations. The source could be either mechanical or electrical or magnetic or their combination.

Various mechanisms have been put forward to explain this phenomena (see, e.g. Hollweg 1990; Narain & Ulmschneider 1990; Ulmschneider *et al.* 1991 and references therein). Since

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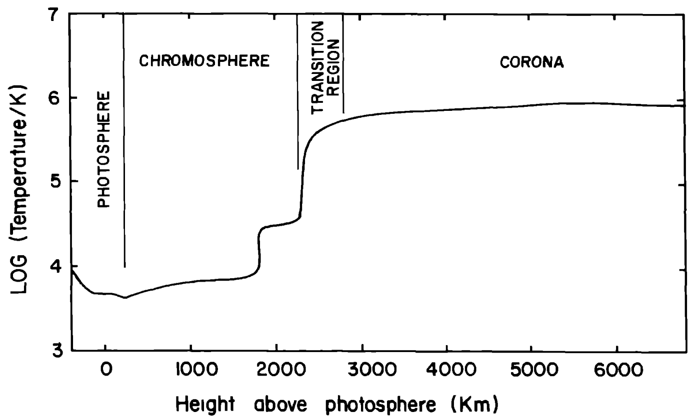

Figure 1. Temperature distribution in solar photosphere, chromosphere and corona.

the publication of these works many new developments have taken place. Our aim is two fold : to exhibit the latest developments in this field in a single article and to concentrate on a single mechanism of heating so that more details could be given. In an earlier work we reviewed the LCR circuit approach of coronal heating (Narain & Kumar 1993) and highlighted its potential. In this article we concentrate on heating by magnetoacoustic waves. Section 2 describes magnetoacoustic waves and their generation. Section 3 deals with propagation aspects and characteristics of wave modes. In section 4 we exhibit dissipation mechanisms. Section 5 is devoted to literature survey. Section 6 contains our conclusions. A comprehensive list of references is given in the end.

2. Magnetohydrodynamic waves and their generation

In the absence of gravity, and for a perfectly conducting, compressible, uniform gas, in a homogeneous magnetic field, there exist three types of waves : (1) Alfvén mode waves, (2) Fast mode waves and (3) Slow mode waves. The types (2) and (3) taken together, are often called magnetoacoustic or magnetosonic waves, and they show gas pressure fluctuations. Alfvén mode waves, to first order, do not show gas pressure variations.

Subphotospheric turbulent motions act as a source of waves. Turbulence can be pictured as a group of eddies of different size and velocity. A turbulent eddy has kinetic energy as well as magnetic energy, which it releases when it mixes with its surroundings at the end of its life. Most of this energy is returned to the ambient medium, but a small fraction gets transformed into propagating waves (Kulsrud 1955). The radiative power is roughly the energy density, ε, in the turbulent motions divided by the decay time scale, τ, for the turbulent motions, multiplied by an efficiency factor. For a emission of multipole order n, following Stein (1981), the radiated power is

$$P \approx \frac{\varepsilon}{\tau} (k\,l)^{2n+1}, \qquad n = 0, 1, 2, \ldots \qquad \ldots (1)$$

where l is the size of the eddy and $k = 2\pi/\lambda$ is the wave vector of the wave. $n = 0$ corresponds to monopole, $n = 1$ to dipole and $n = 2$ to quadrupole emission. The energy density of the turbulent motions is

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$$\varepsilon = \frac{1}{2}\rho u^2 + \delta B^2/8\pi \qquad \ldots (2)$$

where ρ is density of turbulent medium, u the velocity of the eddy and δB the change in magnetic field strength. Obviously, the first term in equation (2) represents kinetic energy density and the second term the magnetic energy density of the eddy. For a magnetic field weaker than or equal to equipartition strength,

$$\varepsilon \approx \rho u^2 \qquad \ldots (3)$$

whereas if the magnetic field dominates the motions, we have

$$\varepsilon = B^2/8\pi. \qquad \ldots (4)$$

The turbulence decay time scale is given by

$$\tau \approx l/u \qquad \ldots (5)$$

and the angular frequency of the generated wave, to a good approximation, is

$$\omega \approx \tau^{-1} \approx u/l. \qquad \ldots (6)$$

Therefore

$$kl \approx (2\pi/\lambda)\, u/\omega \approx u/C, \qquad \ldots (7)$$

where C is the phase velocity of the wave. For an acoustic wave (or the fast mode in a weak magnetic field or the slow mode in a strong one) C equals sound speed c_S whereas for Alfvén waves (or the slow mode in a weak magnetic field or the fast mode in a strong one) C equals the Alfvén speed, v_A [see equation (18)].

The dominant multipole order depends on the background magnetic field strength and the wave type. Monopole emission corresponds to a mass source. In the absence of a magnetic field there are no mass source in the convection zone. When a magnetic field is present, it channels the Alfvén waves, and a strong field channels the slow mode acoustic waves, so that the waves move in one-dimension along the magnetic field. The dipole emission corresponds to a momentum source (i.e., an external force). In a uniform medium there is no external force and hence no dipole emission. In stars, due to gravitational field, some dipole emission occurs. Quadrupole emission corresponds to the action of the turbulent Reynolds stresses. This is the dominant process in stellar atmospheres in the absence of magnetic fields.

In the absence of magnetic fields, turbulent convection produces acoustic waves by quadrupole emission (n = 2) and the acoustic power is given by

$$P_{ac} \approx (\rho u^3/l)\left(\frac{u}{c_s}\right)^5. \qquad \ldots (8)$$

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If there is turbulent magnetic field and it is less than or equal to equipartition magnetic field strength (gas pressure ≈ magnetic pressure) with the turbulent motions the Alfvén and slow mode waves are produced by monopole emission. Using equations (1), (3) and (5) the radiated power is given by

$$P_a \approx (\rho u^3/l)\left(\frac{u}{v_A}\right), \quad \ldots (9)$$

$$P_s \approx (\rho u^3/l)\left(\frac{u}{c_s}\right) \quad \ldots (10)$$

and $$P_f \approx (\rho u^3/l)\left(\frac{u}{v_A}\right)^5. \quad \ldots (11)$$

P_a, P_s and P_f are the powers radiated in form of Alfvén, slow mode and fast mode waves, respectively. In case the turbulent magnetic field dominates the convective motions the corresponding expressions, using equations (1), (4) and (5), are as follows :

$$P_a \approx (B^2 u/l)\left(\frac{u}{v_A}\right) \approx (\rho u^2 v_A/l), \quad \ldots (12)$$

$$P_s \approx (B^2 u/l)\left(\frac{u}{c_s}\right) \approx (\rho u^2 v_A/l)\left(\frac{v_A}{c_s}\right), \quad \ldots (13)$$

$$P_f \approx (B^2 u/l)\left(\frac{u}{v_A}\right)^5 \approx (\rho u v_A^2/l)\left(\frac{u}{v_A}\right)^5. \quad \ldots (14)$$

The wave energy flux so produced is obtained by multiplying emitted power by the eddy size. Thus

$$F \approx Pl, \quad \ldots (15)$$

This is valid if the largest velocities occur within one eddy of the top of the convection zone, because the waves emitted deeper are scattered by overlying eddies. The total rate of wave energy emission is the flux, F, multiplied by the area, A, of the emitted region. That is,

$$L \approx PlA, \quad \ldots (16)$$

These relations are applicable to individual flux tubes and the area of the emitted region is the area of cross section of the magnetic flux tube at the emitted region.

