Bull Astr Soc India (1990) 18, 177-189

Accretion dynamics in binary systems

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1. Introduction

Of the various forms of electromagnetic radiation that we receive from the distant cosmos x-rays are one of the most prominent. Even before the discovery of cosmic x-ray sources, Hayakawa & Matsuoka[1] had suggested that close binary stars may be detectable as x-ray sources, because of the mass accretion (gas from the companion impinging upon the primary) yielding high temperature plasma which can emit thermal x-rays In a similar vein, Novikov & Zeldovich[2] and Shklovsky[3] had proposed that accretion onto neutron stars and black holes in binaries could produce x-rays emitted as a consequence of the liberation of gravitational binding energy released by the infalling matter The discovery[4] in 1971 of the sources, Cen X-3 and Hex X-1 by the UHURU satellite, exhibiting eclipses and periodic Doppler variations of the pulsation period was really the beginning of the new era in astronomy Further discoveries of new x-ray binaries lead to review articles[5-7] already by 1974 and in 1976 Apparao & Chitre[8] concluding their review article on celestial binary x-ray sources commented that though the role of the magnetic field in the accretion of gas has been studied in the context of models for x-ray pulsars, detailed hydromagnetic model which takes into account the effect of radiative transfer and the presence of shocks was yet to be worked out.

Since then the subject has grown immensely with an ever increasing number of discoveries of new sources and correspondingly proposed theoretical models. Paczynski[9] in 1978 expressed a belief that the majority of strong galactic x-ray sources are interacting close binaries with the optical component being O or B type giants nearly filling their Roche lobe and the x-ray emitter being a neutron star (Cen X-3, Vela X-1, SMC X-1) or possibly a black hole like in Cyg X-1 and Cir X-1. Further it was argued that the type II sources which have a very high ratio of x-ray to optical luminosity could very well be compact objects like neutron star or black hole surrounded by massive (thick) accretion disc (the binary companion being probably detached).

Whether it is high mass x-ray binary (XB) or the low mass one, the main mechanism of x-ray emission is attributed to accretion with mass transfer through stellar wind (spherical accretion) for the high mass ones and being through Roche lobe filling (disc accretion) for the low mass ones. In the case of low mass XB the x-ray spectra being soft it is believed that part of the emission comes from the inner regions of the accretion disc and the rest from the neutron star surface. The theory of accretion has attracted a lot of attention in the last two decades and it is still one of the hottest topics in theoretical astrophysics. Almost all the models constructed for accretion have been in the realm of

Newtonian description of gravitation as it was always believed that the gravitational potential at the site of emission is rather small. However, in the description of accretion discs, the effects of general relativity have been considered by several authors[10-12], an integrated treatment of which may be found in refs 13 and 14. Hanawa[14] showed that for x-ray emission from geometrically thin discs, the general relativistic effects are however *not small* with the maximum temperature of the general relativistic model being lower by a factor of three. compared to that of the Newtonian ones.

The situation for considering general relativity in the accretion dynamics got further strengthened with the discovery of quasi-periodic oscillations in the galactic x-ray sources. Paczynski[15] has pointed out the relevance of the flow through r_{ms} the marginally stable orbit for nonmagnetospheric disc accretion onto neutron stars, which requires the discussion of flow properties as described by general relativity.

In case the compact object is a black hole it is very pertinent to have the general relativistic formalism since the disc could reach almost upto 3m (1.5 times the Schwarzschild radius) with the help of external magnetic fields. The same would apply in the case of extremely compact neutron stars too. Whatever the emission mechanism be, it appears clearly that the accretion in binary systems with self-consistent electromagnetic fields in the presence of intense gravitational field is the physical phenomenon whose dynamics has to be properly understood for constructing models.

With this in mind we have been studying this scenario for the past decade starting with the discussion of charged particle trajectories in electromagnetic fields on curved spacetime and eventually arriving at the discussion of the structure of equilibrium disc configurations of plasma. In the following I shall first briefly summarize certain features of the QPOs, and of accretion dynamics of discs and then describe our attempts at understanding the equilibrium disc configurations with self-consistent electromagnetic fields.

