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Production rate of H_2O molecules as indicator of the surface temperature of cometary nucleus

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Abstract. The technique outlined by Feynman to compute the number of molecules evaporating from a liquid surface has been extended to compute the sublimation rate of H_2O molecules from the surface of a cometary nucleus and hence its surface temperature. The application of this technique to a cometary nucleus of typical radius 2.5 km and having a production rate of H_2O molecules as 10^{29} per second yields 263°K as its surface temperature using iteration method. The error involved is estimated as ± 5°K if the radius of the nucleus is changed by 0.5 km from the adopted value of 2.5 km.

Key words : sublimation—comet—molecules

1. Introduction

Feynman, Leighton & Sands (1963) has outlined a technique to estimate the number of molecules evaporating from the liquid surface. The molecule about to evaporate has to overcome the strong attractive forces of the neighbouring molecules on the liquid surface and thus needs extra-energy. In the case of sublimation from a solid surface, the molecule is attracted very strongly by the neighbouring molecules. The work done by the molecule against these attractive forces resulting in its sublimation from the solid surface is thus greater than that if the molecule is to evaporate from the liquid surface.

The work done by the escaping molecule from its liquid or solid state (crystalline/amorphous form) is its heat of vapourization/sublimation which depends on temperature. Thus due consideration must be given to the temperature while computing the total heat content required for sublimation of molecules from the solid phase. We have extended the technique of Feynman to compute the rate of sublimation (per square centimeter per second) of H_2O molecules from the surface of a cometary nucleus. Knowledge of the surface temperature of the cometary nucleus is vital in the modelling of the cometary coma. The production (sublimation) rate and initial velocity at sublimation of the parent molecules directly depend on the surface temperature of the cometary nucleus.

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1992BASI...20..297S

With a view to have a better understanding of the cometary coma, an attempt has been made to compute the surface temperature of the cometary nucleus from the observed rate of production of H_2O (parent) molecules for a particular comet.

2. Sublimation rate of H_2O molecules

The number N of H_2O molecules sublimating per unit area per second from the surface of a cometary nucleus is given by

$$N = \frac{1}{A} \frac{V}{D} e^{-W/KT} \qquad \ldots (1)$$

where V is the average velocity of the molecule at temperature T (°K); A the cross sectional area (πr^2) of the molecule of radius r; D the molecular diameter of the escaping molecule or the thickness of the outermost layer of such embedded molecules on the surface of the cometary nucleus; W the heat of sublimation of the H_2O molecule from the solid phase; K the Boltzmann constant = 1.38×10^{-16} erg/°K; and T the surface temperature of the cometary nucleus.

We have applied this technique to estimate theoretically the sublimation rate of H_2O molecules from the surface of the nucleus of a comet, assuming a typical value of 2.5 km for the radius of its nucleus (Biermann, Giguere & Huebner 1982).

We have used in our computations the following data pertaining to water molecule :

(i) Diameter = 2.72 Å
(ii) Average velocity = $(3KT/m)^{1/2}$; m = mass of H_2O molecule
(iii) Heat of sublimation ($H_{subl.}$) = 12.173 Kcals/gm mole (Petrucci 1989).
(iv) Specific heat of water ice below 273°K at various temperatures. This data taken from CRC Handbook of Chemistry and Physics (1979-89) is shown in figure 1.

For various values of T and for appropriate values of W, the sublimation rates of H_2O molecules from the surface of the cometary nucleus are computed from equation (1). These are tabulated in table 1 and are shown in figure 2.

3. Surface temperature by iteration method

The procedure adopted by us in the computation of the surface temperature of a cometary nucleus by the iteration method is as follows:

1. We have taken a typical value of 2.5 km for the radius of the cometary nucleus. We have considered only the sunlight surface of the nucleus ($2\pi \times$ square of radius).
2. We have taken for an initial value of W, the heat of sublimation ($H_{subl.}$) = 12.173 Kcals/gm mole for water ice at 273°K.
3. Using data pertaining to H_2O molecule, we have computed the surface sublimation rates for various values of T ranging from 240°K to 275°K (figure 2, I).
4. For the observed production rate of 10^{29} sec^{-1} of H_2O molecules for a given comet, we thus obtained a value of 262°K for T.