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3. Propagation of MHD waves

A. General introduction

We are well familiar with propagation of sound in an ideal gas. Compressions and rarefactions in the atmosphere propagate isotropically with the adiabatic sound speed c_s, defined in terms of the undisturbed gas pressure p and density ρ_0 by

$$c_s = (\gamma p/\rho_0)^{1/2} \qquad \dots (17)$$

where γ (= 5/3 for solar atmosphere) is the ratio of specific heats.

But in the case of compressible, conducting fluid immersed in a magnetic field some other types of waves are possible. We find that in the presence of uniform magnetic field ($\mathbf{B}_0$) variation in gas pressure will disturb the magnetic field lines, which behave effectively as elastic strings under a tension of $B_0^2/4\pi$. In a perfect conductor, the magnetic field lines and the fluid motions are frozen together, so any attempt to initiate a sound wave will result in variation in the magnetic field as it is locally compressed or rarefied. Therefore sound is no longer able to propagate with the sound speed c_s and the directionality of the applied magnetic field renders wave propagations anisotropic.

The characteristic speed of such magnetic disturbances is known as Alfvén speed $\mathbf{v}_A$, and is given as

$$\mathbf{v}_A = (\text{magnetic tension/density})^{1/2} = \mathbf{B}_0/(4\pi\rho_0)^{1/2} \qquad \dots (18)$$

The values of c_s and $\mathbf{v}_A$ calculated on the basis of Bilderberg-Continuum-Atmosphere (BCA) are shown in figure 2 (Bray & Loughhead 1974).

Longitudinal vibrations are also possible to occur in a compressible, conducting fluid in a magnetic field. For the wave propagation along the magnetic field there will be no field perturbation, since the waves are free to move in this direction. Therefore, in this case the waves will be ordinary longitudinal sound waves, which propagate with the sound speed c_s. In this case

$$p\rho^{-\gamma} = \text{constant}, \qquad \dots (19)$$

or

$$\nabla p = (\gamma p/\rho)\, \nabla\rho = c_s^2\, \nabla\rho. \qquad \dots (20)$$

On the other hand, for wave propagation normal to the magnetic field direction, a new type of longitudinal wave motion is possible. In addition to the fluid pressure p, there is a magnetic pressure $B^2/8\pi$, in the plane normal to $\mathbf{B}_0$ so that the total pressure is $p + B^2/8\pi$, and the velocity of propagation is v_m. These waves are magnetoacoustic or magnetosonic waves and are described by the following equation :

$$\nabla(p + B^2/8\pi) = v_m^2\, \nabla\rho. \qquad \dots (21)$$

Therefore

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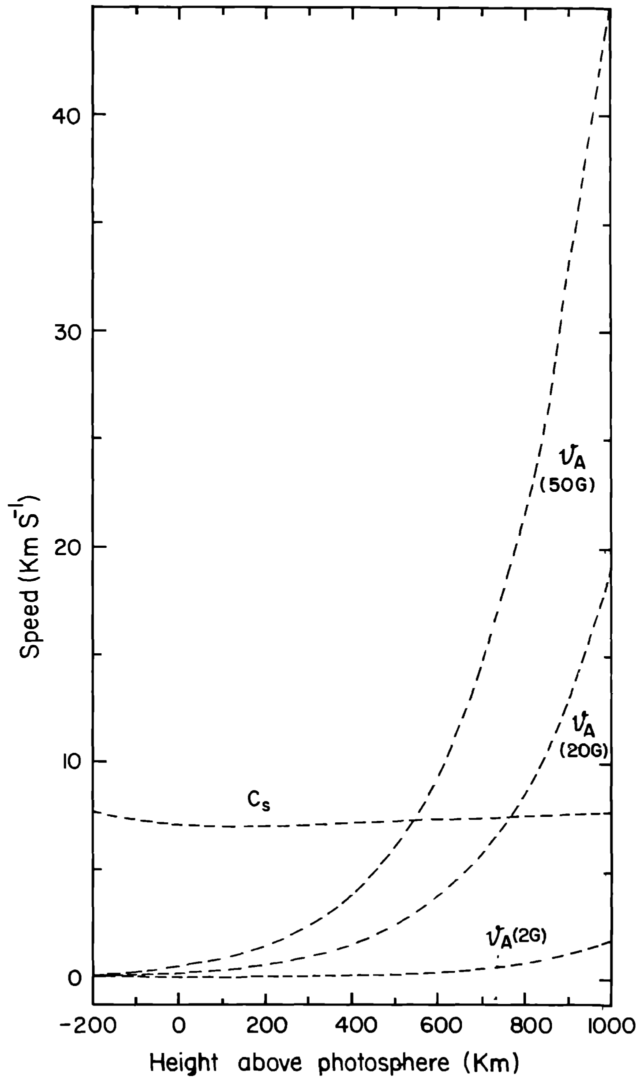

Figure 2. Variation of sound and Alfvén speeds with height and magnetic induction in the solar atmosphere.

$$v_m^2 = \frac{d}{d\rho}\left(p + B^2/8\pi\right)_{(\rho=\rho_0)} = c_s^2 + \frac{d}{d\rho}(B^2/8\pi)_{(\rho=\rho_0)} \qquad \ldots (22)$$

where suffix zero refers to undisturbed state. Since the lines of force in the conducting fluid, are frozen, therefore, $B/\rho = B_0/\rho_0$ (see, e.g. Bittencourt 1988, p. 393). Hence

$$v_m^2 = c_s^2 + \frac{d}{d\rho}(B_0^2\rho^2/8\pi\rho_0^2)_{(\rho=\rho_0)} = c_s^2 + v_A^2. \qquad \ldots (23)$$

This is the expression for propagation velocity of magnetoacoustic waves in a homogeneous plasma.

In the case of inhomogeneous plasma, magnetic field is not uniform throughout the medium. As an example, consider the tube waves having uniform magnetic field inside the

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tube and embedded in an infinite non-magnetic plasma. As pointed out earlier, following three modes are possible.