2. Quasiperiodic oscillations

After the discovery[16] of the millisecond pulsar in 1982, there has been a systematic search for fast pulsars in bright galactic bulge sources and during one such search in 1985, Von der Kils *et al.*[17] discovered quasi-periodic oscillations (QPO) from the source GX 5-1 as a broad peak in the power spectrum of the source flux. The observed periods are between 25 and 50 ms with coherence times between 75 and 25 ms, corresponding to a 4 to 6% rms variation in the source intensity. Similar QPOs were observed in Cyg X-2[18] and Sco X-1[19]. Whereas the frequency of the principal peak in the power spectrum for Cyg X-2 is almost similar to GX 5-1, it varies from 6 to 20 Hz in Sco X-1. Though both GX 5-1 and Cyg X-2 show substantial low freuency noise (l.f.n.) in the former the rms variations caused by the QPO and the l.f.n. both remain approximately constant as the source intensity increases whereas rms variation caused by the QPO remains approximately constant but that due to l.f.n. increases with intensity in the latter. In the case of Sco X-1 the QPOs were observed only during the low intensity intervals and during transitions to the quiescent state, and the dependence of the properties of the QPO on source flux is qualitatively different between the low intensity state and the transition state (being similar to the other two in the former while varies erratically in the latter). This indicates that the origin of QPOs may not be the same for all the three sources[20]. Morfil & Trumper[21] question whether all the discovered QPO sources exhibit various features of the same basic physical process, and point out the similarities to be explained (i) absence of the pulsed emission from the neutron star, (ii) QPO source strength ~ 5% of the total source strength; (iii) QPO frequencies in the range of tens of Hz, (iv) weakening of the phenomenon with increasing total luminosity, and finally the presence of l.f.n. As Lamb points out, a variety of physical models have been discussed: (i) oscillation in the accretion disc around the neutron star; (ii) instability of the accretin flow; (iii) QPO variations in the boundary layer, (iv) oscillations of the neutron star surface; and (v) interaction between the magnetosphere of the neutron star and the accretion disc. However, it has been pointed out that of all the attempts, one class of models called the beat frequency model -first investigated by Warner[22] in the context of cataclysmic variable and subsequently discussed by Alpar & Shaham[23] seems to be promising.

The basic suggestion in this model is that due to coupling between the accretion disc and the compact star, with f_k being the orbital (Keplerian) frequency of the plasma in the disc and f_0 the rotation frequency of the star, the oscillations are caused by the quasi-periodic modulation of the accretion flow at the beat frequency $|f_k - f_0|$. Taking the standard scenario of the rotating accretion disc around a rotating compact star with possible dimensions one can then set up constraints on the location of the disc inner edge as well as on the strength of the magnetic field associated with the star.

Paczynski[15], however, asserts that as none of the observed QPO sources is a pulsar, the associated neutron stars may have no magnetosphere and further the radii of the neutron stars are smaller than 6m, and as shown by Sunyaev & Shakura[24] it is possible to liberate between 69% and 86% of the accretion energy when matter falls from the marginally stable orbit at 6m. As steady-state flow may not be possible when the viscosity parameter[25] $\alpha > 0.03$ they suggest that the unsteady flow would make the boundary layer luminosity variable, possibly giving rise to QPO phenomenon.

From the above discussion it is clear whatever be the physical mechanism that gives rise to x-ray emission and QPO phenomena, the basic feature that needs to be clearly understood is the configuration – equilibrium and stability, of accretion discs of plasma around highly compact objects with consistent electromagnetic fields on curved spacetime.

3. Basic features of accretion dynamics

The two main ways of accretion in binary systems are either through stellar wind of the companion or through the Roche lobe overflow[26]. In the former case the orbiting primary comes in as an obstacle in the wind stream of the companion which has expanded and a bow shaped shock forms around the primary and the gas stream accretes onto the primary in a spherically symmetric fall. On the other hand in the case of Roche lobe overflow the expanding gas from the corona of the secondary moves through the inner Lagrange point L_1 onto the orbiting primary (pulled in by the intense gravity of the primary). As this directed flow will have significant angular momentum, the infall will no longer be radial but forms a disc around the compact primary with the matter in the disc being balanced by the centrifugal and gravitational forces. Unless the binary period is quite long, the inflowing gas through L_1 appears to move almost orthogonally to the line joining the centres of the two stars.