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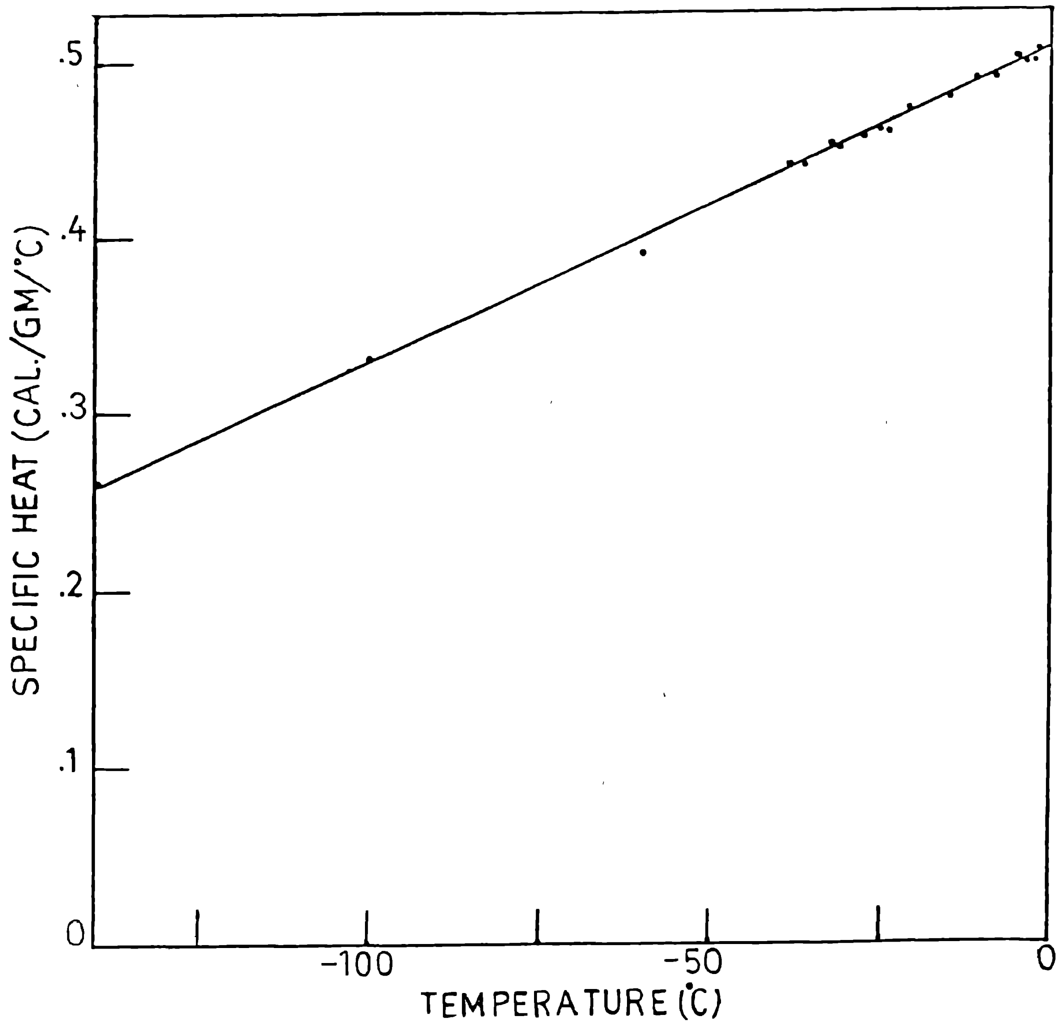

Figure 1. Specific heat of water-ice at various temperatures below 273°K (CRC Handbook of Chemistry and Physics, 1979-89).