(a) LONGITUDINAL OR SAUSAGE MODE

Longitudinal mode involves compressions and rarefactions of the gas within the tube which in turn leads to contractions and expansions of the tube's cross-section. Such area changes imply compressions and rarefactions of the magnetic pressure, so the longitudinal mode involves changes in both the gas pressure and magnetic pressure. Gas compressions are associated with sound speed c_s in the tube, and magnetic compressions with the Alfvén speed v_A. Longitudinal tube waves are essentially acoustic waves, which propagate along the magnetic flux tubes, with the tube speed c_T given by (Roberts 1990),

$$\frac{1}{c_T^2} = \frac{1}{c_s^2} + \frac{1}{v_A^2}$$

or

$$c_T = \frac{c_s v_A}{(c_s^2 + v_A^2)^{1/2}}. \qquad \ldots (24)$$

These waves are shown in figure 3a.

(b) TRANSVERSE OR KINK MODE

Another mode of wave motion, made possible by the coupling of the magnetic field to the coronal gas, is the compression free, transversal Alfvén wave. In this case the field lines are laterally displaced from their straight equilibrium position, like a string of a string instrument, as shown in figure 3b. They vibrate around their equilibrium position due to the inertia of the material tied to them. So we expect that Alfvén waves travel along the magnetic field with Alfvén speed v_A. But it must be remembered that the vibrating tube displaces some of the surrounding gas of density ρ_e and lowers the speed of propagation. It turns out that the transverse mode wave has a characteristic speed c_k, given by

$$c_k = \left(\frac{\rho_0}{\rho_0 + \rho_e}\right)^{1/2} v_A \qquad \ldots (25)$$

which gets reduced below the Alfvén speed by the inertia of tube's surroundings.

(c) TORSIONAL MODE

For simplicity let us consider a tube without stratification. Let the transverse direction be horizontal, and without compression, so that magnetic force is the only restoring force, then the wave motions are the torsional Alfvén waves. The main tube field is distorted in azimuthal direction only. Therefore the torsional Alfvén waves propagate along the tube. These waves are modified by stratification, in which case the tube widens in the z direction. Figure 3c shows a torsional wave.

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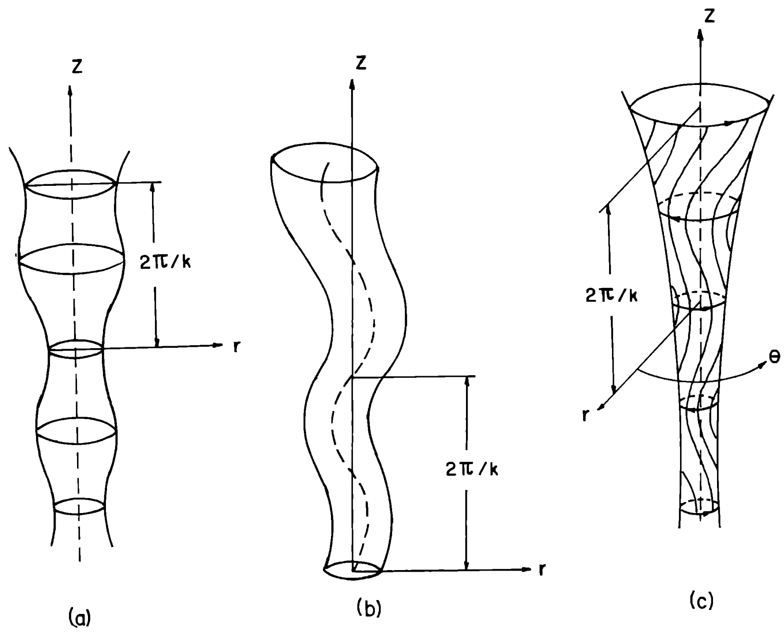

Figure 3. Magnetohydrodynamic wave modes in a flux tube : (a) longitudinal or sausage mode, (b) transverse or kink mode, (c) torsional mode.

It is found that the motion of the material in the longitudinal tube waves is along the magnetic field, whereas for transverse and torsional waves it is perpendicular to the magnetic field. Due to horizontal pressure balance the magnetic flux tubes spread rapidly with height, and at the so called canopy height (somewhere in the middle chromosphere) fill out the entire available space.

It appears that the longitudinal, transverse and torsional waves which propagate along the magnetic flux tubes at photospheric and chromospheric heights go over into the slow, fast and Alfvén wave modes in the locally homogeneous fields above the middle chromosphere. Considerable nonlinear coupling (mode conversion) between the various modes of the wave is expected to occur at these canopy heights.

A medium with a sharp change in physical properties across an interface has the ability to support surface waves, which propagate along the interface. The phase speed of surface waves lies between the Alfvén speeds v_{A1} and v_{A2}, of two media. In fact, in general, there are two surface waves, which we may refer to as slow and fast surface waves. These waves are compressive and so are magnetoacoustic surface waves.

The body waves correspond to slow modes. The slow body mode waves may be viewed as waves that are constrained within the tube, bouncing from side to side of the tube as they propagate along its interior.

B. Basic equations

I. HOMOGENEOUS MEDIUM

The basic equations governing the behaviour of MHD waves in a compressible, non-viscous and perfectly conducting fluid in a uniform magnetic field are (Bittencourt 1988)

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \ldots (26)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\, \mathbf{v} = -\nabla p \pm (\nabla \times \mathbf{B}) \times \frac{\mathbf{B}}{4\pi} \qquad \ldots (27)$$

$$\nabla p = c_s^2 \nabla \rho \qquad \ldots (28)$$

and

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = \frac{\partial \mathbf{B}}{\partial t}. \qquad \ldots (29)$$

Under equilibrium conditions, the fluid is assumed to be uniform with constant density ρ_0, the equilibrium velocity is zero, and throughout the fluid the magnetic induction $\mathbf{B}_0$ is uniform and constant.

In order to develop a dispersion relation for small amplitude waves, let us apply a small amplitude perturbation from the equilibrium values, so that

$$\mathbf{B}(r, t) = \mathbf{B}_0 + \mathbf{B}_1(\mathbf{r}, t) \qquad \ldots (30)$$

$$\rho(\mathbf{r}, t) = \rho_0 + \rho_1(\mathbf{r}, t) \qquad \ldots (31)$$

$$\mathbf{v}(\mathbf{r}, t) = \mathbf{v}_1(\mathbf{r}, t). \qquad \ldots (32)$$

On substituting equations (30) to (32) and (28) into (26), (27) and (29), and neglecting second order terms, we obtain the following linearized equations

$$\frac{\partial \rho_1}{\partial t} + \rho_0\, (\nabla \cdot \mathbf{v}_1) = 0 \qquad \ldots (33)$$

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + c_s^2\, \nabla \rho_1 + \frac{\mathbf{B}_0}{4\pi} \times (\nabla \times \mathbf{B}_1) = 0 \qquad \ldots (34)$$

$$\frac{\partial \mathbf{B}_1}{\partial t} - \nabla \times (\mathbf{v}_1 \times \mathbf{B}_0) = 0. \qquad \ldots (35)$$

To get an equation for $\mathbf{v}_1$, first we differentiate equation (34) with respect to time t, to obtain

$$\rho_0 \frac{\partial^2 \mathbf{v}_1}{\partial t^2} + c_s^2\, \nabla \left(\frac{\partial \rho_1}{\partial t} \right) + \frac{1}{4\pi} \mathbf{B}_0 \times \left(\nabla \times \frac{\partial \mathbf{B}_1}{\partial t} \right) = 0 \qquad \ldots (36)$$

and with the help of equations (33), (35) and (36), we obtain

$$\frac{\partial^2 \mathbf{v}_1}{\partial t^2} - c_s^2\, \nabla\, (\nabla \cdot \mathbf{v}_1) + \mathbf{v}_A \times \{\nabla \times [\nabla \times (\mathbf{v}_1 \times \mathbf{v}_A)]\} = 0 \qquad \ldots (37)$$

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where

$$\mathbf{v}_A = \mathbf{B}_0/(4\pi\rho_0)^{1/2},$$

is the vector Alfvén velocity.

