Bull. Astr. Soc. India (1993) **21**, 421-423

Sourceless Abelian gauge string in a Robertson-Walker universe with general spatial curvature

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Abstract. A model of a sourceless Abelian gauge string was examined by J. R. Morris (1991) in a Robertson-Walker universe with a flat 3-space. We generalise this result to the case of positive or negative spatial curvatures. We find that creation or destruction of the gauge string is possible only if the spatial curvature is zero or positive but no nonstatic solution of the string satisfies the boundary conditions in a Robertson-Walker model with negative spatial curvature.

Key words : gauge string—spatial curvature—cosmological models

1. Introduction

Some cosmologists believe that cosmic strings left over as relics of the big bang may have acted in the early universe as seeds of the galaxy formation. Since our universe is expanding Morris (1991) considered sourceless Abelian gauge string in a Robertson-Walker cosmological model with zero spatial curvature. We generalise this result by taking non zero spatial curvature :

$$ds^2 = dt^2 - \frac{S^2(t)}{\left\{1 + \frac{K(r^2 + z^2)}{4}\right\}^2} (dr^2 + r^2\, d\phi^2 + dz^2) \qquad \dots (1.1)$$

where $S(t) \sim t^\alpha$ $(0 \leq \alpha < 1)$.

For mathematical simplicity we confine our attention to the equatorial plane $z = 0$. On $z = 0$ plane we obtain the following generalisation of the equation obtained by Morris (1991).

$$\ddot{P} + \frac{\dot{S}}{S}\dot{P} - \frac{(1 + \frac{1}{4}kr^2)^2}{S^2}\left[P'' - \frac{P'}{r}\frac{(1 - \frac{1}{4}kr^2)}{(1 + \frac{1}{4}kr^2)}\right] = 0. \qquad \dots (1.2)$$

We have a singular gauge string restricted to the z axis with an associated flux :

$$\Phi_s(t) = \frac{2\pi}{e}[1 - P(0, t)]. \qquad \ldots (1.3)$$

We assume separability of variable like Morris. Hence

$$P(r, t) = F(r)\, G(t). \qquad \ldots (1.4)$$

F and G satisfy the following two equations

$$F'' - \frac{F'}{r}\frac{(1 - \frac{1}{4}kr^2)}{(1 + \frac{1}{4}kr^2)} - \frac{m^2 F}{(1 + \frac{1}{4}kr^2)^2} = 0 \qquad \ldots (1.5)$$

$$\ddot{G} + \frac{\dot{S}}{S}\dot{G} - \frac{m^2}{S^2} G = 0 \qquad \ldots (1.6)$$

where m^2 is the separation constant and may in general be positive, zero or negative.

2. Solution

Case I : $k = -1$

The solution of (1.5) and (1.6) consistent with the boundary conditions is

$$P(r, t) = P_0 + P_1\left(1 - \frac{4}{r^2}\right)^m F_1\left(a, b, c, \frac{4}{r^2}\right) \exp\left\{\frac{\Gamma}{1 - \alpha}\left[1 - \left(\frac{t}{t_0}\right)^{(1-\alpha)}\right]\right\} \qquad \ldots (2.1)$$

where $m > 0$, F_1 is the solution of the hypergeometric equation, P_0 and P_1 are constants and

$$a = \frac{(m + 1) \pm \sqrt{(m + 1)(5m + 1)}}{2}$$

$$b = \frac{(m + 1) \mp \sqrt{(m + 1)(5m + 1)}}{2}$$

$$c = 1.$$

Here we have

$$P(0, t) = P_0 \qquad \ldots (2.2)$$

and the string flux is given by

$$\Phi_s = \frac{2\pi}{e}(1 - P_0) \qquad \ldots (2.3)$$

which is time indepedent.

Case II : $k = +1$

In this case the solution is

$$P(r, t) = P_0 + P_2 \ln (4 + r^2) + P_1\left(1 + \frac{4}{r^2}\right)^n F_1\left(a, b, c, 1 + \frac{4}{r^2}\right)$$
$$\times \exp\left\{\frac{\Gamma}{1-\alpha}\left[1 - \left(\frac{t}{t_0}\right)^{(1-\alpha)}\right]\right\} \qquad \ldots (2.4)$$

where $n^2 = -m^2$, P_2 is another constant and $a = n + 1$, $b = n$, $c = 2n + 1$. Here

$$P(0, t) = P_0 + P_2 \ln 4 + P_1 \exp\left\{\frac{\Gamma}{1-\alpha}\left[1 - \left(\frac{t}{t_0}\right)^{(1-\alpha)}\right]\right\} \qquad \ldots (2.5)$$

and the string magnetic flux is given by

$$\Phi_s(t) = \frac{2\pi}{e}\,[P_0 + P_2 \ln 4 + P_1\, G(t)]. \qquad \ldots (2.6)$$

We may combine the first two constant terms and call it P_0. Since $G(t_0) = 1$ and $G(\infty) = 0$ we can have either creation or destruction of the string by adjusting the values of P_0 and P_1.

4. Conclusions

We found from above that creation or destruction of the gauge string is possible if the spatial curvature is zero or positive but no nonstatic solution of the string is possible when the spatial curvature is negative.

The mathematical details of the problem will be published elsewhere.

Acknowledgements

Our thanks are due to DST and UGC for financial assistance.

Reference

Morris J. R., 1991, Phys. Rev., 44D, 1015.