Bull. Astr. Soc. India (1992) **20**, 21-40

Chemical evolution of the solar neighbourhood*

N. C. Rana**
Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007

Received 1992 May 21; accepted 1992 July 28

Abstract. A simple model of chemical evolution of the solar neighbourhood is presented with reference to evolution and frequency distribution of a particular element, namely iron. The history of star formation in the solar neighbourhood is consistent with no variation of its rate by more than a factor of two. The calculated and observationally estimated age-metallicity relation agree reasonably well. The initial mass function of stars in the solar neighbourhood is found to follow a power law for masses greater than about 1.5 $M_\odot$ with an index of about -1.67.

Key words : chemical evolution—solar neighbourhood—age-metallicity relation—mass functions—star formation rates—*G*-dwarf problem

1. Introduction

The solar neighbourhood is defined to be a small part of the Milky Way, the Galaxy, extending about a kiloparsec region centred about the sun. The Galaxy consists of a disc, a spheroid and a halo, densely populated by successively older stars and star clusters. The Disc supports the most organised motion of the young stars and gas from which stars of all mass range form and evolve.

The time scale on which stars are formed depends on the physical conditions of the interstellar medium (ISM), namely, whether there is a lot of magnetic field or turbulence trapped inside or not, the size, density and temperature of the cloud, etc. However, this time scale usually varies from at the most one or two Gy to at the least one My or even less than that. Then, the star moves on to the main sequence, during which the star burns hydrogen in its central region. The main sequence lifetime of a star depends crucially on its mass, and to lesser extent on its initial chemical composition. Ordinary solar type of stars of G2V spectral class have a lifetime of about 14 Gy on the main sequence. This is comparable to the supposed age of the Disc, which is about 9-13 Gy. Stars less massive than the sun would have their main sequence lifetimes larger by a factor of about m^{-4}, where m is the mass of the star in units of the solar mass $M_\odot$. The stars more massive

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*Based on the review talk given at the 14th Meeting of ASI, held at PRL, Ahmedabad, during Jan 29-Feb 1, 1991.

**On leave from Tata Institute of Fundamental Research, Bombay.

than the sun have very short lifetimes, ranging from about 1 Gy for $m = 2.5$, 0.1 Gy for $m = 5.5$, 10 My for about $m = 20$, and to about 3.6 My for stars of mass about 100 $M_{\odot}$ (see figure 1, which is compiled from the tables given in Maeder & Meynet 1989, and Maeder 1990).

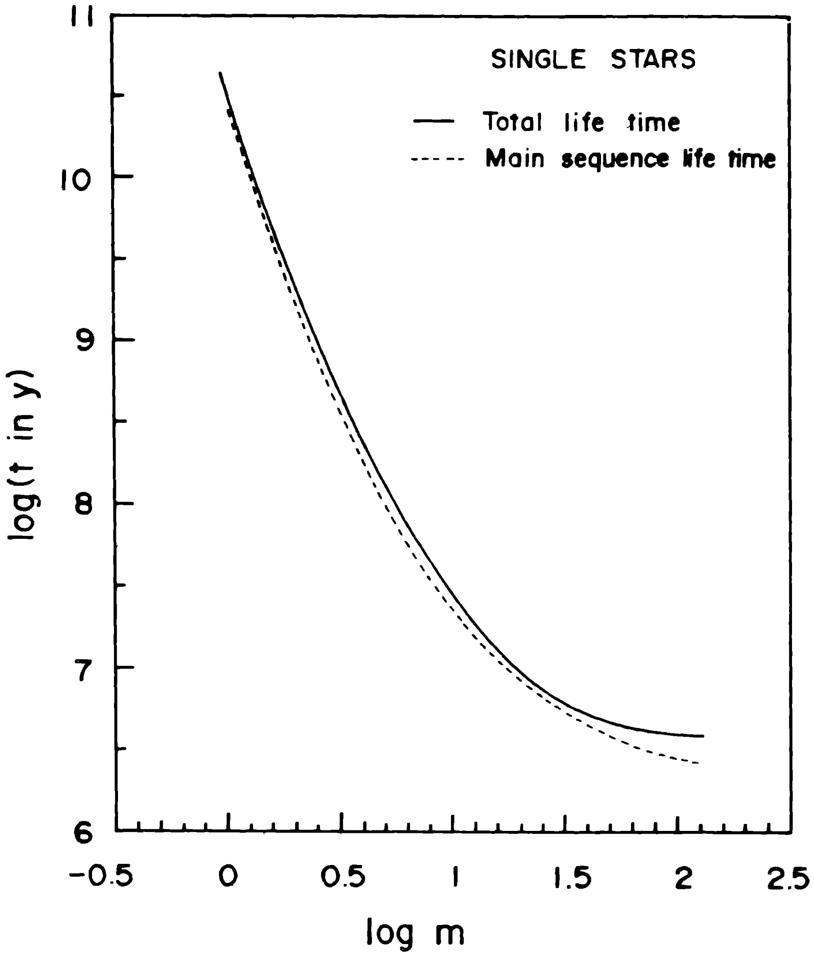

Figure 1. The mainsequence (solid line) and the total (dashed line) life time of stars in years as functions of the initial stellar mass m expressed in solar mass units, the data being compiled essentially from Maeder & Meynet (1989) and Maeder (1990).

Therefore, on time scales shorter than about 1 Gy, all stars more massive than about 2.5 $M_{\odot}$ are born and dead leaving behind a small white dwarf of mass 0.6-1.3 $M_{\odot}$ for the initial mass of the star in the range of 1.0-6.5 $M_{\odot}$, or a neutron star of mass 1.4-3 $M_{\odot}$ for its progenitor's initial mass 8-60 $M_{\odot}$, and possibly a black hole of mass 3 $M_{\odot}$ or greater for the more massive stars. Obviously, the progenitor star throws away a lot of its mass in the form of winds, nova-like outbursts and/or supernova explosions; and these material containing the newly processed elements in the core and outer shell regions of the stars eventually enrich the interstellar medium. Even if we assume a constant rate of occurrence of supernova in our Galaxy to be one per century and each throwing away $\sim$ 10 $M_{\odot}$ of matter containing a good fraction of newly processed elements, over the age of the Disc, the accumulated proportion of the heavy elements (called the abundance of heavy elements, say by mass fraction in this instance) will amount to a few percents of the total baryonic mass of the Disc. In figure 2, we have shown the latest model calculations by Maeder (1992) of the mass fractions of the newly processed and ejected elements and of the possible dead remnants of stellar evolution assuming an initial solar composition (namely, the mass fraction of heavy elements to be 0.02) of the star for a very wide range of initial mass of the stars (M). The disposed off gas into ISM from the evolved stars continue to enrich the ISM with increasing abundances of helium, and elements heavier than boron, which are mainly produced in stars, but the pool of hydrogen, deuterium, lithium and beryllium and boron in the ISM are continually destroyed during the stellar burning. So, the newer stars that are born in such environment of the ISM continually being enriched with helium and heavy elements (the latter also referred to as 'metals' in the astrophysicist's language) would have their initial composition changing gradually with time, and the cycle would go on and on. This entire process is by definition called the *chemical evolution of the Galaxy*.

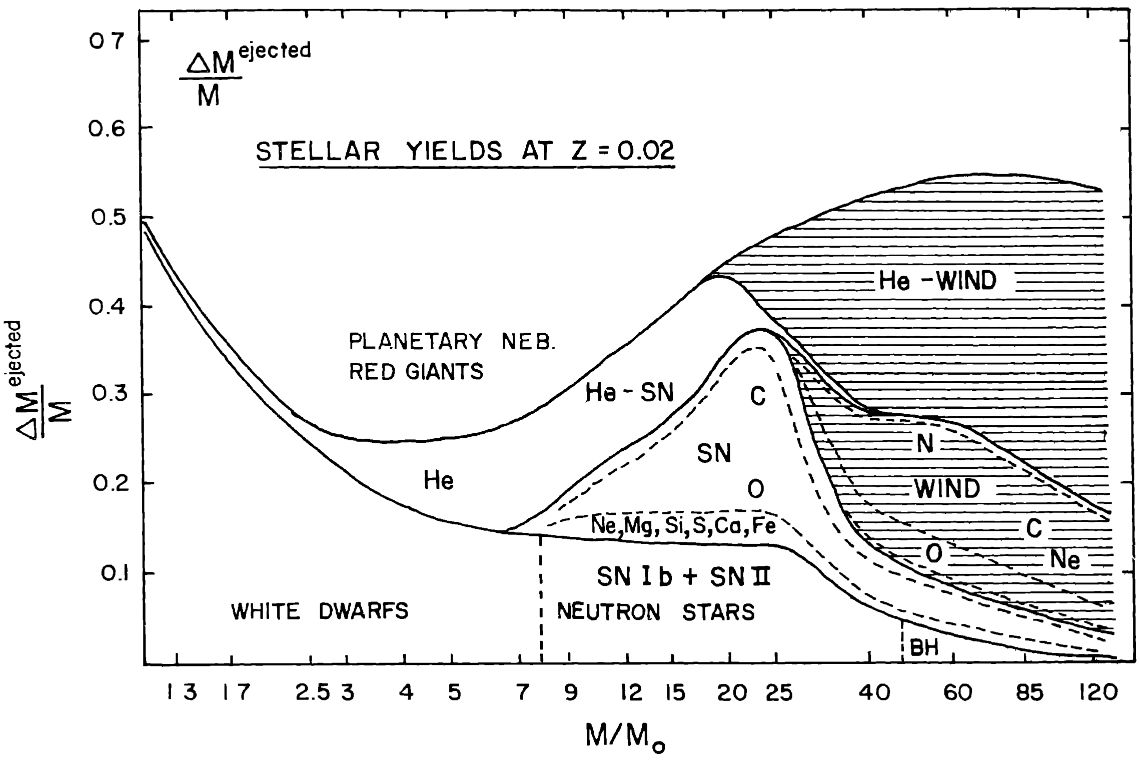

Figure 2. A representative set of model calculations of the mass fractions of newly ejected heavy elements and of the remnant compact object as functions of the initial stellar mass (taken from Maeder 1992).