Let the plane wave solutions be of the form

$$\mathbf{v}_1(\mathbf{r}, t) = \mathbf{v}_1 \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t) \qquad \ldots (38)$$

where $\mathbf{v}_1$ is the velocity amplitude.

In equation (37) we can replace the operator ∇ by $i\mathbf{k}$ and the time derivatives by $-i\omega$, so that equation (37) becomes

$$-\omega^2\mathbf{v}_1 + c_s{}^2(\mathbf{k}\cdot\mathbf{v}_1)\,\mathbf{k} - \mathbf{v}_A \times \{\mathbf{k}\times[\mathbf{k}\times(\mathbf{v}_1\times\mathbf{v}_A)]\} = 0. \qquad \ldots (39)$$

Since for any three vectors, we have the vector identity

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\,\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\,\mathbf{C} \qquad \ldots (40)$$

so equation (39) takes the form

$$-\omega^2\mathbf{v}_1 + (c_s^2 + v_A^2)(\mathbf{k}\cdot\mathbf{v}_1)\,\mathbf{k} + (\mathbf{k}\cdot\mathbf{v}_A)\,[(\mathbf{k}\cdot\mathbf{v}_A)\,\mathbf{v}_1 - (\mathbf{v}_A\cdot\mathbf{v}_1)\,\mathbf{k} - (\mathbf{k}\cdot\mathbf{v}_1)\,\mathbf{v}_A] = 0. \qquad \ldots (41)$$

This is the general equation of wave motion in a uniform magnetic field.

Case I : Propagation parallel to the magnetic field—For wave propagation along the magnetic field ($\mathbf{k} \parallel \mathbf{B}_0$), we have $\mathbf{k}\cdot\mathbf{v}_A = kv_A$, so equation (41) becomes

$$(k^2v_A^2 - \omega^2)\,\mathbf{v}_1 + \left(\frac{c_s^2 - v_A^2}{v_A^2}\right)k^2(\mathbf{v}_1\cdot\mathbf{v}_A)\,\mathbf{v}_A = 0. \qquad \ldots (42)$$

In this case two types of wave motion are possible.

When $\mathbf{v}_1$ is parallel to $\mathbf{B}_0$ and $\mathbf{k}$, equation (42) exhibits a longitudinal mode, with the phase speed

$$\omega/k = c_s. \qquad \ldots (43)$$

This is an ordinary longitudinal sound wave, in which the velocity of mass flow is in the direction of propagation.

The other possibility is that $\mathbf{v}_1$ be perpendicular to $\mathbf{B}_0$ and $\mathbf{k}$, i.e., the wave is a transverse wave. In this case $\mathbf{v}_1\cdot\mathbf{v}_A = 0$, and equation (42) gives the phase velocity of this transverse wave, as below

$$\omega/k = v_A. \qquad \ldots (44)$$

This wave mode is known as shear Alfvén wave or slow Alfvén wave.

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Case II : Propagation perpendicular to magnetic field—When the wave propagation is perpendicular to the magnetic field (i.e., $\mathbf{k} \perp \mathbf{B}_0$), then $\mathbf{k} \cdot \mathbf{v}_A = 0$, and equation (41) becomes

$$-\omega^2 \mathbf{v}_1 + (c_s^2 + v_A^2)(\mathbf{k} \cdot \mathbf{v}_1)\,\mathbf{k} = 0$$

or,

$$\mathbf{v}_1 = (c_s^2 + v_A^2)(\mathbf{k} \cdot \mathbf{v}_1)\,\mathbf{k}/\omega^2, \qquad \ldots (45)$$

which implies that $\mathbf{v}_1$ is parallel to $\mathbf{k}$, so that $\mathbf{k} \cdot \mathbf{v}_1 = kv_1$, and we have a longitudinal wave whose phase velocity is given by

$$\omega/k = (c_s^2 + v_A^2)^{1/2}. \qquad (46)$$

This represents a magnetoacoustic wave, also known as compressional Alfvén wave or fast Alfvén wave.

Case III : Propagation in an arbitrary direction—Let us choose a cartesian coordinate system such that y-axis is normal to the plane formed by the direction of propagation $\mathbf{k}$ and the magnetic induction $\mathbf{B}_0$. Suppose that $\mathbf{B}_0$ is along z-axis and let θ be the angle between $\mathbf{k}$ and $\mathbf{B}_0$. Now we have

$$\mathbf{k} = k(\hat{x} \sin\theta + \hat{z} \cos\theta), \qquad \ldots (47)$$

$$\mathbf{v}_A = v_A \hat{z}, \qquad \ldots (48)$$

and $$\mathbf{v}_1 = v_x' \hat{x} + v_y' \hat{y} + v_z' \hat{z}. \qquad \ldots (49)$$

Hence

$$\mathbf{k} \cdot \mathbf{v}_A = kv_A \cos\theta, \qquad \ldots (50)$$

$$\mathbf{k} \cdot \mathbf{v}_1 = k(v_x' \sin\theta + v_z' \cos\theta), \qquad \ldots (51)$$

and $$\mathbf{v}_A \cdot \mathbf{v}_1 = v_A v_z'. \qquad \ldots (52)$$

Substituting the above values in equation (41) and rearranging the terms, we get the following equations :

$$v_x'(-\omega^2 + k^2 v_A^2 + k^2 c_s^2 \sin^2\theta) + v_z'(k^2 c_s^2 \sin\theta \cos\theta) = 0 \qquad \ldots (53)$$

$$v_y'(-\omega^2 + k^2 v_A^2 \cos^2\theta) = 0 \qquad \ldots (54)$$

and $$v_x'(k^2 c_s^2 \sin\theta \cos\theta) + v_z'(-\omega^2 + k^2 c_s^2 \cos^2\theta) = 0 \qquad \ldots (55)$$

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corresponding to x-, y-, and z-components, respectively. It is obvious from equation (54) that there is a linearly polarized wave, involving vibrations in the direction perpendicular to **k** and $\mathbf{B}_0$ ($v_y' \neq 0$), which propagates with the phase speed,

$$\omega/k = v_A \cos\theta. \qquad \ldots (56)$$

This mode is known as oblique Alfvén wave. Clearly for propagation along the magnetostatic field ($\theta = 0$) $\omega/k = v_A$ while for propagation across the field $\omega/k = 0$ (i.e., the wave disappears).