Let the configuration be as shown in figure 1.

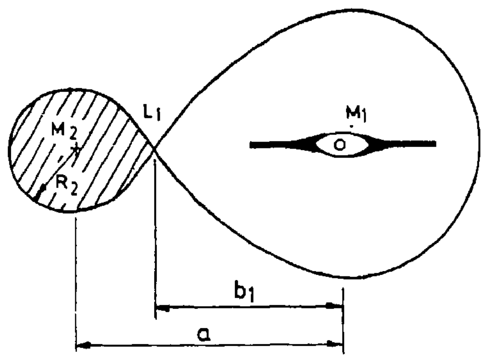

Figure 1. Disc accretion in binaries.

The incoming gas orbits the primary in the binary plane at a distance R_c such that the Keplerian orbit at R_c has the same angular momentum as the transferring gas had on passing through L_1:

$$V_\varphi(R_c) = (G M_1/R)^{1/2} \quad \ldots(1)$$

with

$$R_c V_\varphi = b_1^2 \omega. \quad \ldots(2)$$

Here b_1 is the distance of L_1 from the centre of M_1 and $\omega = 2\pi/P$, P being the binary period. Simplifying and using Kepler's law

$$4\pi^2 a^3 = G(M_1 + M_2) P^2, \quad \ldots(3)$$

one gets

$$R_c = (1 + q) b_1^4/a^3, \quad \ldots(4)$$

q being the mass ratio M_2/M_1.

This gives one the necessary condition on dimensions of the disc once the mass and radius of the primary are known, along with the binary period. As the disc is composed of matter with bulk motion there could be dissipative processes giving rise to a redistribution of angular momentum and subsequent infall of the material from the disc onto the primary surface. As the gas element starts at distances quite far from the primary, with very little binding energy, the total disc luminosity in steady state is

$$L_{disc} = G M_1 \dot{M}/2R, \quad \ldots(5)$$

$\dot{M}$ being the rate of accretion. This disc luminosity is just about half the accretion luminosity which is obtained when all the kinetic energy of infalling matter is given up as radiation at the stellar surface, $L_{acc} = GM_1\dot{M}/R$. Thus about half the kinetic energy of matter would be lost due to disc luminosity and the dissipative processes produce torque that transport angular momentum outwards and finally away from the disc perhaps into the binary orbit. However, the time scales over which this happens could be long enough (in fact it depends on the nature of viscosity) so that one can conceive of disc configurations existing in equilibrium around the primary with the inner edge at R_c and the outer edge inside the Roche lobe of the primary. Most of these discs are generally assumed to be thin (vertical extent far less than the radial extent) whereas there do exist discussions of thick discs with the inner edge being blown up by the radiation pressure. In either case the gas in the inner regions of the disc would get heated up to temperatures $\sim 10^5$-10^6 K and become a source for x-rays. Further these soft x-rays scattering with hot relativistic electrons of the plasma achieve greater energy by inverse Compton scattering and appear as hard x-rays, observed in the x-ray binaries.

There are several models for compact x-ray sources in terms of accretion discs, one of the earliest being due to Pringle & Rees[27]. Subsequently several improvements were made and a detailed review of this may be found in Lightman *et al.*[28]. All these models come under the nomenclature of standard accretion disc models (SADM), or α-model as they all assume the same viscosity law namely that the transverse stress $t_{r\pi} = \alpha\ P$ where P is the pressure and α a dimensionless parameter (<1). In these models the disc is supposed to consist of three regions:

(i) inner region $p_r \gg p_g$, $n^{es} \gg n^{ff}$; (ii) middle region $p_r \ll p_g$, $n^{es} \gg n^{ff}$, (iii) outer region $p_r \ll p_g$, $n^{es} \ll n^{ff}$.