In this article, we shall confine ourselves to the study of chemical evolution of the solar neighbourhood for the growth of the element iron, the one best studied so far. In real terms, it means dealing with the history of star formation in the solar neighbourhood, the evolution of the gas fraction, the mass functions of stars, the rates of occurrence of supernovae, pulsars and white dwarfs, the frequency distribution of stars with respect to their measured surface abundances of iron, and the evolution of iron in the ISM by its mass fraction with time.

2. The basic problems

There are a number of basic problems with the study of chemical evolution of the solar neighbourhood, or for that matter, of the whole Galaxy. Most of these problems did come up to surface from the very first systematic studies of the subject, that began in the early sixties (Schmidt 1963), the only difference being that we shall view them from the point of view of today's situation.

The first problem may be put as follows. Stars ought to have formed from gas. It was Schmidt (1959), who proposed that the star formation rate (SFR, Ψ) in a system should be a monotonically increasing function of its gas content (M_g); he particularly envisaged an empirical power law between the two quantities :

$$\Psi \propto M_g^k, \quad k \cong 1\text{-}2. \qquad \ldots(1)$$

It is therefore, natural to expect that the SFR would be continuously decreasing with time, as every time stars form, only the massive ones die, but the majority of the stars being less massive than the sun cannot simply die during the past 13 Gy, the supposedly age of the Galactic Disc. Their mass content is growing steadily with time at the expense

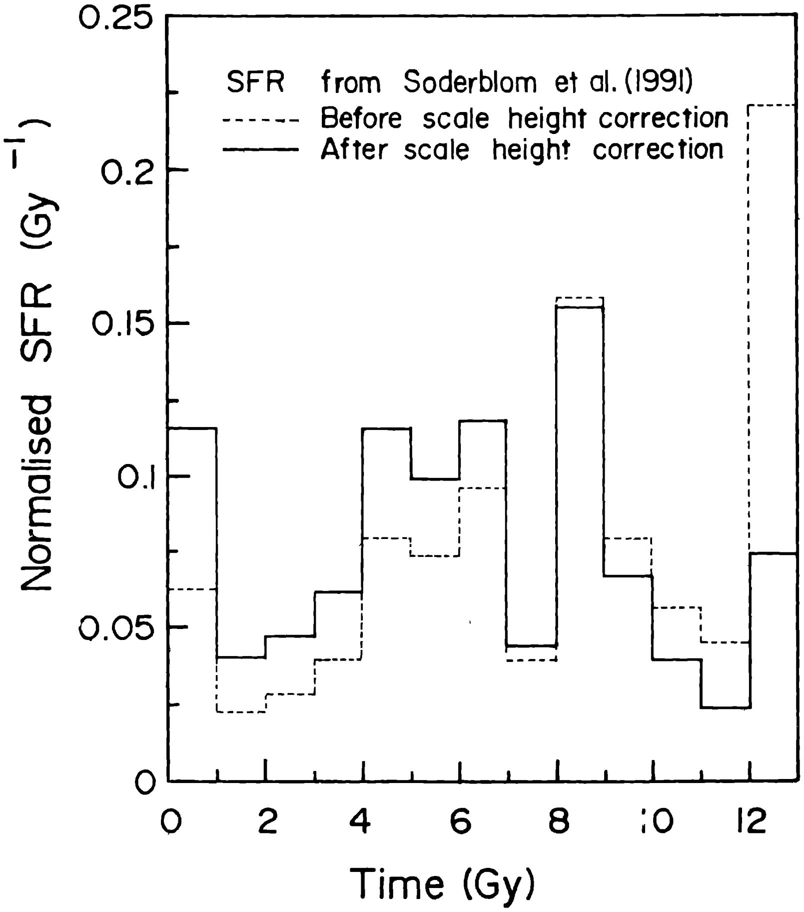

Figure 3. The time evolution of the star formation rate in the solar neighbourhood, for the assumed present age of the Galactic Disc to be 13 Gy, the dashed histogram being for the original volume-limited distribution due to Soderblom *et al.* (1991), the solid histogram being for the vertical scale height corrected SFR due to Rana & Basu (1992).

of the total reserve of the gas. In fact, we see hardly 10 to 15% of the total mass that is locked into stars, to be in the form of gas. One would then expect, if Schmidt's empirical law has some truth in it, the SFR should have fallen by a factor of 7 to 100 during the past 13 Gy. Now, from the observations, one estimates that there are approximately 60-100 billion stars in the Milky Way, and that the age of the Disc is about 13 Gy. Combining these two facts one would infer that the average rate of star formation in the Galaxy could be 5-8 stars/y. Since the mass of an averge star is about 0.8 $M_\odot$ or so, the average star formation rate in the Galaxy should be $\Psi \cong$ 4-6 $M_\odot$/y, which is roughly the *same* as the observationally estimated present day rate of star formation in the Galaxy (Turner 1984). In a more recent investigation by Soderblom *et al.* (1991), it is found that in the solar neighbourhood, the SFR has remained reasonably constant throughout the past 13 Gy (see figure 3). Then the question arises as to why the intuition-based law of Schmidt is so blatantly violated.

There is even some problem with our understanding of the present rate of star formation in the Galaxy. It is very simple to demonstrate that, given a gas cloud, the time scale of its contraction due to self-gravity τ_f is quite independent of its size or mass, and depends only on its mass density ρ_g, namely,

$$\tau_f \sim (G_N \rho_g)^{-1/2} \qquad \ldots(2)$$

where G_N is the Newtonian constant of gravitation. Since the general interstellar density of matter is no less than a few hydrogen atoms per cm^3, the above time scale of star formation is at the most O(10^7 My). But then, there is at present O (10^9 $M_\odot$) of gas located in the Disc of the Galaxy, which would imply the present day rate of star formation to be O (10^2 $M_\odot/y$), that is roughly two orders of magnitude higher than the observed rate. Or in other words, the problem is how to effectively delay the process of the current star formation in the ISM to a time scale of a few Gy, rather than a few tens of My. In the eighties, there have been a lot of discussion about how the star forming processes might be lengthened by the presence of magnetic field, turbulence and rotation in the interstellar clouds, nevertheless there still remains a gap between the observed and the theoretically expected rates of star formation (Goodman & Myers 1991).

The third problem relates to matching between the observed and the expected (due to a simple model) frequency distributions of surface abundance of iron in the local G-dwarf stars. It was Schmidt (1963) who first demonstrated that the so-called simple model of chemical evolution (which will be described in a later section) predicts far too many iron-poor stars than one observes in the solar neighbourhood, a problem canonically known as the G-dwarf problem. Later on, Pagel & Patchett (1975) took a bigger sample (volume-limited to within about 25 pc from the sun) and did a thorough job only to reconfirm the problem. Pagel (1989) reanalysed the same sample data (see figure 4 for the cumulative distribution, and later figure 9 for the differential distribution) using a better calibration of iron abundance (Cameron 1985), the problem has still been there. Figures 4a and 4b (adapted from Basu & Rana 1992a), illustrate how the simple model with zero initial metallicity (the solid curves) fail to represent the data. Further on, the vertical scale height corrections were applied to Pagel's sample to give what are referred to 'modified distribution' in figures 4a and 4b. This correction does certainly alleviate the severeness of the G-dwarf problem to a certain degree only (Somer-Larsen 1991; Rana 1991). Time and again, a number of solutions have been proposed, yet one

