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Dynamical evolution of binary systems

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"Of everything We have created pairs that you may reflect"
(The Quran 51 . 50)

Like all other speakers, I am also very grateful to the organizers for giving me the opportunity of giving an invited talk at this workshop which has been organized at the time of superannuation of Prof. K. D. Abhyankar who has contributed a lot to the progress of this department I wish him many years of fruitful activity.

I have chosen the words 'Binary systems' rather than 'Binary stars' in the title, as I wish to draw attention also to some aspects of the dynamics of binary galaxies and indicate the underlying units in the areas of binary stars and binary galaxies.

I propose to discuss first the interaction of binary stars with single stars and the role of binary stars in star clusters. Then I shall mention some aspects of galaxy interactions which can be viewed as analogues of binary-single body encounters on a bigger scale.

1. Interaction of binary stars with field stars

The study of the effects of a stellar enconter on a binary star has been a subject of extensive study both numerically and analytically. (For reviews see Heggie 1988, Valtonen 1988).

A system of three stars with arbitrary masses can be specified by $7 \times 3 = 21$ parameters (six parameters denoting the initial position and velocity of a star, and the seventh parameter denoting its mass). By choosing the centre of mass of the system as origin, six parameters can be eliminated out of 21. The various choices of the remaining 15 parameters lead to a bewildering variety of solutions. Hills (1975) carried out more than 14,000 numerical experiments. Later Hut & Bahcall (1983) performed more than a million orbit calculations. Valtonen & Huang (1988) have extensively investigated the case of a close binary moving in a circular orbit and the third star making an encounter in a parabolic orbit.

The results show that generally the tightly bound binaries become more tight and the loosely bound ones become more loose. Heggie, who has been a pioneering worker in this area, classified binaries into hard (tightly bound) and soft (loosely bound) (Heggie 1975a, b). If $m v^2/2$ is the local mean kinetic energy of a star, a hard binary is defined as that for which $|E| > m v^2/3$ where E is the binding energy of the binary; while for a soft binary $|E| < m v^2/3$. With these definitions, the fundamental law of binary dynamics can be stated as : 'Hard binaries become harder and soft binaries become softer.' The division between hard and soft binaries occurs at the semi-major axis of the binary given by (Saslaw 1985)

$$a \approx 4 \left(\frac{m}{m_\odot} \frac{100 \text{ km}^2 \text{ sec}^{-2}}{< V^2 >} \right) \text{A.U.} \qquad (1)$$

The studies also show that the rate of hardening of a binary is independent of the hardness. The intensity of the encounter increases with the hardness but the number of close encounters decrease with the hardness. On account of these two opposing tendencies, the rte of hardening is unaffected by the hardness. Hard binaries interacting with field stars having a Maxwellian distribution of velocities, harden at a constant rate given by

$$\frac{dE}{dt} = \frac{-5.1 G^2 m^3 n}{\sigma}, \qquad (2)$$

where m is the mass of a star, n the number density and σ the one-dimensional velocity dispersion.

Close approaches of a field star with a hard binary can lead to its capture and the formation of a triple system which ultimately breaks up. Usually the least massive of the three stars escapes.

2. Evolution of binary stars in star clusters

Binaries in star clusters, whether primordial or formed by three body encounters, exchange energy with the neighbouring field stars as well as with the more numerous distant stars. The relative contribution of close and distant encounters depends on the velocity distribution of the field stars and the orbital parameters of the binary. It has been found that typically close and distant encounters contribute equally to energy changes.

Soft binaries in clusters tend toward a stationary equilibrium distribution, the disruption of soft binaries being balanced by the formation of new ones by three-body encounters. Hard binaries, on the other hand, tend to accumulate. The number of hard binaries formed in triple encounters in time t is of the order of (Spitzer & Hart 1971)

$$N_{\text{hb}} \approx \frac{1.5}{N \ln(N/2)} \frac{t}{T_{\text{R}}} \qquad (3)$$

where N is the total number of stars and T_{R} the relaxation time.

A more accurate treatment which takes into account also the formation of hard binaries from the hardening of soft binaries yields for the rate of hard binary formation per unit volume (Heggie 1980):

$$\frac{dn_{\text{hb}}}{dt} \approx \frac{100\, G^5 m^5 n^3}{< V^2 >^{9/2}}$$

$$\approx 2 \times 10^{-13} \left(\frac{n}{10^4 \, p_c^{-3}} \right)^3 \left(\frac{m}{m_\odot} \right)^5 \left(\frac{100 \text{ km}^2 \text{ sec}^{-2}}{< V^2 >} \right)^{9/2} p_c^{-3} \text{ yr}^{-1} \qquad (4)$$

It can be seen from this equation that in the central core where the number density is high, hard binaries would form at a more rapid rate.