5. We have taken the first next value for $W = H_{\mathrm{subl}} + \Delta H_1$, where ΔH_1 is the heat required to raise the temperature of 1 gm mole of water ice from 262°K to 273°K.
6. Repeating steps (3) and (4) we obtained T = 264°K (figure 2, II).
7. For the second next value for W, we have taken $W = H_{\mathrm{subl}} + \Delta H_2$, where ΔH_2 is the heat required to raise the temperature of 1 gm mole of water ice from 264°K to 273°K.
8. Repeating steps (3) and (4) again, we obtained T = 263.4°K (figure 2, III).

The rapidity with which the values of T converge to 263°K by the above iteration process, suggests that the value of the surface temperature is very close to 263°K.

3.1. Effect on the computed value of T due to variation in the radius of the cometary nucleus (r_N)

Once the value of the surface temperature T is fixed by the method discussed in section 3 (implying that W is fixed), we may evaluate the error in T if the radius of the cometary

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1992BASI...20..297S

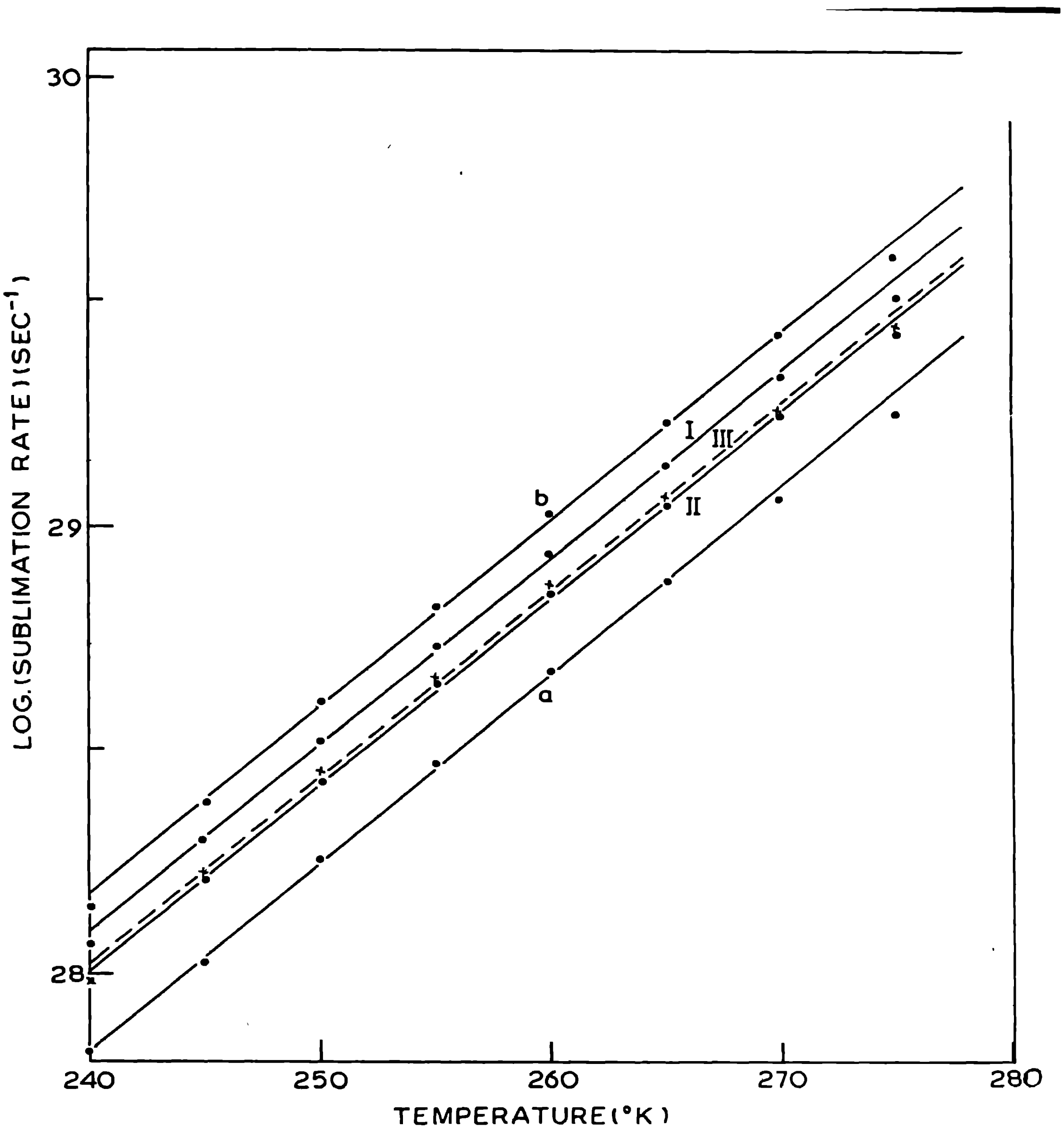

Figure 2. Variation of sublimation rate of H_2O molecules from the sunlit surface of cometary nucleus (radius : 2.5 km) with surface temperature.

nucleus is different from 2.5 km. The graphs marked (a) and (b) in figure 2 are obtained (table 1) if the radius of the cometary nucleus is taken as 2 km and 3 km respectively. For the same production rate of 10^{29} sec^{-1} of H_2O molecules, the respective values of T from the graphs (a) and (b) are 268°K and 259°K.

4. Conclusion

The observed production rate of the parent molecule (*e.g.*, H_2O, NH_3, etc.) may be used as a tool to estimate the surface temperature of cometary nucleus at the instant of observation. The error involved in the computed surface temperature by the above technique would be