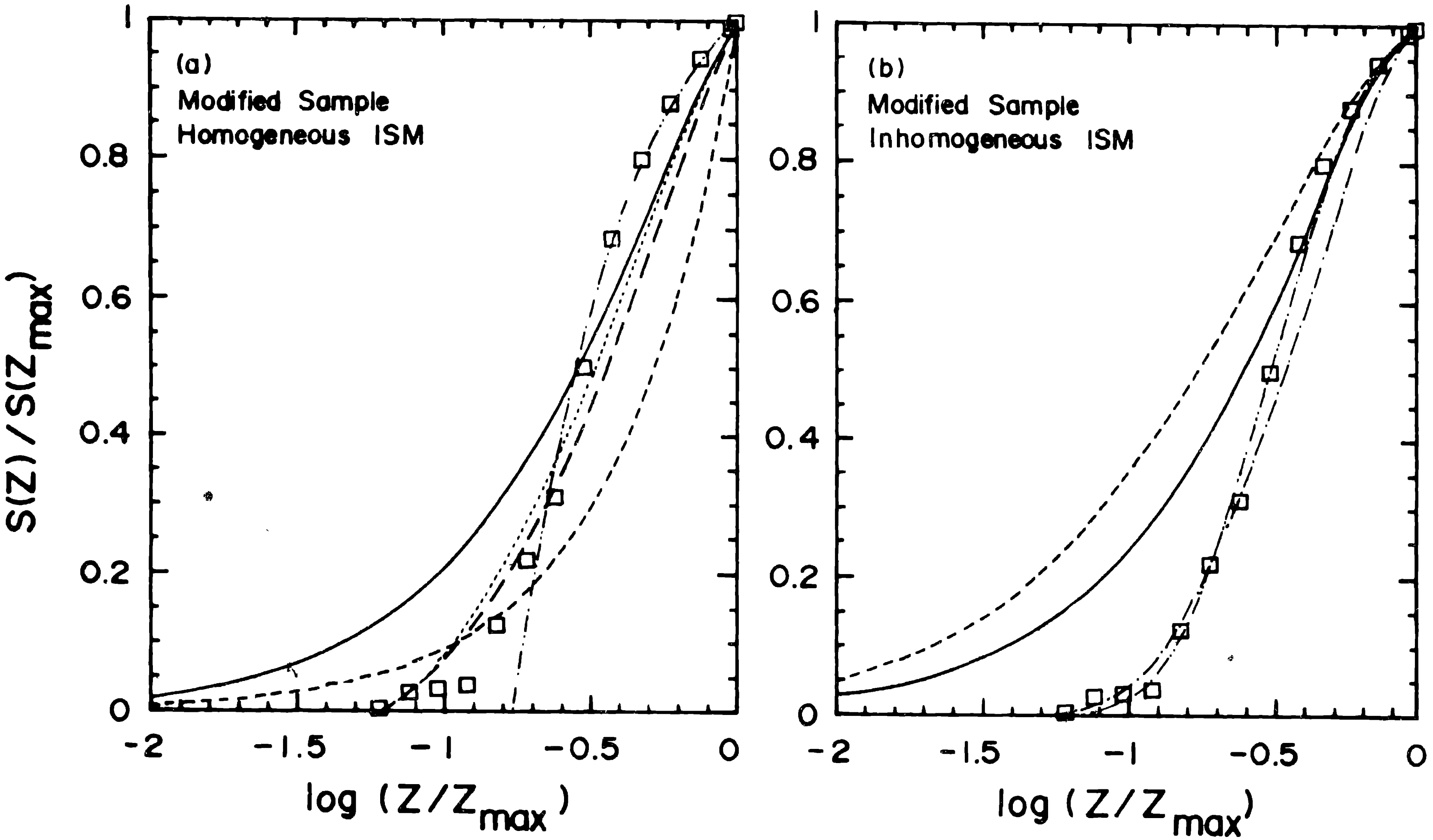

Figure 4. The famous G-dwarf problem. The boxes represent the cumulative frequency distribution of metallicity of the G-dwarf stars, after correcting Pagel's data base for the possible inflation of the vertical scaleheights. The prediction of the simple model with no initial metallicity is shown by the solid curve. Other curves represent other models including our preferred ones (the ones with double dots sandwitched between long dashes). The component figures (a) and (b) represent the best fits for chemically homogeneous and inhomogeneous models (adapted from Basu & Rana 1992a).

does not seem to feel very comfortable about any one of these solutions, which are, of course, shown by dotted, dashed, long-dashed and dot-dashed lines in figures 4a and 4b.

The fourth problem concerns the problem of local dark matter. It is found (Oort 1928; Bahcall 1989) that stars in the solar neighbourhood show more random motions (velocity dispersion) than the total visible matter can actually account for. About 50% of matter is found to be missing, and must exist in the form of invisible or *dark* matter.

Another key problem is related to the iron balance in the solar neighbourhood. Considering the observed rates of supernovae, the amount of iron released by the supernova events seems to exceed too far from a detailed model of chemical evolution of iron in the solar neighbourhood, that would suggest. The factor of discrepancy seemed to be about a factor of two, if not three. Even the initial growth rate of both oxygen and iron at the beginning of the disc formation seems to be far slower compared to the time scale of the formation of the disc itself. This problem has been time and again addressed by several authors in the field, and it's till very recently, no satisfactory solution existed.

3. A simple model of chemical evolution

The so-called simple model of chemical evolution has been presented time and again in the literature, but a tautology of the same cannot be avoided simply due to the fact that we will have to refer to the equations and relevant quantities a number of times in the present article. We follow Tinsley (1980), or Rana (1991), to write down the basic assumptions and the equations. The only difference of the present work with the other is that we would like to demonstrate that most of the problems stated in the previous section can be resolved within the framework of a simple model, and more importantly, in a consistent manner.

3.1. *The basic assumptions and equations*

The basic assumptions of the simple model of chemical evolution are as follows:

(i) The gas is chemically homogeneous at all times.

(ii) The region of the disc is closed from the radial and azimuthal directions, but open in the direction perpendicular to the plane of the disc.

(iii) The region has a non-zero initial metallicity and initial stellar and baryonic dark matter components. In the beginning, the disc was chemically homogeneous.

(iv) The initial mass function of stars is constant.

(v) The instantaneous recycling of gas is valid. This assumption is often referred to as the instantaneous recycling approximation (IRA), which, in short, means that all stars that are born are classified into two groups: one in which stars do not die within the time span of the disc evolution ($m \leqslant 1.0$), and the other in which stars live so short compared to the age of the disc that their evolution is considered to be almost instantaneous ($m \geqslant 2.0$). The evolution of stars in between the above range of mass do violate the assumption of IRA.

We define the gas fraction by mass (μ), stellar fraction by mass (σ), and the baryonic dark matter fraction by mass (δ) as $\mu = \Sigma_g/\Sigma_T$, $\sigma = \Sigma_* /\Sigma_T$, and $\delta = \Sigma_d/\Sigma_T$ respectively, where Σ_g, Σ_*, Σ_d and Σ_T are the surface densities of gas, star, baryonic dark matter, and total matter respectively. We denote the initial values by putting a subscript '0', and the current values by '1'. The basic differential equations governing the chemical evolution are then given by

$$\frac{d\Sigma_g}{dt} = -(1 - \mathrm{R})\Psi_s + \psi_f, \qquad \text{...(3a)}$$

$$\frac{d\Sigma_*}{dt} = (1 - R - D)\Psi_s, \qquad \text{...(3b)}$$

$$\frac{d\Sigma_d}{dt} = D\Psi_s, \qquad \text{...(3c)}$$

$$\frac{d(Z\Sigma_g)}{dt} = -Z(1 - R)\Psi_s + (1 - R)y\Psi_s + Z_f\Psi_f \qquad \text{...(3d)}$$

with the auxiliary condition that

$$\Sigma_T = \Sigma_g + \Sigma_* + \Sigma_d. \qquad \text{...(3e)}$$

Here Ψ_s and Ψ_f are the surface mass densities of the star formation rate, and of halo infall rate, respectively. Z and Z_f are respectively the interstellar and infalling gaseous halo abundances by mass fraction of the element under consideration (here iron is the element). The quantities R, D and y represent respectively the so-called 'return fraction of gas', 'baryonic dark matter fraction' per generation of stellar recycling, and the 'yield' of the element per generation of stellar recycling. These quantities are determined from the IMF and the details of stellar ejection into the ISM of the element considered. If IMF is constant and if the instantaneous recycling approximation (IRA) is assumed, the values of R, D, and y become constants, and would then be defined as

$$\begin{aligned} D &= \int_{-1.1}^{2} r(m)\,\xi(\log m)\; d\log m \Big/ \int_{-1.1}^{2} m\,\xi(\log m)\; d\log m, \\ (1 - R - D) &= \int_{-1.1}^{2} m[\xi(\log m) - \phi_{ms}(\log \mathrm{m})]\; d\log m \Big/ \int_{-1.1}^{2} m\,\xi(\log m)\; d\log m, \\ (1 - R)y &= \int_{-1.1}^{2} p_z(m)\,\xi(\log m) \Big/ \int_{-1.1}^{2} m\,\xi(\log m)\; d\log m, \end{aligned} \qquad \text{...(3)}$$

where $\xi(\log m)$ and $\phi_{ms}(\log m)$ are the IMF and the PDMF (the present day mass function) of the main sequence stars respectively, and $r(m)$ and $p_z(m)$ are respectively the remnant stellar mass and the total ejected mass of element considered, by a star of mass m (in unit of $M_\odot$). The theoretical stellar evolution models are supposed to provide $r(m)$ and $p_z(m)$ (see figure 2, for example). However, here R, D and y are being considered as constants.

3.2. *Yield of iron and other relevant quantities*

In the present work, we consider the growth of one element only, namely iron, because of our better understanding of the AMR, SFR, G-dwarf metallicity distributions, and the yield with reference to iron, than any other element. For example, in order to calculate the yield of iron per generation of stars in the solar neighbourhood, we take 0.6 $M_\odot$ of iron produced in each supernova of type Ia (Nomoto *et al.* 1984; Khokhlov *et al.* 1992), 0.15 $M_\odot$ and 0.075 $M_\odot$ in each SN of types Ib and II respectively (Nomoto *et al.* 1990). The SN type Ia is assumed to be due to either single star deflagration/detonation or slightly delayed detonation of the degenerate core of a progenitor star having its initial mass between 7.5 and 9.5 $M_\odot$ (Woosley 1991; Khokhlov *et al.* 1992), while stars having initial mass between 9.5 and 12 $M_\odot$ do not produce any iron, and progenitor mass range of 12 to 20 $M_\odot$ lead partly to SN Ib and SN II, and most stars having mass between 20 and 45 $M_\odot$ end up possibly exclusively as the type II SN (Nomoto *et al.* 1990). These are still very much model dependent quantities. We shall later consider another scenario of SN explosions of the type Ia. However, for the present, we take the above numbers in their face values, and use the observed ratios for the rates of occurrence of supernovae of types Ia, Ib, and II in the spiral galaxies of the Milky Way type to be 0.28(± 0.11) : 0.27(± 0.15) : 1.04(± 0.30) respectively from Evans *et al.* (1989).