Hard binaries play a very important role in the dynamical evolution of a globular cluster. The effects of stellar encounters in the cluster tend to increase the central density of the cluster without limit. On the other hand, the energy liberated by the hard binaries in the process of the hardening tends to expand the cluster and counteract the collapse of the cluster. Hard binaries in clusters act as an energy source, some what as do nuclear reactions in stars (Binney & Tremaine 1987). Many of the globular clusters in our galaxy have probably undergone core collapse and are currently re-expanding (Djorgovski & King 1986; Elson *et al.* 1987). Goodman & Hut (1989) suggest that primordial binaries could provide the necessary energy for the re-expansion of the cluster.

It is of great interest to investigate the ultimate fate of a hard binary whose components get more and more tightly bound with the passage of time. If the stars are of large size, the process can culminate in the coalescence of the pair. On the other hand, if the stars are compact objects, the components can approach each other to much smaller separations without touching each other, the relative velocity of the pair reaching a high value in the process. Close encounter with a field star in such a case, would convert part of this internal energy of the binary to the translational energy of the pair and can lead to the ejection of the binary from the cluster.

If all the three stars are of the same mass m, 1/3 of the energy released is carried off by the recoil of the binary and 2/3 by the field star. If $|\Phi_c|$ is the central potential depth of the cluster, the field star will escape if its recoil energy is greater than $m|\Phi_c|$. The binary itself will usually be ejected from the cluster if $|E| > 15\, m|\Phi_c|$ (Goodman 1984).

Hard binaries, especially those involving compact white dwarfs, may become strong x-ray and γ-ray sources. Binaries also form in a star cluster by tidal capture and some of these can lead to stellar mergers (Lee & Ostriker 1986).

3. Binaries in the Galaxy

Since soft binaries become softer as time elapses, we should expect to find larger separations for younger binaries in the Galaxy. This is supported by observations. There are no binaries older than the sun with separations greater than 4000 A.U., while binaries younger than the sun may have separations of about 180,000 A.U. The rate of disruption appears too large to be explained by the effects of field stars. Probably molecular clouds and spiral arms are the most effective agents (Poveda 1988).

Triple stars, consisting of an inner binary revolving around the third star, are common in the Galaxy. If P_0 and P_i are the outer and inner periods, the evolution of the orbits can be well described analytically for $P_0/P_i > 50$. For most known triple stars $P_0/P_i \approx 1000$. In these systems, the inner orbit undergoes periodic changes in eccentricity (Valtonen 1988).

4. Analogues of binary-single body encounters in galaxy interactions

We have seen that an encounter between a binary star and a single star is an inelastic encounter involving exchange of energy between the binary and the star. In interacting galaxies we come across this phenomenon of inelasticity on a considerably magnified scale leading to rapid merger of galaxies (see review by Alladin *et al.* 1988).

Let me draw attention to three aspects of galaxy interactions : (a) Formation of bridges and tails; (b) mass distribution in merger remnants; and (c) dynamical friction These phenomena can also be interpreted by extrapolation of the law of binaries . hard binaries tend to become harder and soft binaries tend to become softer.

Consider a galaxy as a multitude of binaries, each star forming a binary with the galactic centre around which it revolves. Stars in the core of the galaxy, are tightly bound to the galactic centre. These may be considered as forming hard binaries with the galactic centre. On the other hand, the stars in the outer parts of the galaxy, may be considered as forming soft binaries with the galactic interior. In an encounter between two galaxies, the soft binaries become softer. This implies that the outer stars, move away from the galactic centre. This indeed is the case. Under favourable conditions, this leads to the formation of bridges and tails in disc galaxies (Toomre & Toomre 1972)

In galactic encounters leading to the merger of two galaxies, it has been found that the final coalesced object has a higher central concentration of mass and a more extensive envelope than its progenitors (*e.g.* White 1978, Roos & Norman 1979). As expected from the law of binaries, the inner stars of the galaxies move more inward and the outer stars move more outward, with the result that the merger remnant has higher central density and lower peripheral density than the progenitors.

