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Temperature and brightness distributions in the components of close binary system

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Abstract. A theoretical model of close binary system for the gravity-darkening due to tidally and nonuniformly rotating Röche components has been formulated using the fourth order of r/R where r is the radial distance and R is the distance between the centres of the binary stars. This model provides expressions to calculate the temperature and brightness distributions along the surface of the components.

Key words : Roche components—nonuniform rotation—tidal effect—surface temperature—surface brightness

1. Introduction

A comprehensive model to explain the effect of rotation and the gravity darkening on the spectral characteristics, the temperature, and brightness distributions of a binary system is still lacking. The factors that need to be considered are reflection, limb-darkening, turbulence, and meridional circulation currents within the stellar atmosphere. Ireland (1966, 1967) has discussed the importance of gravity-darkening in the case of uniformly as well as nonuniformly rotating Roche model. He has shown that nonuniform rotation can significantly change the temperature and brightness distributions along the surface of a star. Peraiah (1969) extended the analysis to the case of close binary-system. He showed that tidal and rotational effects have much influence on the distributions of temperature and brightness along the surface of the components. Here he considered only the radial component of the surface gravity. Subsequently Peraiah (1970) considered the case of total gravity. To calculate potential due to tidal forces of the secondary component, Peraiah (1970) used only the second order term of Legendre polynomial $P_2(\theta)$ which contains second order in r/R, the ratio between the radial distance r and the distance R between the centres of the stars of the binary system (Kopal 1959). In the present paper, this ratio has been extended to the fourth order.

We assume the system to be under radiative as well as hydrostatic equilibrium. The origin of the coordinate system is taken to be the centre of the primary; x-axis is the line joining the centres of the stars; the axis of rotation of the primary component is the z-axis. The z-axis is taken perpendicular to the orbital plane which is also the equatorial plane of the primary component. Any point on the surface of the primary is given by the polar co-ordinates (r, θ, ϕ).

2. Formulation of the model

Let a binary system have masses m_1 (primary component) and m_2 (secondary component). For a Roche model of mass m_1 rotating according to the law (Ireland 1966)

$$\Omega = b_1 + b_2 w^2, \qquad \ldots(1)$$

where $w (= r \sin \theta)$ is the distance r measured from the axis of rotation, the equation of hydrostatic equilibrium can be written as

$$\text{grad } p = \rho_1 \text{ grad } \psi, \qquad \ldots(2)$$

when the equation of equipotential is simply given by

$$\psi = \text{constant}, \qquad \ldots(3)$$

In equation (2), p is the total pressure, ρ_1 the density of the gas and ψ the combined potential of gravitational and rotational forces. Now, the equation of hydrostatic equilibrium can be written as

$$dp = \rho_1 \, dv + \frac{1}{2} \rho_1 \, \Omega^2 \, d(w^2), \qquad \ldots(4)$$

where v is given by (Kopal 1959)

$$v = \frac{Gm_1}{r} + \frac{Gm_2}{R} \sum_{j=2}^{4} \left(\frac{r}{R}\right)^j P_j(z), \qquad \ldots(5)$$

$$z = \cos \phi \sin \theta, \qquad \ldots(6)$$

a being the gravitational constant, r the distance from the primary's centre, R the distance between the centres of gravity of the two components and $P_j(z)$ the Legendre polynomials. The first term of equation (5) is the gravitational potential due to the primary component and the second term due to the secondary (Kopal 1959). The right-hand side of equation (4) can be expressed as a perfect differential. The Legendre polynomials. $P_j(z)$ $(j = 2, 3, 4)$ in equation (5) can be calculated from the relation (Macrobert 1966).

$$(j + 1) \; P_{j+1}(z) - (2j + 1)z \; P_j(z) + j \; P_{j+1}(z) = 0, \; (j = 0, 1, 2, 3, 4, \ldots\ldots) \ldots(7)$$

if we remember

$$P_0(z) = 1, \; P_1(z) = z \qquad \ldots(8)$$

To the order of accuracy, the maximum value of j in equation (5) has been taken $j = 4$.