Now, if the present day mass function (PDMF) of stars in the solar neighbourhood is taken from Basu & Rana (1992b), and if a constant rate of star formation over the past

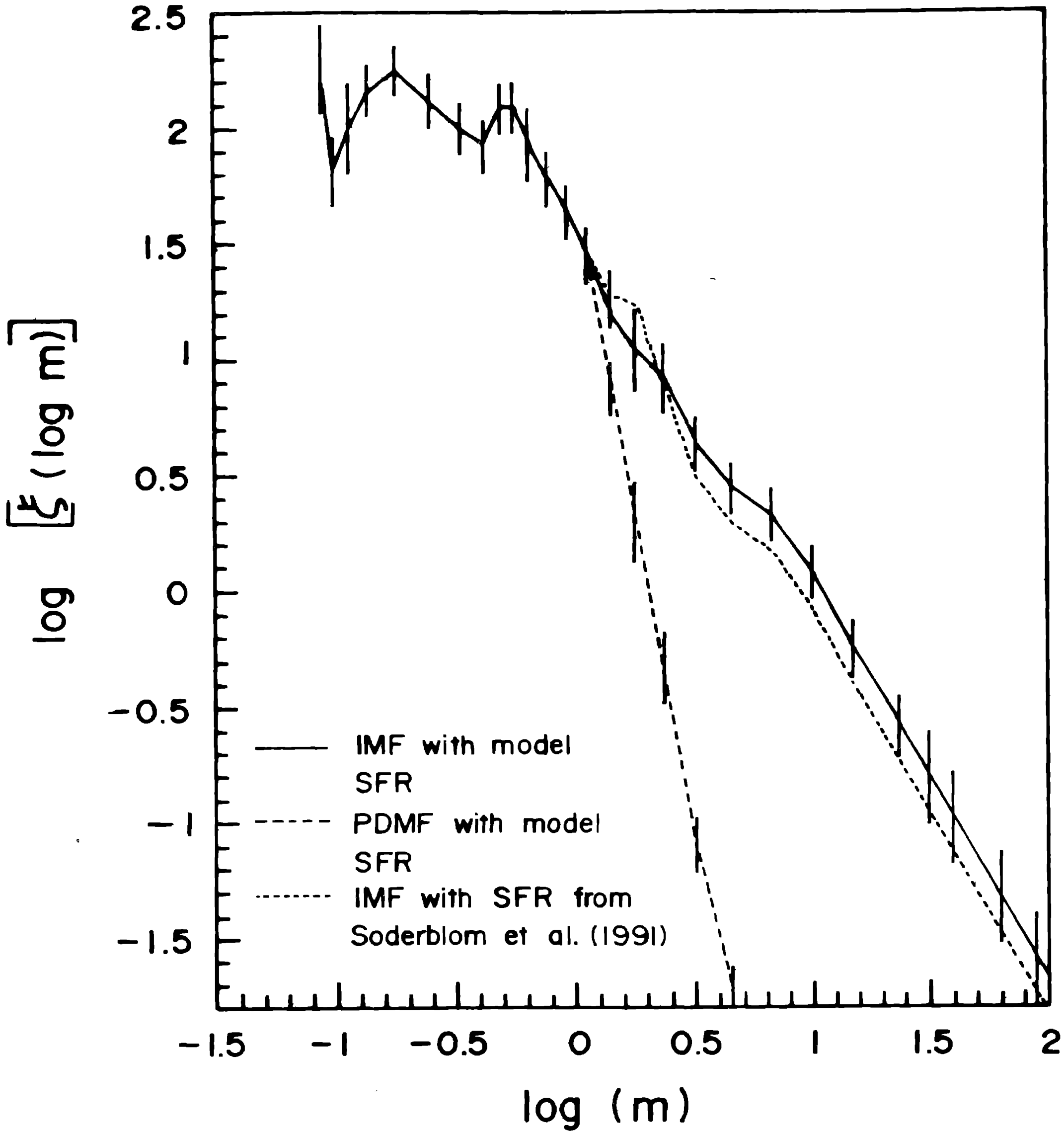

Figure 5. The present day mass function (PDMF) and the initial mass function (IMF) are shown. The PDMF is taken from Basu & Rana (1992a), and the IMF is calculated in this work in consistency with equations (4)-(6). The IMF corresponds to a power law at stellar mass greater than about 1.5 $M_\odot$, with an exponent of − 1.67.

13 Gy is assumed, it is possible to derive the-initial mass funciton $\xi(\log m)$. It is found that such an IMF reproduces correctly not only the observed relative ratios of the above mentioned types of SN progenitors, but also the values of the yield of iron $y = 0.565(Z_\odot)_{\mathrm{Fe}}$, the returned fraction of gas to the ISM per generation of star formation $R = 0.206$, the remnant fraction $D = 0.06$, and the total surface densities of stars $\Sigma_{\mathrm{IMF}} = 56.4^{+20}_{-15}$ $M_\odot$ pc^{-2} for those which are born so far, and $\Sigma_* = 39^{+12}_{-9}$ $M_\odot$ pc^{-2} accounting for those stars that are now present in the solar neighbourhood of the Disc (see figure 5).

As regards the gas component of the local interstellar medium, it consists of neutral and molecular hydrogen, helium, and small amounts of ionised hydrogen, heavier elements, CO and other molecules. There are also a few dust grains accounting for no more than about 2% of the gas content. Following Rana (1991), and supplemented further by Deul (1988)'s data on H I distribution, we take the total gas content to be about 6.6 $M_\odot$ pc^{-2}, which amounts to a fraction of 0.12-0.13(= μ_1) compared to the total surface density of matter $\Sigma_{\mathrm{T}} = 54(\pm 6)$ $M_\odot$ pc^{-2} (Gould 1990; Kuijken & Gilmore 1991). The self-consistence demands further that the required initial surface density of mass of gas in the solar neighbourhood to be 51.3 $M_\odot$ pc^{-2}, thus justifying the baryonic form of matter alone accounting for local problem of dark matter.

3.3. *The metallicity distributions, SFR, AMR, and vertical scale height corrections*

The metallicity distribution of stars in the sample of Pagel (1989) as well as the sample of stars used for the construction of the history of star formation in the solar neighbourhood by Soderblom *et al.* (1991) are essentially volume limited to within about 25-30 pc of the sun. Pagel's sample contains 133 stars, and the latter sample has 177 stars. In fact, for consistency, the PDMF derived in Rana & Basu (1992) suggests for about 150-200 unresolvable stellar systems appearing as G-dwarfs within the above mentioned volume.

We normalize both the samples to a total number of stars as unity, that is, they are expressed as cumulative stellar number fraction $S([\mathrm{Fe/H}])$ and $S(t)$ (with $t = 0$ at the beginning of the disc formation and $t = t_{\mathrm{d}} = 13$ Gy for the present epoch), and then graphically eliminate the variable S. This requires an *a priori* knowledge of the initial and terminal metallicities $[\mathrm{Fe/H}]_0$ and $[\mathrm{Fe/H}]_1$, which are by far not very definitely known quantities. From the population studies, one usually assumes an initial iron abundance in the solar neighbourhood of the Disc to be somewhere between $[\mathrm{Fe/H}] = -1.35$ dex and -0.80 dex (Morrison *et al.* 1990, for example), and the present interstellar abundance of iron characterized by that of the young open clusters, Hyades cluster in particular, corresponds to $[\mathrm{Fe/H}] = 0.12(\pm 0.06)$ dex (Nissen 1988). Thus, under the variations of these initial and final metallicities together with the usual $\sqrt{N}$ error in the statistical count of stars in the individual metallicity bins and an uncertainty of about ± 0.15 dex in the measurement of metallicity for individual stars, an age-metallicity relation is derived by Rana & Basu (1992). These are shown by points with cross marks in figure 6 with 1σ error limits. for comparison, the data of Twarog (1980), Carlberg *et al.* (1985), Nissen *et al.* (1985), Strobel (1991) and Meusinger *et al.* (1991), all being duly normalized to the adopted age of the Disc for the present work, namely 13 Gy, are also shown in figure 7. The remarkable feature about the AMR thus derived by Rana & Basu (1992) is that the growth of iron abundance is nearly exponential with respect to time (or equivalently,