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1992BASI...20..297S

Table 1. Variation of sublimation rate (sec^{-1}) of H_2O molecules with temperature

S.No.	Temp (°K)	Velocity (10^4 cm/sec)	*N* $W = H_{subl}$ = 12.173	*N* $W = H_{subl.} + \Delta H_1$ =12.271	*N* $W = H_{subl.} + \Delta H_2$ = 12.254	$W = 12.254$ $r_N = 2$ km	$W = 12.254$ $r_N = 3$ km
1.	240	5.766	1.168 (28)	9.493 (27)	9.855 (27)	6.307 (27)	1.419 (28)
2.	245	5.826	1.986 (28)	1.622 (28)	1.682 (28)	1.077 (28)	2.423 (28)
3.	250	5.885	3.308 (28)	2.713 (28)	2.812 (28)	1.800 (28)	4.049 (28)
4.	255	5.944	5.400 (28)	4.485 (28)	4.607 (28)	2.949 (28)	6.635 (28)
5.	260	6.002	8.663 (28)	7.157 (28)	7.408 (28)	4.741 (28)	1.068 (29)
6.	265	6.059	1.364 (29)	1.131 (29)	1.170 (29)	7.489 (28)	1.685 (29)
7.	270	6.116	2.112 (29)	1.758 (29)	1.818 (29)	1.163 (29)	2.617 (29)
8.	275	6.173	3.221 (29)	2.690 (29)	2.779 (29)	1.779 (29)	4.002 (29)

N : Number of sublimating H_2O molecules per second from the sunlit surface of cometary nucleus of radius 2.5 km.
W : In units of kcals/gm mole.
$q(p) : q \times 10^p$.

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1992BASI...20..297S

within ± 5°K if the value of the radius of the cometary nucleus under investigation differ by 0.5 km from the assumed value of 2.5 km.

The above method assumes uniform outgassing from the sunlit cometary nucleus and hence should be used with caution. The observed anisotropy of the outflowing gases from the cometary nucleus, has been interpreted as and given support to the existence of the spotty "active regions" on the cometary surface. The temperature derived by the above iteration method by taking into account the area of the active regions only, would then be the temperature of the active regions wherefrom the H_2O molecules are outflowing.

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