Equations (53) and (55) constitute a system of two simultaneous equations for v_x' and v_z'. To obtain a solution for which v_x and v_z are nonzero, the determinant of the coefficients of this system must vanish. Hence

$$\begin{vmatrix} (-\omega^2 + k^2 v_A^2 + k^2 c_s^2 \sin^2\theta) & (k^2 c_s^2 \sin\theta\cos\theta) \\ (k^2 c_s^2 \sin\theta\cos\theta) & (-\omega^2 + k^2 c_s^2 \cos^2\theta) \end{vmatrix} = 0 \qquad \ldots (57)$$

which gives following dispersion relation,

$$\omega^4 - (c_s^2 + v_A^2)\,\omega^2 k^2 + k^4 c_s^2 v_A^2 \cos^2\theta = 0. \qquad \ldots (58)$$

On solving this equation, we get following two real solutions,

$$\left(\frac{\omega}{k}\right)^2 = \frac{1}{2}\{(c_s^2 + v_A^2) \pm [(c_s^2 + v_A^2)^2 - 4c_s^2 v_A^2 \cos^2\theta]^{1/2}\}. \qquad \ldots (59)$$

The higher frequency mode with positive sign is known as fast magnetoacoustic wave while the other with negative sign is slow magnetoacoustic wave. Thus,

$$v_f^2 = \frac{1}{2}\{(c_s^2 + v_A^2) + [(c_s^2 + v_A^2)^2 - 4c_s^2 v_A^2 \cos^2\theta]^{1/2}\} \qquad \ldots (60)$$

and

$$v_s^2 = \frac{1}{2}\{(c_s^2 + v_A^2) - [(c_s^2 + v_A^2)^2 - 4c_s^2 v_A^2 \cos^2\theta]^{1/2}\} \qquad \ldots (61)$$

are the phase speeds of fast and slow mode MHD waves, respectively.

The fast-mode waves can propagate in any direction. For weak magnetic fields (say, 2 Gauss), the fast mode waves move with the velocity of sound wave ($v_f \approx c_s$) whereas for strong fields (say, 50 Gauss or more), they move with the velocity of Alfvén waves ($v_f \approx v_A$). In both cases the propagation velocity is independent of the direction of motion. At intermediate fields, the velocity depends weakly on the direction (Osterbrock 1961).

In weak magnetic fields, the direction of motion of the material is longitudinal to the direction of the field and the waves are essentially acoustic waves. But, in strong fields, the direction of motion is perpendicular to the direction of the field, and the waves are essentially magnetoacoustic.

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The slow-mode waves can propagate only in direction close to the direction of the magnetic field. For a very weak field (say, 2 Gauss) as well as for a strong field (say, 50 Gauss or more), the only allowed direction of propagation is exactly along the field, but for an intermediate field (say, 20 Gauss), the allowed direction of propagation lies within a cone with half angle ≈ 27°.

For weak magnetic fields, the slow-mode waves move with the velocity of Alfvén waves ($v_s \approx v_A$), whereas for strong magnetic fields, they move with the velocity of sound waves ($v_s \approx c_s$). The direction of the motion of the material is perpendicular to the direction of propagation of the wave, in case of the weak magnetic fields. But for the strong magnetic fields, the direction of the motion of the material is along the field.

II. INHOMOGENEOUS MEDIUM (CYLINDRICAL FLUX-TUBE)

Consider a cylindrical flux tube of radius r_0 with a uniform magnetic field B_0 in z direction. Inside the tube the gas density is ρ_0, and pressure is p. Let the plasma outside the flux tube has density ρ_e, pressure p_e, and sound speed c_e. The density varies in the transverse direction being uniform inside the duct and having discrete jump at duct's boundary. Neglecting electrical resistivity, gravity, and background flow velocity (i.e., setting these quantities to zero) static equilibrium is defined by

$$p + \frac{B_0^2}{8\pi} = p_e \qquad \ldots (62)$$

which may also be written as

$$\rho_0(c_s^2 + \gamma v_A^2/2) = \rho_e c_e^2. \qquad \ldots (63)$$

Considering, small velocity perturbation $\mathbf{v}(\mathbf{r}, \theta, z, t)$ within the tube and $\mathbf{v}_1(\mathbf{r}, \theta, z, t)$ outside the tube, with corresponding perturbation in pressure δp, δp_e, in magnetic field δB and density $\delta\rho_0$, $\delta\rho_e$. The tube perturbation equation reduces to (Wilson, 1980 and references contained therein)

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial t^2} - (c_s^2 + v_A^2)\,\nabla^2\right)\Delta + v_A^2 c_s^2 \frac{\partial^2}{\partial z^2}\nabla^2\Delta = 0 \qquad \ldots (64)$$

where $\Delta = \nabla \cdot \mathbf{v}$, and the perturbation is of the form of (Wilson 1980),

$$\Delta = R(r)\exp(in\theta + ikz - i\omega t) \qquad \ldots (65)$$

where n is azimuthal wave number and k the longitudinal wavenumber. Hence equation (65) becomes,

$$[\omega^2 (c_s^2 + v_A^2) - v_A^2 c_s^2 k^2]\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) - \frac{n^2}{r^2}R\right] + [\omega^4 - \omega^2 k^2 (c_s^2 + v_A^2) + c_s^2 v_A^2 k^4]\,R = 0 \qquad \ldots (66)$$

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provided $\omega^2(v_A^2 + c_s^2) - c_s^2 v_A^2 k^2 \neq 0$. Above equation may be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \left(n_0^2 - \frac{n^2}{r^2}\right)R = 0 \qquad \ldots (67)$$

with
$$n_0^2 = \frac{(\omega^2 - v_A^2 k^2)(\omega^2 - c_s^2 k^2)}{(c_s^2 + v_A^2)\,\omega^2 - c_s^2 v_A^2 k^2}. \qquad \ldots (68)$$

The solutions of equation (67) are linear combination of the modified Bessel functions $J_n(|n_0 r|)$ and $K_n(|n_0 r|)$.