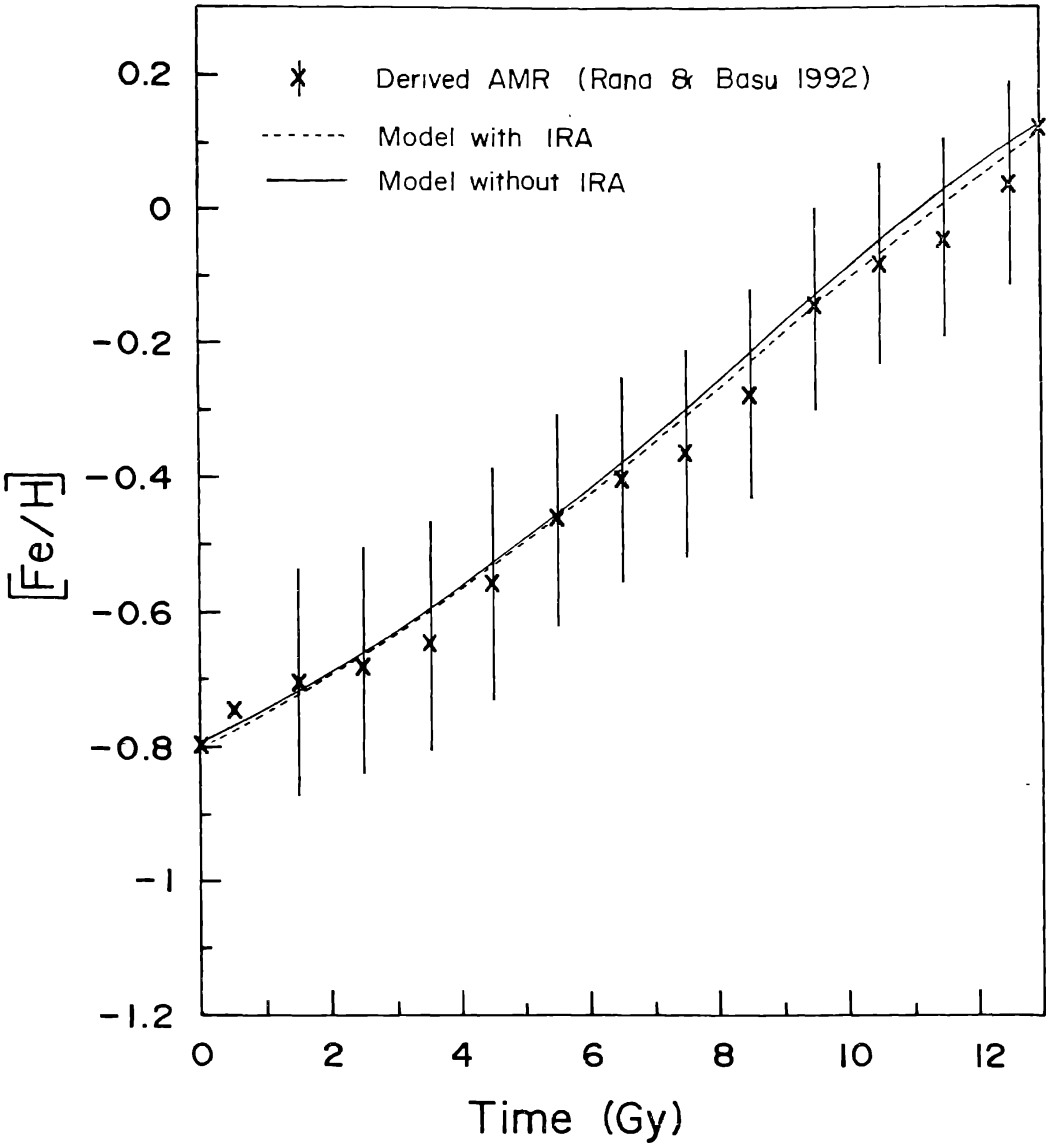

Figure 6. The age-metallicity relation of the local dwarf stars. The data marked by crosses with 1σ error bars are taken from Rana & Basu (1992) as derived from a unique combination of figure 3 and figure 9, by eliminating the normalised stellar number function *S*. The continuous line is the prediction of the simple model described in the text, and the dashed one results in if the approximation of instantaneous recycling is relaxed.

linear for [Fe/H] as a function of t), and that it deviates visibly from the trend of convexity that one notices so prominently in the original data set of Twarog (1980), or even from its latest modified version given by Meusinger *et al.* (1991). As it will be apparent in subsequent sections of this article, this change of trend in the derived AMR by Rana & Basu (1992) helps resolve many of the problems of chemical evolution of the solar neighbourhood.

Since the stellar samples of Pagel (1989) and Soderblom *et al.* (1991) were essentially the volume limited ones, the metal poor stars (or equivalently, the older stars) remained considerably underabundant, because the vertical component of the velocity dispersion (σ_z) of stars increase with their age, resulting in an inflation in their vertical scale heights

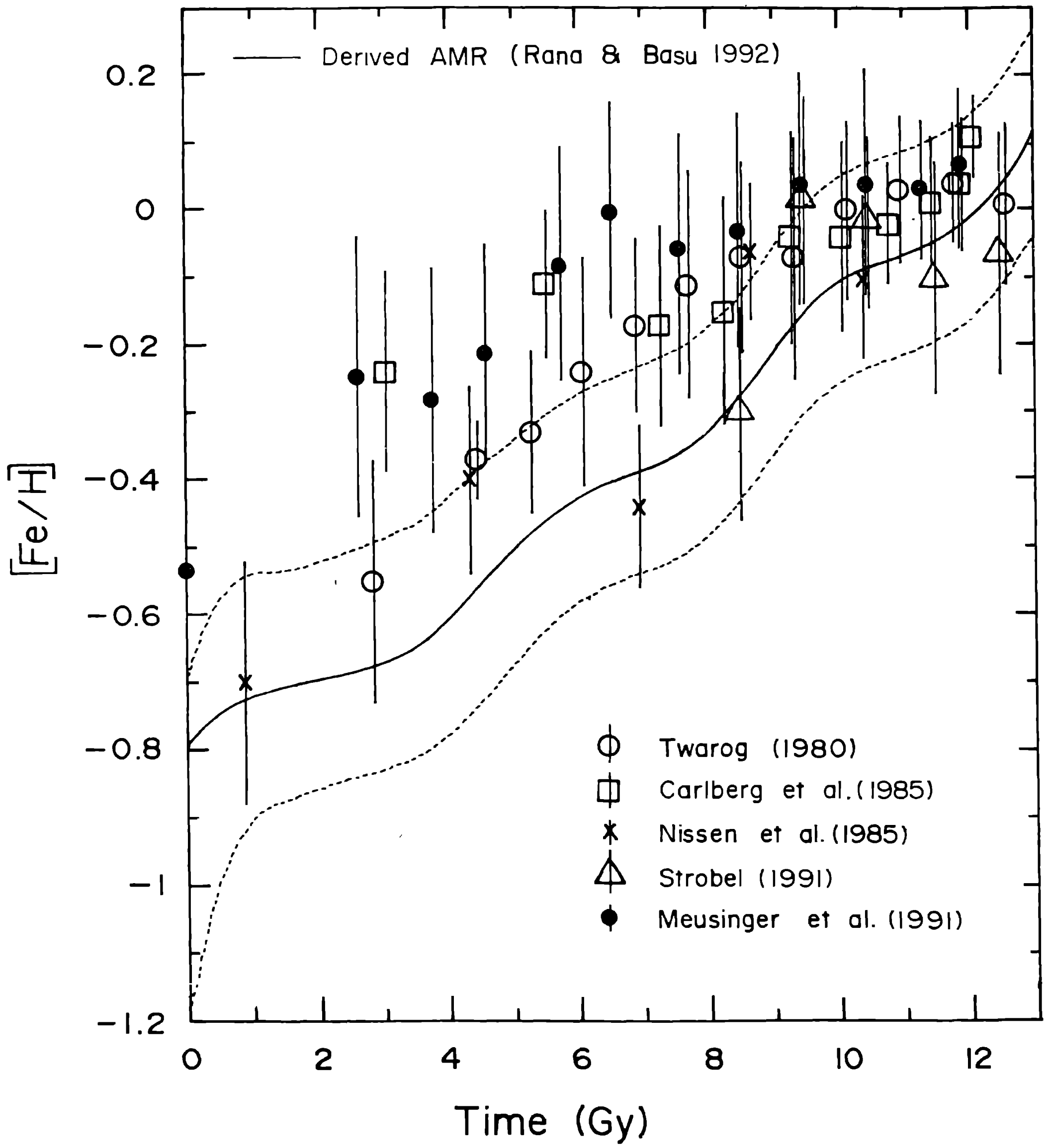

Figure 7. The data of figure 6 are compared with other independent determinations. The dotted lines represent the boundaries of 1σ error from the mean values.

$\langle h_z \rangle$. These are quite extensively discussed in Rana (1991) and Basu & Rana (1992a). For an isothermal disc,

$$h_z \propto \sigma_z^2 \propto \left(1 + \frac{t'}{\tau_0}\right)^{2n}, \qquad \ldots(4)$$

where t' is the age of a star under consideration, and n could be anything between 1/3 and 1/2 (Wielen 1977; Wielen & Fuchs 1983; Villumsen 1985). Taking data from Robin & Crézé (1986) for the vertical scale height variations with the age of stars, we show in figure 8, the excellent fit that we obtain for $n = 1/3$ and 95 pc for the constant of proportionality, say h_{z0}, the value of h_z at the time of birth of a star in the Disc. However, we also compare this with the more recent data available in Meusinger *et al.* (1991). Data