Putting $j = 1$, 2 and 3 in equation (7) and using equation (8), we shall get

$$P_2(z) = \frac{1}{2}(3z^2 - 1),$$

$$P_3(z) = \frac{z}{2}(5z^2 - 3),$$

$$P_4(z) = \frac{1}{8}(35z^4 - 30z^2 + 3), \qquad \ldots(9)$$

From equations (1), (2) and (4), we can write

$$\psi = \frac{Gm_1}{r} + \frac{Gm_2}{R}\sum_{j=2}^{4}\left(\frac{r}{R}\right)^j P_j(z) + \frac{1}{2}b_1^2 r^2 \sin^2\theta$$

$$+ \frac{1}{2}b_1 b_2 r^4 \sin^4\theta + \frac{1}{6}b_2^2 r^6 \sin^6\theta. \qquad \ldots(10)$$

If Ω_e and Ω_p be the equatorial and polar angular velocities, then from equation (1) we have

$$b_1 = \Omega_p, \quad b_2 = (\Omega_e - \Omega_p)/r_e^2, \qquad \ldots(11)$$

r_e being the equatorial radius, we set

$$x = \Omega_e/\Omega_p, \quad f = r_e^3\ \Omega_e^2/Gm_1, \qquad \ldots(12)$$

where x is the ratio of the equatorial to the polar angular velocities and f the ratio of the centrifugal to the gravitational forces at the equator. Now we can write from equation (10) (putting $\theta = 0°$) as:

$$(\psi)_{\text{pole}} = \frac{Gm_1}{r} + \frac{Gm_2}{R}\left[-\frac{1}{2}\left(\frac{r_p}{R}\right)^2 + \frac{3}{8}\left(\frac{r_p}{R}\right)^4\right]. \qquad \ldots(13)$$

If we equate equation (10) to equation (13), then the equation of the stellar surface can be put in the form:

$$\alpha\rho^7 + \beta\rho^5 + \gamma\rho^4 + \delta\rho^3 - \zeta\rho + 1 = 0, \qquad \ldots(14)$$

where

$$\alpha = \alpha_1 \sin^6\theta,$$

$$\beta = \beta_1 \sin^4\theta + J_3,$$

$$\gamma = J_2,$$

$$\delta = \gamma_1 \sin^2\theta + J_1,$$

$$\zeta = 1 - Q_1 + 3Q^3,$$

$$\alpha_1 = \frac{f(x-1)^2}{6x^2} \cdot \left(\frac{r_p}{r_e}\right)^7,$$

$$\beta_1 = \frac{f(x-1)}{2x^2} \cdot \left(\frac{r_p}{r_e}\right)^5.$$

$$\gamma_1 = \frac{f}{2x^2} \cdot \left(\frac{r_p}{r_e}\right)^3,$$

$$J_1 = Q_1(3\sin^2\theta\cos^2\phi - 1),$$

$$J_2 = Q_2(5\sin^3\theta\cos^3\phi - 3\sin\theta\cos\phi),$$

$$J_3 = Q_3\,(35\,\sin^4\,\theta\,\cos^4\,\phi - 30\,\sin^2\,\theta\,\cos^2\,\phi + 3).$$

$$Q_1 = \frac{1}{2}\left(\frac{r_p}{r_e}\right)^3 \mu_1,\; Q_2 = \frac{1}{2}\left(\frac{r_p}{r_e}\right)^4 \mu_2,$$

$$Q_3 = \frac{1}{8}\left(\frac{r_p}{r_e}\right)^5 \mu_3,$$

$$\mu_j = \frac{m_2}{m_T}\left(\frac{r_e}{R}\right)^{2+j},\; (j = 1,\; 2,\; 3), \qquad \ldots(15)$$

where r_p/r_e is given by

$$\left(\frac{r_e}{r_p}\right)^5 - u\left(\frac{r_e}{r_p}\right)^4 - \frac{\mu_1}{2}\left(\frac{r_e}{r_p}\right)^2 + \frac{3}{8}\,\mu_3 = 0, \qquad \ldots(16)$$

with

$$u = 1 + \frac{f(x^2 + x + 1)}{6x^2} + \mu_4$$

$$\mu_4 = \mu_1\,J' + \mu_2\,J'' + \mu_3\,J''',$$

$$J' = \frac{1}{2}\,(3\,\cos^2\,\phi - 1).$$

$$J'' = \frac{1}{2}\,(5\,\cos^3\,\phi - 3\,\cos\,\phi),$$

$$J''' = \frac{1}{8}\,(35\,\cos^4\,\phi - 30\,\cos^2\,\phi + 3).$$

$$\ldots(17)$$

The surface gravity g is given by

$$g \equiv -\,\mathrm{grad}\;\psi \equiv (g_r,\; g_\theta,\; g_\phi), \qquad \ldots(18)$$