In case of discontinuity at the position $r = r_0$, which separates the two regions, the equilibrium quantities take constant but different values. After some detailed calculations, it may be shown that the modes of vibration satisfy the dispersion relation (Wilson 1980; Edwin & Roberts 1983; Edwin & Zhelyazkov 1992).

$$\rho_0(k^2 v_A^2 - \omega^2)\, m_e \frac{K_n'(m_e r_0)}{K_n(m_e r_0)} + \rho_e \omega^2 n_0 \frac{J_n'(n_0 r_0)}{J_n(n_0 r_0)} = 0 \qquad \ldots (69)$$

where
$$m_e = k^2 - \frac{\omega^2}{c_e^2}.$$

The above dispersion relation is obtained with the assumption that $m_e^2 > 0$ or $\omega^2 < k^2 c_e^2$. This implies that motions outside the tube are radially evanescent, declining in r as we move away the tube. In other words, the environment of the tube is slightly disturbed by the wave motions. Indeed, there is no indication of the basic speeds of c_T and c_k, the characteristic speeds of tube's vibrations in the longitudinal ($n = 0$) and kink ($n = 1$) modes, and fluting mode ($n \geq 2$). But there are solutions with phase speeds close to c_T and c_k provided the tube is thin (i.e., $k^2 r_0^2 \ll 1$).

We have assumed $m_e^2 > 0$, but there is no restriction upon n_0^2. Waves with $n_0^2 < 0$ are termed as surface mode waves and those with $n_0^2 > 0$ are termed as body mode waves (Roberts 1990).

4. Damping of MHD waves

When the fluid is not perfectly conducting, but has a finite conductivity, or if viscous effects are present the MHD oscillations will be damped. Denoting the kinematic viscosity (viscosity divided by mass density) of the fluid by η_k, and the magnetic viscosity by η_m the linearized set of equations are (Bittencourt 1988)

$$\frac{\partial \rho_1}{\partial t} + \rho_0(\nabla \cdot \mathbf{v}_1) = 0 \qquad \ldots (70)$$

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + c_s^2 \nabla \rho_1 + \mathbf{B}_0 \times (\nabla \times \mathbf{B}_1)/4\pi - \rho_0 \eta_k \nabla^2 \mathbf{v}_1 = 0 \qquad \ldots (71)$$

$$\frac{\partial \mathbf{B}_1}{\partial t} - \nabla \times (\mathbf{v}_1 \times \mathbf{B}_0) - \eta_m \nabla^2 \mathbf{B}_1 = 0. \qquad \ldots (72)$$

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Although for a compressible fluid the use of simple viscous force term $\rho_0\eta_k\nabla^2\mathbf{v}_1$ is not really allowed but it is expected to give the correct order of magnitude behaviour.

For plane wave solutions, the differential operators $\partial/\partial t$ and ∇ may be replaced, respectively by $-i\omega$ and ik so that the set of differential equations (70) to (72) become a set of algebraic equations. Thus, we have

$$\rho_1 = \rho_0(\mathbf{k}\cdot\mathbf{v}_1)/\omega, \qquad \ldots (73)$$

$$\omega\mathbf{v}_1 = (\rho_1/\rho_0)\, c_s^2\mathbf{k} + \mathbf{B}_0\times(\mathbf{k}\times\mathbf{B}_1)/(4\pi\rho_0) - i\eta_k k^2\mathbf{v}_1 \qquad \ldots (74)$$

$$\mathbf{B}_1 = -\mathbf{k}\times(\mathbf{v}_1\times\mathbf{B}_0)/(\omega + i\eta_m k^2). \qquad \ldots (75)$$

Equation (74), together with equations (73) and (75), gives

$$-\omega^2(1 + i\eta_k k^2/\omega)\,(1 + i\eta_m k^2/\omega)\,\mathbf{v}_1 + (1 + i\eta_m k^2/\omega)\; c_s^2(\mathbf{k}\cdot\mathbf{v}_1)\,\mathbf{k}$$

$$-\mathbf{v}_A\times[\mathbf{k}\times\{\mathbf{k}\times(\mathbf{v}_1\times\mathbf{v}_A)\}] = 0 \qquad \ldots (76)$$

where equation (18) for $\mathbf{v}_A$ has been used. Comparing equation (76) with the corresponding non-viscous equation we find similar result except that ω^2 must be multiplied by the factor $(1 + i\eta_k k^2/\omega)$ $(1 + i\eta_m k^2/\omega)$ and c_s^2 must be multiplied by the factor $(1 + i\eta_m k^2/\omega)$.

Case A : Alfvén waves—The dispersion relation (relation between ω and k) for the propagating transverse Alfvén waves in the viscous medium may be written as

$$k^2v_A^2 = \omega^2(1 + i\eta_k k^2/\omega)\,(1 + i\eta_m k^2/\omega)$$

$$= \omega^2[1 + i(\eta_k + \eta_m)\,k^2/\omega - \eta_k\eta_m k^4/\omega^2]. \qquad \ldots (77)$$

Let us assume that the convection terms corresponding to the kinematic and magnetic viscosity are small so that the last term in equation (77) may be neglected. Therefore

$$k^2v_A^2 \cong \omega^2[1 + i\,(\eta_k + \eta_m)\,k^2/\omega]$$

$$\cong \omega^2[1 + i\,(\eta_k + \eta_m)\,\omega/v_A^2] \qquad \ldots (78)$$

where k has been replaced by ω/v_A in the right-hand side, to a first approximation. Equation (78) now gives

$$k \simeq \frac{\omega}{v_A}[1 + i\,(\eta_k + \eta_m)\,\omega/v_A^2]^{1/2}$$

$$\simeq \frac{\omega}{v_A} + \frac{i\,(\eta_k + \eta_m)\,\omega^2}{2\,v_A^3}. \qquad \ldots (79)$$

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The positive imaginary part in the expression for $k(\omega)$ implies wave damping which may be seen from what follows. Writing $k = k_1 + ik_2$ (with k_1 and k_2 as real numbers) a plane wave propagating along z-axis may be represented by

$$\exp(ikz) = \exp(-k_2 z) \cdot \exp(ik_1 z). \quad \ldots (80)$$

The right-hand side of equation (80) represents a damped wave propagating along z-axis with wave number k_1 with its amplitude decreasing exponentially. The amplitude falls to $1/e$ of its initial value in a distance of $1/k_2$, called damping length for the wave. The damping of Alfvén waves increases rapidly with frequency or wave number ($k \propto \omega^2$), but decreases rapidly with increasing magnetic field intensity [c.f., equation (79)]. Further the damping increases with the fluid (kinematic) and magnetic viscosities.

Case B : Acoustic waves—The propagation of sound waves in a viscous medium takes place according to the following dispersion relation

$$k^2 c_s^2 = \omega^2 (1 + i\eta_k k^2/\omega). \quad \ldots (81)$$

Assuming the resistive and viscous correction terms to be small, we have

$$k = \frac{\omega}{c_s} + \frac{i\eta_k \omega^2}{2c_s^3} \quad \ldots (82)$$

which shows that damping (attenuation) of sound waves also increases rapidly with frequency but decreasing with increasing sound velocity. As expected, the attenuation increases with increasing fluid viscosity.