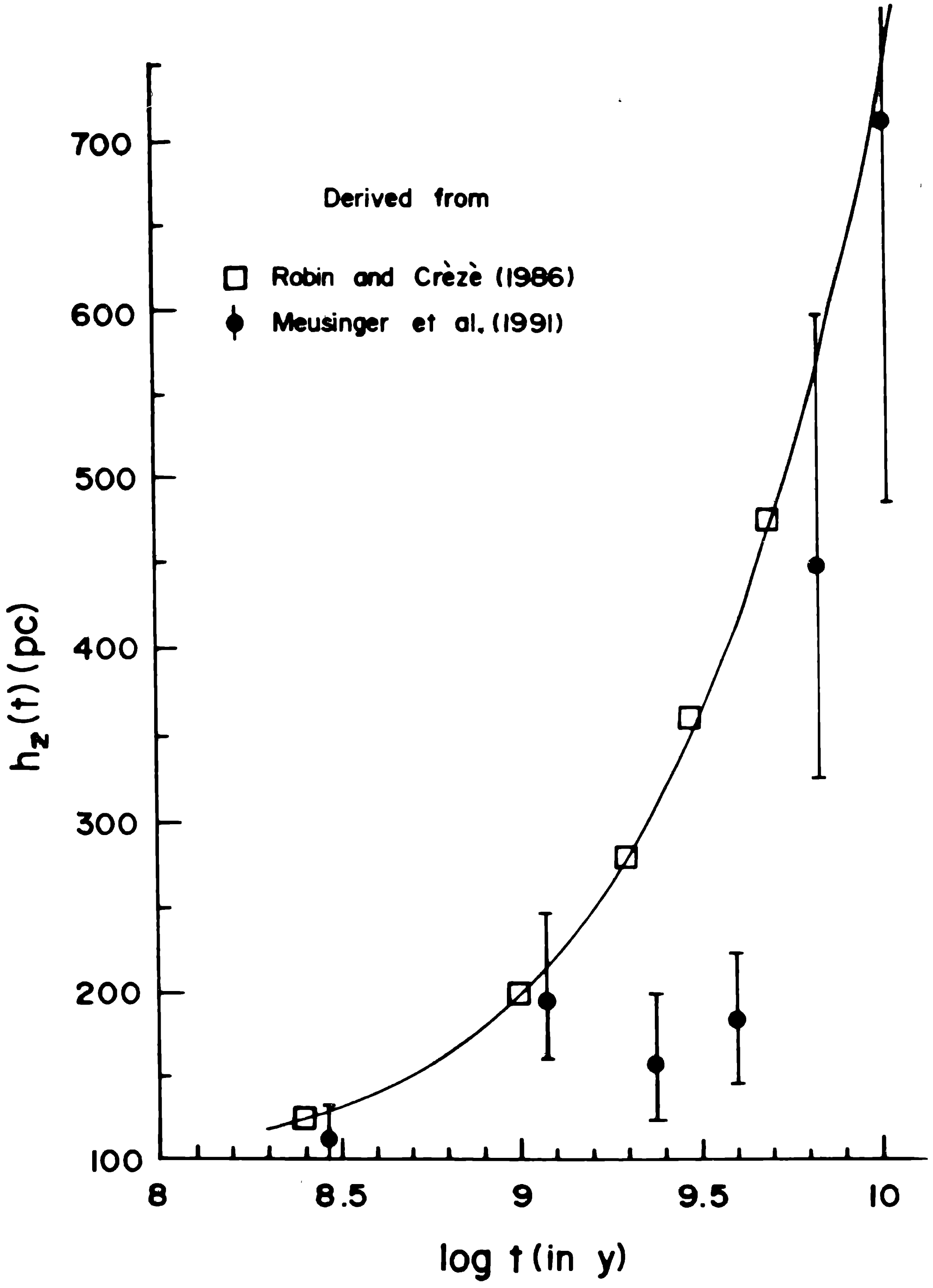

Figure 8. The existing data on the inflation of the vertical scale height of the local dwarf stars in the Disc are compared with the solid curve representing equation (4).

taken from Robin & Crézé are found to be consistent with the known average scale height of the gas distribution in the solar neighbourhood (65 pc for the molecular hydrogen and 130 pc for the atomic hydrogen). According to Amendt & Cuddeford (1991), the Galactic Disc in the solar neighbourhood may be considered as approximately isothermal, and Meusinger *et al.*'s data do not seem to agree with it. However, in this respect, we go ahead with equation (4) supplemented by the AMR of the present work, for correcting Pagel's and Soderblom *et al.*'s data for the possible inflation of the vertical scale height at lower metallicities, and for the sake of consistency, the same above form of the scale height evolution [equation (4)] has been used in deriving the PDMF and IMF for the present work.

Using our AMR, we determine the age of a star corresponding to its metallicity, and correct the data samples of Pagel (1989) and Soderblom *et al.* (1991) for the possible scale height inflation in the direction normal to the surface of the disc [equation (4)]. These scale height corrected (and renormalized) data are respectively shown in figures 9 and 10 by the histograms drawn in solid lines, against their original data shown by the dotted lines.

The correction factor for the oldest stars seems to have exceeded 8, which means that 7 out of 8 stars that originated within about 25 pc of the sun at the time of the

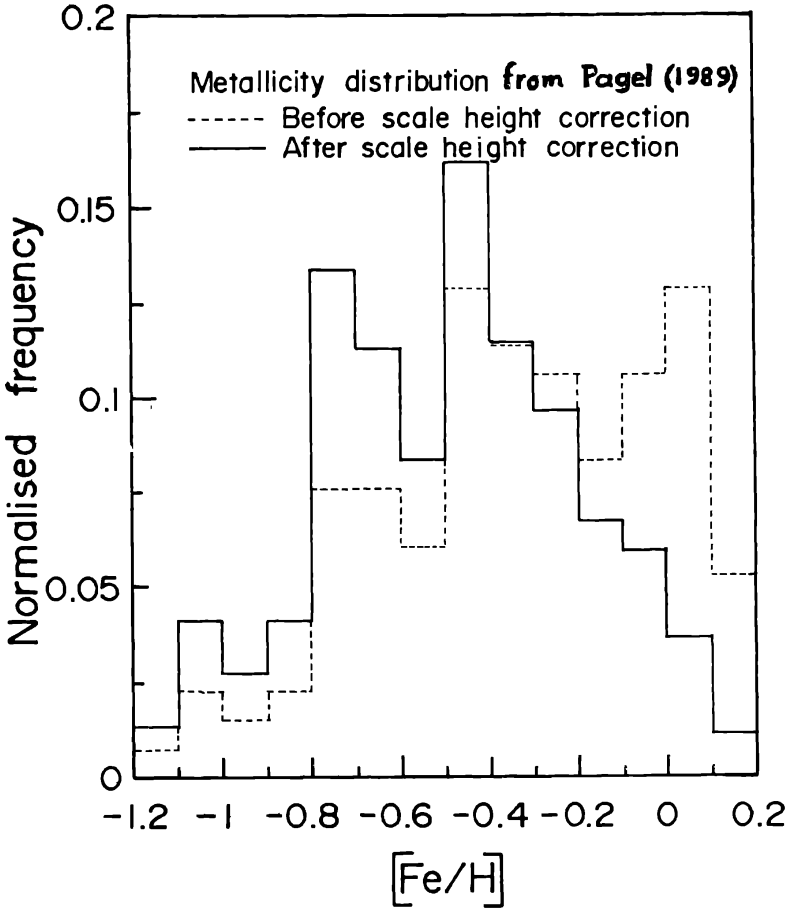

Figure 9. The normalised frequency distribution of the local G-dwarf stars with respect to their surface abundance of iron. The volume-limited data, shown by the dotted histogram, are taken from Pagel (1989), and after duly correcting for the possible inflation of the vertical scale heights, shown by the solid histogram, the data are taken from Rana & Basu (1992). The continuous curve represents the prediction of the simple model described in this work.

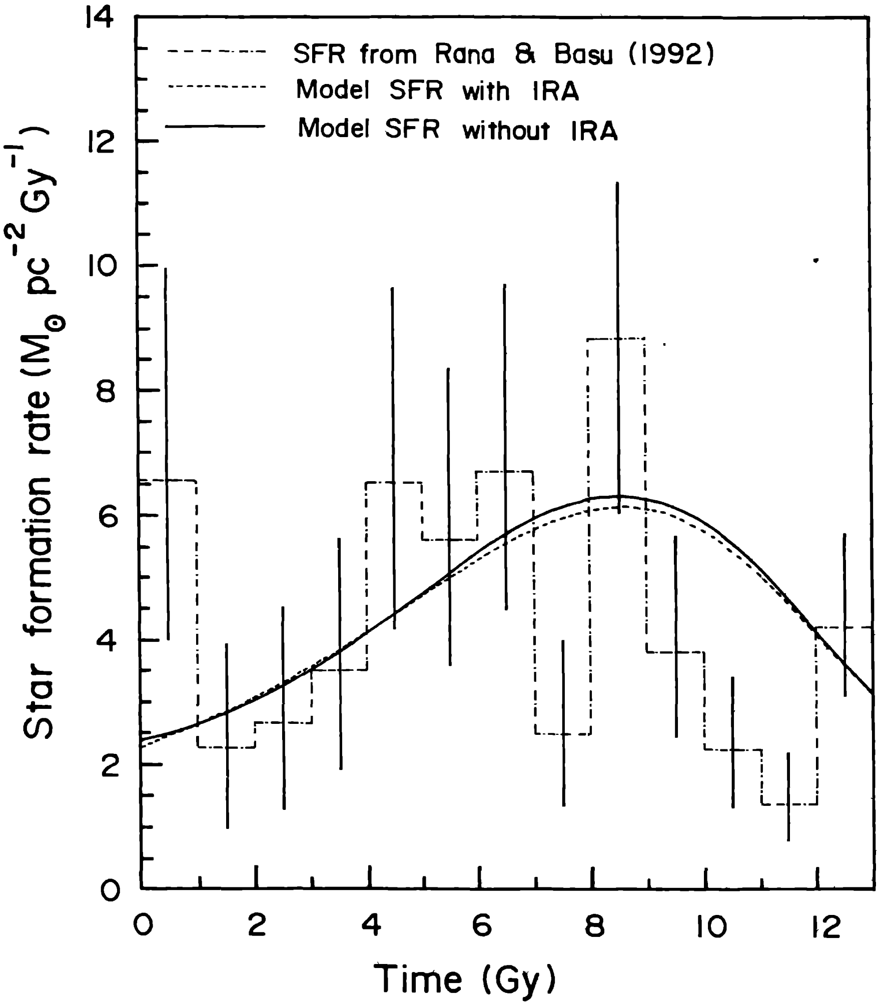

Figure 10. The time variation of SFR in the solar neighbourhood (as shown in figure 3) is compared with our model prediction, the continuous curve being due to the best fit obtained in accordance with equations (5) and (6), a form originally proposed by Rana & Wilkinson (1986, 1988).

formation of the disc have already migrated vertically outward (the radial migrations, if any, are assumed to be in balance, of course). Pagel's data had also been independently corrected for possible vertical escapes of metal poor stars by Somer-Larsen (1991), which do not differ in a significant way from that of ours.