A satellite galaxy moving in the outer parts of the primary galaxy generally experiences dynamical friction and spirals inward (White 1979). In this case, while the outer stars in the primary galaxy form soft binaries with the galactic centre, the satellite galaxy, on account of its much larger mass than a star, generally forms a hard binary with the galactic centre of the primary galaxy. The satellite therefore tends to become a harder binary by spiralling inward. If however the satellite mass is sufficiently small it can behave under certain circumstances as a soft binary component and move outward. The results of N-body simulations show that if the primary is a disc galaxy of mass M, rotating in the same sense as that in which the satellite moves, merger is inhibited if the satellite is of mass less than 0.05 M. Also retrograde companions within a self-gravitating disc may be ejected from the system (for a review, see Byrd 1988). Thus the results of N-body simulations are consistent with the inferences that can be drawn qualitatively from an extrapolation of Heggie's laws.

In binary single-star encounters, energy changes are relatively much smaller than the angular momentum changes. In the case of encounters between two galaxies also, the same is the case. If p is the distance of closest approach of the two galaxies, V the collision velocity, M' the mass of the perturbing galaxy, the energy change of a star in the test galaxy, is a second order effect in

$$\eta \equiv 2GM'/p^2V, \qquad \ldots(5)$$

while the angular momentum change is a first order effect in η (Knobloch 1978). N-body simulations of interacting galaxies, also show that escaping stars carry away a lot of angular momentum but little energy.

A comparative study of the similarities in the areas of binary stars and binary galaxies in a quantitative manner would be of much interest. This may lead to a unified treatment of the dynamical effects.

5. Binary-binary interaction

An area of much current interest with great scope for research is interaction between two binary stars. During the later stages of the collapse of a binary-rich core of a cluster, binary-binary encounters are likely to be more important than binary-single star encounters. Four-body interactions are even more complex than three-body encounters and investigation of the cross-sections and reaction rates for binary-binary scattering has only just begun (Binney & Tremaine 1987).

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Discussion

Kulkarni : You said that in the case of a galaxy density of stars near the centre will go on increasing (harder) while the density of the peripheral stars will decrease. So will there be a region in between where density will remain constant over a very long time?

Alladin : I mentioned that in galaxy interactions leading to mergers, the merger remnant has higher central density and lower peripheral density than the progenitors. In the intermediate region of the merger remnant, the density is likely to be more or less the same as the density in the intermediate regions of the progenitors.

Kulkarni : If third body is not there, what happens to the binary?

Alladin : If the third body does not encounter the binary, obviously it cannot change its binding energy. But some energy change can still occur in the binary on account of the interaction of the binary with the distant stars of the cluster.

Radhakrishnan : The hardening of a binary is the effect of a perturbation. In the absence of a perturber is it not more natural for the tidal forces to soften the binary?

Alladin : In the absence of a perturber, the binary does not become softer. Stellar tidal effects bring the two stars closer and can even lead to merger (see Lee & Ostriker 1986).

Pandey : Is the coalescence of stars in globular clusters in its final phases cataclysmic or slow adiabatic process?

Alladin : I don't know. I think that the coalescence will generally be a slow adiabatic process. If the binary is composed of compact objects, it is more likely to be ejected from the cluster before coalescence occurs.

Abhyankar : Are Heggie's laws of statistical nature or do they have analytical basis?

Alladin : Heggie's laws are of statistical nature. Analytic expressions are available for special cases.

Abhyankar : Is the problem of the effect of the general potential field of the galaxy or cluster on a binary considered so far?

Alladin : M. D. Weinberg, S. L. Shapiro & I. Wasserman (1987, *Ap. J.* **312**, 367) have studied the effect of the galactic tidal field on binaries.

Kochhar : Is the analogy between Heggie's laws and results of binary interactions in galaxies merely a convenient way of remembering the galaxy results, or is it more fundamental?

Alladin : In my opinion, it is more fundamental.

Sapre : Do galaxy collisions have a role to play in the formation and evolution of spiral and elliptical galaxies?

Alladin : Yes, galaxy collisions do play an important role in the formation and evolution of galaxies. A merger of two spiral galaxies leads to the formation of an elliptical galaxy. A dwarf galaxy, merging with a disc galaxy, can contribute to its bulge. It has even been suggested that the reverse Hubble sequence may be an evolutionary sequence [Roos (1985) *Ap. J.* **294**, 479].