Case C : Magnetoacoustic waves—For longitudinal magnetoacoustic waves propagating across $\mathbf{B}_0$, the dispersion relation is of the form

$$k^2 c_s^2 (1 + i\eta_m k^2/\omega) + k^2 v_A^2 = \omega^2 (1 + i\eta_k k^2/\omega)(1 + i\eta_m k^2/\omega) \quad \ldots (83)$$

which is much more complicated than that of the non-viscous case, namely $\omega/k = (c_s^2 + v_A^2)^{1/2}$. Assuming, as usual, the kinematic, and magnetic viscosities to be small the term involving the product $(\eta_k \eta_m k^4/\omega^2)$ may be neglected. Now equation (83) gives

$$k^2 (c_s^2 + v_A^2) \cong \omega^2 \left[1 + \frac{ik^2}{\omega}\left(\eta_k + \eta_m \left(1 - \frac{c_s^2}{c_s^2 + v_A^2}\right)\right)\right] \quad \ldots (84)$$

Replacing k^2 by $\omega^2/(c_s^2 + v_A^2)$ in the right-hand side of equation (84), finally we get

$$k \cong \frac{\omega}{(c_s^2 + v_A^2)^{1/2}} + \frac{i\omega^2}{2(c_s^2 + v_A^2)^{3/2}}\left[\eta_k + \frac{\eta_m}{(1 + c_s^2/v_A^2)}\right] \quad \ldots (85)$$

which shows that the attenuation of magnetoacoustic waves also increases with frequency,

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kinematic and magnetic viscosities. It decreases with increasing magnetic field strength ($v_A \propto B_0$).

5. Survey of literature

Recently Narain & Ulmschneider (1990) reviewed entire literature including heating by magnetoacoustic waves up to the year 1989. To avoid repetition we review the work done in this field afterwards.

Musielak (1991) reviews theories of acoustic and MHD wave generation in subphotospheric convection zones and points out that more realistic theories should take into account the magnetic flux tubes of finite width with a non-uniform horizontal structure and the interaction of the magnetic flux tubes with the external medium. He concludes that it is still not clear how much energy is generated in the stellar convection zones and what is its role in the heating of stellar chromospheres and coronae.

Subphotospheric convection consists of a nearly uniform, warm diverging up flow in which cool, converging, filamentary down drafts is embedded. This convective flow generates acoustic waves in the non-magnetic regions and torsional, kink and sausage magnetic flux tube waves in regions where magnetic fields are present. As acoustic waves propagate upward, they reach a height (in the middle chromosphere) where the photospheric magnetic flux tubes cover the entire surface and gas pressure equals the magnetic pressure. At this height, about one-third of the acoustic flux is converted to a fast magnetoacoustic wave flux and the rest is reflected. These fast mode waves are severely reflected as they propagate upwards. They may suffer total internal reflection because of the increase in Alfvén speed with height. The difficulty in getting enough energy to the upper chromosphere and corona may be overcome by waves ducted along the magnetic flux tubes. Such waves are produced directly by the convective flow and may also be produced by coupling to the acoustic wave flux incident on the chromospheric magnetic canopy (Stein & Nordlund 1991). Dissipation of organized wave energy into thermal energy by viscosity, resistivity, and conductivity requires the development of small scale structure. The slow and fast modes dissipate by forming shocks. In a collisionless regime fast modes may dissipate by Landau damping. The nonlinear transfer of energy from one mode to another seems to play quite crucial role in the heating of chromosphere and corona.

Califano *et al.* (1990) study the general problem of wave propagation and absorption in non-uniform, magnetized plasmas, within the framework of normal mode analysis and incompressible MHD. Electrical resistivity is the sole dissipation mechanism considered by them. The aim has been to investigate the formation of small spatial scales that are prerequisite for an efficient dissipation and heating. They show the existence of a new class of resistive (non-resonant) solutions which are characterized by the explicit appearance of resistivity in their asymptotic form and by the formation of small scales over the entire inhomogeneous (non-uniform) region. This feature distinguishes them from the more familiar resonant solutions, that obey ideal asymptotic boundary conditions and develop large gradients only at particular spatial locations. The smallest damping length in resonant and non-resonant cases is comparable. Its lowest value could be as low as a fraction of the scale of non-uniformity. The damping length is found to decrease with increasing degree of Gegenbauer polynomials which represent various normal modes. The authors conclude that the non-resonant modes seem to be good candidates for heating the stellar plasma.

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Since the incompressible case is not suitable for stellar chromospheres and coronae Califano *et al.* (1992) attempted the compressible case which is closer to conditions existing in the stellar atmosphere. For simplicity, the normal mode analysis has been restricted to 1-dimensional case (i.e., slab geometry where the relevant physical quantities depend on a single coordinate normal to the magnetic field direction). As a consequence of the interaction of the propagating wave with the background plasma (a non-uniform medium) small spatial scales arise in a natural way. The non-uniform medium supports slow magnetosonic and shear Alfvén waves in presence of electrical resistivity as the dissipative mechanism. The fast mode waves are not found to exist in this case. Under isothermal conditions the slow modes propagate with very little damping and hence do not appear to be likely candidate for the resistive heating of the stellar atmosphere.

To provide additional support to the existence of small spatial scales numerical simulation has been attempted by Malara *et al.* (1992), within the framework of incompressible MHD. For shear Alfvén waves with phase mixing as the damping mechanism the existence of small spatial scales is confirmed. Numerical simulation for the more physical compressible case might confirm the conclusions arrived at by these authors, more convincingly.

Due to density discontinuity on the loop surface, there can be a magnetosonic surface mode with evanescent radial vector. This feature makes a surface wave a good candidate for coronal loop heating. Since the frequency of the magnetosonic wave falls within the shear Alfvén continuum, energy dissipation by phase mixing can take place. Following Assis & Tsui (1991 a, b) the dispersion relation of the fast surface wave is

$$\omega_f^2 = k_z^2(B_1^2 + B_2^2)/[4\pi(\rho_1 + \rho_2)] \qquad \dots (86)$$

where the subscripts 1, 2 represent physical quantities in the region $x < 0$ and $x > 0$. The surface waves frequency ω_f falls away between two Alfvén frequencies $\omega_{A1} = B_1/(4\pi\rho_1)^{1/2}$ and $\omega_{A2} = B_2/(4\pi\rho_2)^{1/2}$. For reasonable coronal parameters they find a heating rate of order 10^{-4} erg cm^{-3} s^{-1} which is about 10% of the required heating rate. Therefore these waves are not principal source for loop heating.