3.4. *The history of star formation and the SFR*

The evolution of the SFR due to Soderblom *et al.* (1991), after the due correction for the scale height inflation appears to be more uniform than what it was before such corrections were applied. The undulations in the SFR cannot be taken too seriously as the authors themselves claim that the error in their use of calibration for age determination by the degree of chromospheric activity in stars is possibly quite large.

In an attempt to understand the apparent constancy of the star formation rate over the past 13 Gy, Rana & Wilkinson (1986, 1988) had suggested an empirical formula for the surface mass density of SFR, to be given by

$$\Psi_s \propto \Sigma_{H_2}^k, \qquad p_2 \equiv \frac{\Sigma_{H_2}}{\Sigma_{HI} + \Sigma_{H_2}} \propto Z^b, \qquad \ldots(5)$$

$$\text{or,} \quad \Psi_s = C(\Sigma_T \mu X_H)^k Z^{kb}.$$

where $k = 1.2 \pm 0.2$, $b = 1.3 \pm 0.3$, C is a constant, and Z is the abundance of oxygen by mass fraction. Similar, but not identical, correlations were later demonstrated by Basu (1988), and Tosi & Diaz (1990). Since the relation between the rate of evolution of iron is known to have been faster than that of oxygen (Clegg *et al.* 1981), the value of b with respect to iron abundance is expected to be somewhat lower. However, one can verify that, given the form of SFR as in equation (5) and Twarog's AMR, the SFR does not vary by more than a factor of 3 during the past 13 Gy (Rana & Wilkinson 1986, and in particular, figure 10 of the present work).

The present work has further been extended for studying the metallicity gradient and the radial distributions of gas, total matter and SFR across the Galaxy in Basu & Rana (1992b). It explains reasonably well the metallicity gradient, giving about 0.045 dex/kpc for iron, and fits the total gas distribution to within χ^2 per degree of freedom to be 1.0.

4. Plausible solutions to the major problems

In the present work, we tried in a consistent way to reproduce the AMR of the present work, fit the cumulative metallicity distributions of the scale height corrected Pagel's data, and use the above form [equation (5)] of the SFR with variable values for k and b in order to derive the IMF, the yield of iron y and the returned fraction R, by successive iteration of solving the differential equations (3a-d). A convergence was successfully achieved for

$$k = 1.15 \pm 0.05, \quad b = 1.13 \pm 0.07, \quad R = 0.206, \ (1 - R)y = 0.45$$

$$[\mathrm{Fe/H}]_0 = -0.80, \ [\mathrm{Fe/H}]_1 = +0.12 \ \text{and} \ \Psi_1 = 3.56 \ M_\odot \ \mathrm{pc}^{-2}. \qquad \ldots(6)$$

It assumed all the material to have been originally in the form of gas only, with $\Sigma_{g0} = 51.3 \ M_\odot \ \mathrm{pc}^{-2}$, which resolves the problem of dark matter in the solar neighbourhood. In figure 4,we have plotted the shape of the SFR (the dashed curve) that fitted Sodderblom *et al.*'s data quite agreeably for the above values of k and b. The SFR peaks at about $t = 9$ Gy with the peak value of the SFR at 5.53 $M_\odot \ \mathrm{pc}^{-2}$. The resulting IMF is shown in figure 5 alongwith the PDMF. This solution resolves the classical G-dwarf problem due to the imposition of a finite initial metallicity a solution similar to the one proposed long back in 1971 by Truran and Cameron.

The star formation rate still depends on the gas content with a Schmidt type of index k = 1-2 (in the present case, approximately 1.15, that is, closer to a linear dependence), but with a difference that the SFR also depends quite strongly on the metallicity, with once again a power law type of dependence having a value of the exponent, $kb = 1.30$. Since the gas fraction is continuously decreasing with time (in a close model such as the present one) and the metallicity is increasing with time, the time variation of SFR cannot vary by a very large factor. This SFR has an added advantage over one of the simple Schmidt type that all metal-poor galaxies, or even the outskirts of the typical spiral galaxies with low metallicity, cannot have the process of star formation at a significant level, even if they contain a lot of gas in them. Our next door neighbours, LMC and SMC could be cited as examples for such a case. In the cases of tidal interactions between two galaxies, it is the enhanced concentration of gas that would trigger a burst of star formation, even if then metallicity is not too high. The greatest advantage of having a value of $k > 1$ is however that it can produce the right trend of the radial variation of metallicity in a galaxy without calling for any horizontal or vertical infall of metal-free or metal-poor gas from the outskirts or halo of the Galaxy, a point which has been demonstrated in Rana & Wilkinson (1986).

The observed birth rates of pulsars and white dwarfs are also found to be consistent with the above IMF and a constant SFR (Basu & Rana 1992b, c). The self-consistency of the entire scheme of chemical evolution demands further than the value of k should be 1.15 and that of kb as 1.30, with the constant of proportionality having a value of 0.284, if Σ_g and Z are expressed in units of $M_\odot$ pc^{-2} and solar photospheric abundance of iron, respectively (Basu & Rana 1992c).

Again, since we are using the yield of iron from the theoretical models of stellar evolutions and the IMF derived consistently with the same law of SFR as used in the model of chemical evolution, a simple closed-box model of chemical evolution with the instantaneous recycling approximation is found to resolve all the major problems that were listed in section 2. Even if we relax IRA, the results do not vary by more than 3% or so (Rana & Basu 1992), and this is evident from figures 6 and 10.

5. What if the alternative theories of SN Ia are accepted?

A large number of theories claim that the SN type Ia results from reaching the threshold mass for the Chandrasekhar limit for a stable WD by either accreting gas from its companion or coalescing a companion WD by emitting gravitational waves. These scenarios are plausible simply because more than 80% of all stars are locked up in multiple stellar systems, leaving behind at most 20% of all stars to be the single stars. It is also found that the multiple stellar systems do appear more or less as binary systems in a hierarchical manner, that is, a triple star system usually consists of a close binary, which forms another binary system with a distant third star; a quadruple system may consist either of a widely separated two component binary systems with each component binary being a close pair, or of a widely separated fourth star and a close triple star system, in which the triple star system has a widely separated third star from a yet closer pair; and so on. The mass ratios of a typical binary is found to be 1 : 4 for low mass stars ($m < 1$, but they become gradually equal mass partners for higher masses; and there are examples of practically equal mass binaries for most stars above masses $> 10\ M_\odot$. Now, the evolution of binary stars are significantly different from that of the single stars, particularly after the main-sequence phase is over for the more massive partner; either both of them end up as CO-rich WDs (white dwarfs), or a pair of WD and NS (neutron star), or a pair of NSs, or BHs (black holes), or no remnants at all. In the post main sequence phase of the less massive component, it might fill the Roche lobe and an overflow of matter to the other component, which may be a WD, or NS or a BH, occurs. In the process, a WD may reach the Chandrasekhar mass limit and explode as a SN Ia, or even a pair of WDs in close binary orbits may emit gravitational waves at a high rate leading finally to coalescence of the two WDs and ending up as a type Ia supernova. The time scales on which such processes are likely to operate is of the order of a few Gy.

Then, the assumption of IRA will no longer be valid if such delayed processes are to be included.

So far as the type Ib and type II SNs are concerned, they are assumed to be the end product of the evolution of stars more massive than 8 (could be even 12) $M_\odot$. The exact course of their evolution depends so much upon how they lose mass due to winds, or whether they are part of a close binary system, or how the process of convection of heat is modeled through what is called the phenomenon of overshooting, and so on, that at present, all models are grossly imperfect. There are even cases where the numerical results could not have been reproduced under identical initial conditions by the competing groups working on such models.