where g_r, g_θ, g_ϕ are the components of g in the three orthogonal directions defined by the curvilinear co-ordinates $(r,\; \theta,\; \phi)$, so,

$$g_r = -\frac{\partial\psi}{\partial r} = \frac{Gm_1}{r_p^2}\cdot\frac{1}{\rho^2}\left[1 - f_r\,(\theta)\sin^2\theta - 2\,J_r\,(\theta)\,\rho^3\right], \qquad \ldots(19)$$

$$g_\theta = -\frac{1}{r}\,\frac{\partial\psi}{\partial\theta} = \frac{Gm_1}{r_p^2}\cdot\frac{1}{\rho^2}\left[-f_r\,(\theta)\sin\theta\cos\theta - 6\,\rho^3\,K\cos\theta\cos\phi\right], \qquad \ldots(20)$$

$$g_\phi = -\frac{1}{r\sin\theta}\,\frac{\partial\psi}{\partial\phi} = \frac{Gm_1}{r_p^2}\cdot\frac{1}{\rho^2}\left[6\,\rho^3\,K\sin\phi\right], \qquad \ldots(21)$$

with

$$f_r(\theta) = \frac{\Omega^2 r^3}{Gm_1} = 2\gamma_1\,\rho^3 + 4\,\beta_1\,\rho^5\,\sin^2\,\theta + 6\,\alpha_1\,\rho^7\,\sin^4\,\theta,$$

$$J_r(\theta) = J_1 + \frac{3}{2}\rho J_2 + 2\rho^2 J_3,$$

$$K = Q_1 \sin\theta\cos\phi + \frac{1}{6}\rho\, Q_2\, K_2 + \frac{1}{6}\rho^2 Q_3 K_3,$$

$$K_2 = 15 \sin^2\theta \cos^2\phi - 3,$$

$$K_3 = 140 \sin^3\theta \cos^3\phi - 60 \sin\theta\cos\phi.$$

Therefore, the total surface gravity g is given by

$$g = (g_r^2 + g_\theta^2 + g_\phi^2)^{1/2}, \quad \ldots (23)$$

and hence the polar surface gravity g_p is

$$g_p = \frac{Gm_1}{r_p^2}\left\{(1 + 2\,Q_1 - 12\,Q_3)^2 + 9\,Q_2^2\right\}^{1/2}, \quad \ldots (24)$$

The effective temperature T_e at any point on the tidally and rotationally distorted component can be obtained from

$$\frac{T_e}{T_p} = \left(\frac{g}{g_p}\right)^{1/4}, \quad \ldots (25)$$

where T_p is the polar temperature for the case of black body radiation.

The distribution of brightness is (Kopal 1959) given by

$$\frac{H}{H_p} = 1 + \frac{b}{4}\left(\frac{g}{g_p} - 1\right), \quad \ldots (26)$$

where H is the brightness at any point and H_p is that at the pole. The value of b is given by

$$b = \frac{a}{1 - e^{-a}},$$

$$a = \frac{hc}{\lambda kT} \quad \ldots (27)$$

where h is the Planck's constant, c the velocity of light, k the Boltzman constant, λ the wavelength and T the temperature.

Equations (25) and (26) are used to calculate the temperature and brightness distributions. For calculating these values equations (14) to (27) are employed in the following manner : a few appropriate values for x, f, m_2/m_1 and r_e/R are selected. r_e/r_p has been calculated from equations (16) and (17). This value of r_e/r_p is used in the equation (15) to calculate the coefficients α, β, γ, δ, ζ which, in turn, are substituted in the equation (14) to solve for ρ. By substituting ρ in the equations (23) and (24), g/g_p is calculated, whose substitution into the equations (25) and (26) enables us to calculate the temperture and the brightness distributions.

If we put $P_3(z) = 0$, $P_4(z) = 0$ in the relation (5) [i.e. $\mu_2 = \mu_3 = 0$ in equation (15)] and $g_\theta = g_\phi = 0$ in equations (20) and (21), the formulations deduced here reduces to those of Peraiah's paper (1969) and if we put $P_2(z) = 0$, $P_3(z) = 0$, $P_4(z) = 0$ [i.e. $\mu_1 = \mu_2 = \mu_3 = 0$ in equation (15)], the formulations reduce to those of Ireland's paper (1967). If $(r/R)^4$ in equation (5) is neglected [i.e. $\mu_3 = 0$ in equation (15)], then the secondary component, of which tidal force is considered, is a point mass to the order of accurracy.

In the next papers, other important factors already mentioned will be included into the model.

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