Davila & Chitre (1991) and Chitre & Davila (1991) propose that the resonant absorption and subsequent dissipation of acoustic waves impinging on the chromospheric magnetic canopy could be a viable process for heating the solar chromospheric layers. They examine this idea by assuming the chromospheric layers to be adiabatic, inviscid, behaving as perfectly conducting ideal fluid that is stratified under a constant gravitational field in the vertical $\hat{z}$ direction and is pervaded by a non-uniform magnetic field $\mathbf{B}_0 \simeq [B_{0x}(z), B_{0y}(z), 0]$. With Alfvén speed $v_{Ax} \simeq 10^6$ cm s^{-1} $\simeq c_s$, $k_x^2/k_y^2 \simeq 0.5$, the angular frequency $\omega \simeq 10^{-2}$ s^{-1}, initial mass density $\rho_0 \simeq 10^{-12}$ g cm^{-3} and rms velocity amplitude $\simeq 2 \times 10^5$ cm s^{-1} they get a heating flux of 1×10^5 erg cm^{-2} s^{-1}. The scale height of the canopy is taken to be about 1000 km. This heating flux is sufficient to account for the observed radiation losses in the upper chromosphere. The field configuration of low-lying loops in the chromospheric magnetic canopy is shown in figure 4. In order to account for the enhanced heating seen in vertical magnetic flux tubes in the solar network the proposed model needs modification.

Davila (1991) tries to explain convincingly some of the misconceptions in the field of resonant absorption. For example, incompressible models implicitly assume a plasma with thermal pressure of the order of the magnetic pressure. Because of this the incompressible limit is incompatible with the magnetic pressure dominated state of the solar corona. This

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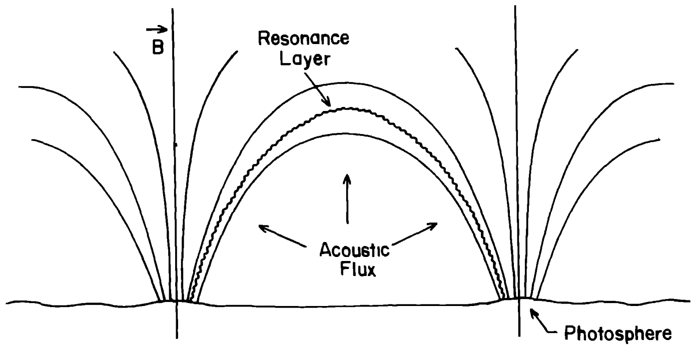

Figure 4. Typical magnetic field geometry for low-lying loops in the chromospheric network.

can lead to incorrect conclusions regarding the relationship between the heating rate and observable parameters like plasma velocity near the resonance layer. EUV observations of the turbulent power spectrum at the base of the corona are not available but they are urgently required. Although the heating rate is independent of the dissipation mechanism for reasonable values of the coefficients, the amplitude of the velocity inside the resonance layer and the width of the layer, both depend on the magnitude of the dissipation coefficient. He concludes that, in spite of some unresolved issues, the observations of the heating rate and non-thermal line broadening in the solar corona are consistent with heating by resonance absorption mechanism.

Erdélyi & Marik (1991) calculate semi-theoretically the temperature as a function of height (above temperature minimum in chromosphere) for weak magnetoacoustic shock waves of various periods (50s-300s). They find that the temperature increases with height, in agreement with observations.

Hasan (1991) studies heating in intense magnetic flux tubes using numerical techniques. His results show that overstable oscillations set up in the tube, are probably unimportant in heating the chromosphere and corona.

The propagation and damping of slow mode MHD shock waves in a cylindrical magnetic flux tubes embedded in a field-free homogeneous fluid has been studied by Mas & Insertis (1991) in the thin flux tube approximation, analytically. An expression for the energy losses in the weak shock case has been derived. Numerical results are not exhibited.

Ruderman (1991) has investigated the propagation of a MHD surface wave on a single magnetic surface in a cold plasma in presence of ion viscosity. For typical coronal conditions the ion collision interval is of the order of 2s therefore for MHD description to be valid the disturbances are assumed to have periods greater than 2s. It is further assumed that the wavelength of the disturbances is much smaller than the scale height over which the medium can be considered homogeneous. This assumption restricts the characteristic disturbance periods to values smaller or of the order of one minute.

In the framework of linear theory the damping length (the distance in which the wave amplitude decreases e-fold) is given by

$$L_d \approx 5 \times 10^{-17}\, v_{A+}^3 T^2 \Gamma_*^{-1} \text{ cm} \qquad \dots (87)$$

where v_{A+} is the Alfvén speed in the region $z > 0$ and $\Gamma_* = \Gamma\rho_0 + (v_{A+}/\mu)$ is the dimensionless damping decrement, with μ as viscosity coefficient. T is the wave period in seconds. With

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$B \approx 10$G, $T \approx 30$s and $\Gamma_* \approx 0.1$, $L_d \approx 1.5 \times 10^6$ km. This implies that the damping of the wave in the corona is not significant.

In case of nonlinear wave damping the distance, L_n, in which the wave energy flux decreases e^2 fold is, given by

$$L_n \approx 0.115\, T^2 A^{-1} W^{-1} \text{ cm} \qquad \ldots (88)$$

where A is the amplitude of velocity oscillations and W is some function of plasma parameters. With $T = 60$s, one gets $L_n \approx 5 \times 10^5$ km. Thus the nonlinearity is a very effective mechanism for enhancing the wave damping.

Edwin & Zhelyazkov (1992) re-examine the dissipation of ducted, fast magnetoacoustic waves by ion viscosity and electron heat conduction in a radiating, optically thin atmosphere. They show that damping lengths do not become unacceptably large if the magnetic field strength $B \geq 10$G, as stated by Gordon & Hollweg (1983). Their table II shows that it should be possible to heat strong magnetic regions such as coronal active region loops as well as quiet and weak field regions. In agreement with Gordon & Hollweg they find that most of the wave energy loss occurs inside the duct, that is in the denser region. Further there is significant dissipation of energy, in the duct's exterior, of wave travelling along and near to the surface of the duct. Reasonable dissipation lengths (i.e., about 2-3 wavelengths) were found for periods 5-15s and wavenumbers of 2-5 $\times 10^{-8}$ cm^{-1} for 10G magnetic fields and particle densities of 5×10^9 cm^{-3}. For the coronal data of Sahyouni *et al.* (1987) the lower bound should be 74s instead of reported 11.8s. A few more errors of the paper of Sahyouni *et al.* (1987) are also pointed out.

6. Conclusions

Our study leads us to the following conclusions :

1. The estimation of MHD flux generation suffers from significant uncertainties because of our poor knowledge of turbulence occurring in stellar convective zones and also because of the neglect of the interaction of the generated waves with the turbulence. Correspondingly the estimation of heating in stellar chromospheres and coronae is uncertain. Realistic numerical modeling seems quite promising for this purpose.

2. The interaction and mode coupling, in which other wave modes are converted to slow and fast (magnetosonic) modes which can dissipate by forming shocks, is expected to play an important role in the stellar atmosphere because some of the modes are totally reflected before reaching chromospheric and coronal heights.

3. Formation of small spatial scales in inhomogeneous stellar plasmas, which is necessary for the efficient dissipation of waves, now appears to be an established fact.

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