Under such circumstances, we prefer to depend more on observational quantities and statistics than on uncertain theoretical models. It would be safe to assume that the type Ib and type II supernovae are the end products of all stars more massive than 8 $M_\odot$, leaving behind a NS if the progenitor mass lies between 8 and 60 $M_\odot$, and a BH for more massive ones. We further assume that all type Ia supernovae are delayed merging of WDs or accreting WDs resulting from Roche lobe overflow of matter from its evolved companion, the amount of delay (say, τ_d) is left as a free parameter to fit the data best. The ejection of iron by a typical type Ia SN is found to be about 0.60 $M_\odot$, in order to reproduce their light curves in the way their radiation output is seen to decline. In order to find out how much iron the SN types Ib and II would produce, we assume that the amount of ^{56}Ni released in a SN is proportional to their peak brightness simply because ^{56}Ni decays promptly into ^{56}Co with a mean life of about a week. Of course, later ^{56}Co decays into ^{56}Fe on a time scale of 3 months, the light curve declines exponentially till the dust emissions etc. take over. From Evans *et al.*, we take the absolute B-magnitude at maxima. They are -19.8, -18.6, and -18.0 for the SN types Ia, Ib and II respectively. On normalizing the absolute peak luminosity for type Ia SN to 0.60 $M_\odot$ of iron, we get 0.20 $M_\odot$ for type Ib SN, and 0.11 $M_\odot$ for type II. The frequency of occurrence of these SNs follow the relative ratios of 0.28 : 0.27 : 1.04 for the types Ia, Ib and II respectively (Evans *et al.* 1989). Now we can normalize the integrated IMF over the mass range 8 to 60 $M_\odot$ to the proportionate share for the present day observed rate for the types of SN Ib and II combined together, which will automatically fix the present day rate of occurrence of the SNe of type Ia. We find that our IMF gives a total number of SNe Ib + II as 0.29 stars/pc^2, giving a constant rate of 0.0223 events of SNe Ib + II per pc^2 per Gy in the solar neighbourhood for an assumed constant SFR. This also fixes the present rate of SN Ia in the solar neighbourhood at 0.0048 events pc^{-2} Gy^{-1}.

It is also now fairly easy to calculate the yield of iron due to SN types Ib + II, say $y_{\rm Ib2}$, which turns out to be 0.037 $M_\odot$ pc^{-2}/57 $M_\odot$ pc^{-2} $\equiv 0.00065$ by mass fraction. Assuming the solar abundance of iron to be $Z_\odot = 0.00184$ (corresponding to $({\rm Fe/H})_\odot = 7.67$ dex due to Grevesse *et al.* 1989), and $R = 0.20$,

$$y_{\rm Ib2}(1 - R) = 0.35\ Z_\odot. \qquad \ldots(7)$$

This yield is already very close to the required value of y for fitting the data, say the AMR, in a simple model of chemical evolution. Under the present circumstances, if we allow the SNe of type Ia to be releasing iron without any delay (just by single star explosion, say), the contribution to the yield by type Ia SNe turns out to be $y_{\rm Ia}(1 - R) = 0.35\ Z_\odot$. Therefore, we can no longer accommodate the contributions by the

SNe type Ia at a constant rate, but it is certainly possible to make a provision for matching the observed high rate of SNe of type Ia for the present epoch with adequate suppression for its rate in the past. The delay effect can be used to alleviate this canonical problem. We introduce a new quantity called the rate of yield of iron due to SNe type Ia, assumed to be a constant and denoted by, say, y'_{1a}, and modify equation (3d) to take the following form

$$\frac{d(Z\Sigma_g)}{dt} = -Z(1-R)\psi_s + (1-R)y_{1b2}\,\psi_s + (1-R)y'_{1a}\int_{\tau_d}^{t_d}\psi_s(t')dt' + Z_f\,\psi_f, \qquad \ldots(8)$$

where τ_d is the delay between the time of birth of a potential candidate for the progenitor of a would-be SN of type Ia and the time of its actually becoming a type Ia SN. For no infall, we can now compute by iteration the required value of τ_d in order to match with the observed ratios for the rates of SNe incidence at the present epoch. We finally obtain a value for τ_d as 4 Gy, and this also fixes the value of $(1 - R)y'_{1a}$ at $0.036\ Z_\odot\ \mathrm{Gy}^{-1}$. The entire combination of all the observed data are found to agree with the above sets of values together with the same values of k, b, etc. as we obtained for the previous case. The growth of iron is slightly different, and is well reproduced within the error limits of the relevant observed data.

In this second case, it is quite difficult to accommodate the type Ia supernovae as exploding single stars, unless of course, the quantities of iron ejected by SNe of types Ib and II are somewhat reduced from their values that have been calculated from the relative intensities of blue light at the peak of SNe of various types, as detailed above.

6. Conclusion

A closed-box, simple model of chemical evolution or a slight modification of simple model is still good enough to explain all the observed aspects of chemical evolution of iron in the Galaxy, the solar neighbourhood in particular, including the mass functions of stars, the inflation of the vertical scale heights of stars with age, and the observed birth rates of SNe, pulsars and white dwarfs, in a quite consistent manner.

References

Amendt A., Cuddeford P., 1991, ApJ, 368, 79.
Anders E., Grevesse, N., 1989, Geochim. Cosmochim. Acta, 53, 197.
Bahcall J. N., 1989, A. Rev. Astr. Astrophys., 27, 629.
Basu S., 1988, M.Sc. project, Dept. of Physics, University of Poona, unpublished.
Basu S., Rana N. C., 1992a, Astrophys. Space Sci., 393, 373.
Basu S., Rana N. C., 1992b, ApJ (in press).
Basu S., Rana N. C., 1992c, in preparation.
Bhat C. L., Houston B. P., Issa M. R., Mayer C. J., Wolfendale A. W., 1985, Nature, 314, 511.
Bronfman L., Cohen R. S., Alvarez H., May J., Thaddeus P., 1988, ApJ, 324, 248.
Burton W. B., Gordon M. A. 1978, A&A, 63, 7.
Cameron L. M., 1985, A&A 147, 47.
Carlberg R. G., Dawson P. C., Hsu T., VandenBerg D. A., 1985, ApJ, 250, 262.
Clegg R. E., Lambert D. L., Tomkin J., 1981, ApJ, 250, 262.
Clegg R. E., Lambert D. L., Tomkin J., 1981, ApJ, 250, 262.

Duel E. R., 1988, in : Comets to Cosmology, *Proc. 3rd IRAS Conf.*, ed. A. Lawrence, Springer-Verlag, p. 89.
Evans R., van den Bergh S., McClure R. D., 1989, ApJ, 345, 752.
Goodman A. A., Myers P. C., 1991, Harvard-Smithsonian Astrophysics Preprint.
Gould A., 1990, MNRAS, 244, 25.
Iben I., Tutukov A. V., 1984, ApJ, 282, 615.
Janes K. A., 1979, AJS, 39, 135.
Khokhlov A., Müller E., Höflich P., 1992, A&A, 253, L9.
Kuijken K., Gilmore, G., 1991, ApJ, 367, L9.
Maeder A., 1990, AAS, 84, 139.
Maeder A., 1992, A&A (in press).
Maeder A., Meynet G., 1989, A&A, 210, 155.
Meusinger H., Reimann H. -G, Stecklum B., 1991, A&A, 245, 57.
Morrison H. L., Flynn C., Freeman K. C., 1990, AJ, 100, 1191.
Nissen P. E. 1988, A&A, 199, 146.
Nissen P. E., Edvardsson B., Gustafsson B., 1985, in : Production and Distribution of C, N, O Elements, eds I. J. Danziger, F. Matteucci & K. Kjr ESO, Garching, p. 131.
Nomoto K., Thielemann F. K., Yokoi K., 1984, ApJ, 286, 641.
Nomoto K., Shigeyama T., Tsujimoto T., 1990, in : Evolution of Stars: the Photospheric Abundance Connection, IAU Symp. 145, ed. G. Michaud, Kluwer, Dordrecht.
Oort J. H., 1928, Bull. Astr. Inst. Neth., 4, 269.
Pagel B. E. J., 1989, in : Evolutionary Phenomena in Galaxies, eds J. E. Beckman & B. E. J. Pagel, Cambridge Univ. Press, Cambridge, p. 201.
Pagel B. E. J., Patchett B. E., 1975, MNRAS, 172, 13.
Rana N. C., 1987, A&A, 184, 104.
Rana N. C., 1991, A. Rev. Astr. Astrophys, 29, 129.
Rana N. C., Basu S., 1992, A&A (in press).
Rana N. C., Wilkinson D. A., 1986, MNRAS, 218, 496.
Rana N. C., Wilkinson D. A., 1988, MNRAS, 231, 509.
Robin A., Crézé M., 1986, A&A, 157, 71.
Scalo J. M., 1986, Fund. Cosmic Phys., 11, 1.
Schmidt M., 1959, ApJ, 129, 243.
Schmidt M., 1963, ApJ, 137, 758.
Soderblom D. R., Duncan D. K., Johnson D. R. H., 1991, ApJ, 375, 722.
Somer-Larsen J., 1991, MNRAS, 249, 368.
Strobel A., 1991, A&A, 247, 35.
Tinsley B. M., 1980, Fund. Cosmic Phys., 5, 287.
Tosi M., Diaz A. I., 1990, MNRAS, 246, 616.
Truran J. M., Cameron A. G. W., 1971, Astrophys. Space Sci., 14, 179.
Turner B. E., 1984, Vistas Astr., 27, 303.
Twarog B. A., 1980, ApJ, 242, 242.
Villumsen J. V., 1983, ApJ, 274, 632.
Villumsen J. V., 1985, ApJ, 290, 75.
Wielen R., 1977, A&A, 60, 263.
Wielen R., Fuchs B., 1983, in : Kinematics. Dynamics and the Structure of the Milky Way, ed. W. L. H. Shuter, D. Reidel, Dordrecht, p. 91.
Woosley S. E., 1991, in : Proc. Les Houches Summer School on Supernovae, eds J. Audouze, S. Bludman, R. Mochkovitch & J. Zinn-Austin, Elsevier.
Woosley S. E., Weaver T. A., 1986, A. Rev. Astr. Astrophys, 24, 205.