Here p_r and p_g denote the radiation pressure and gas pressure; n^{es} and n^{ff} are the opacities due to electron scattering and free-free absorption. Both these characteristics were first suggested by Shakura & Sunyaev[24] who further showed that the energy production rate Q is zero at $r = r_i$ the inner edge of the disc and reaches a maximum at $r = (49/36)$ m and decreases as r^{-3} for large r. The total luminosity L,

$$L = 4\pi \int_{r_i}^{r_o} Q\, r\, dr = \frac{1}{2} \dot{M} M\ G/r_i, \qquad \ldots(6)$$

is independent of the nature of the dissipative forces and depends only on the accretion rate $\dot{M}$ and the inner radius r_i. This obviously shows that having the inner edge closer to the central compact star would increase the luminosity. The dynamics of the disc as envisaged in SADM is governed by the laws of conservation of mass, angular momentum, energy, vertical momentum, nature of viscosity, and the law of radiative transfer from inside the disc to its upper and lower surfaces. The gas (plasma) in the disc is generally assumed to be in orbit mainly by rotation with Kepleriam angular velocity thus keeping in balance against the gravitational pull of the central body while the self-gravitation of the disc is being neglected. When the disc is thick, the velocity in the vertical direction is assumed to be subsonic and the vertical structure is governed by the law of hydrostatic balance. The energy produced owing to the friction is transferred to the disc surfaces and the medium is considered to be optically thick with the opacity due to the Thomson scattering and free-free absorption. In order to discuss the stationary state one also needs an equation of state relating the matter density and the total pressure $(p_g + p_r)$ and an equation connecting the radiation density with the thermodynamic properties of the gas. Stability of such disks was considered by many authors[24-29] and it was generally found that the inner regions of the disc are secularly and thermally unstable. However, as this disc structure did not consider the effect of pressure gradient forces the model had to be changed and with the inclusion of such forces the motion is no longer Keplerian and the disc would get thicker. Abramowicz *et. al.*[30] showed that the disc will have a vertical structure with the inner edge forming a cusp between r_{ms} and r_{mb}. Chakraborty & Prasanna[31] explicitly showed that the existence of a cusp at the inner edge between r_{ms} and r_{mb} on the equatorial plane is apparent only in the general relativistic formulation, where as in the purely Newtonian picture it is not seen. With thick discs the accretion rate could become supercritical leading to critical luminosities approaching the Eddington luminosity. Several attempts[32] have been made to construct thick discs that are dynamically stable in that $dl/dr > 0$ and $d\Omega/dr < 0$, $l(r)$ and $\Omega(r)$ being the specific angular momentum and angular velocity respectively. As viscosity is the agency to transport angular momentum, convection is likely to be the agency for vertical transport of energy. Robertson & Taylor[33] have found that for a wide range of viscosity laws discs which are thermally unstable have convective instabilities too Regions dominated by radiation pressure seem to be unstable under convection but convection itself is likely to occur only if the gas pressure is dominant. Though a simple analysis suggests that convection carrying a sufficiently large fraction of energy may remove thermal instability, a more detailed study has shown that radiation always carries most of the energy and convection only increases the critical value of radiation pressure for instability without actually removing it.

However, inspite of all these studies no clear understanding of the disc dynamics has been possible as neither the nature of viscosity nor the role of electromagnetic fields had been considered. It is true that in the α-models it was assumed that the viscosity may be due to the small scale magnetic fields or turbulence but no analytic expression was utilised explicitly including the electromagnetic fields. In fact as pointed out by Lightman *et al.* at the high temperatures attained close to the compact object, the particle mean free paths are so long that a fluid dynamical treatment is not really self-consistent unless collective effects are operative. As they pointed out the only way to overcome this difficulty is by taking into account the effects of interstellar magnetic fields which even if initially negligible, due to the stretching of field lines during inflow that makes the magnetic energy density vary as r^{-4}, will become dynamically important.

Bisnovatyi Kogan & Blinnikov[34] and Ichimaru[35] have considered the effect of magnetic field on the accreting plasma and found that there could be an increase in the efficiency of radiation emission and that the turbulence is generated mainly by the differential rotation of plasma which decays through current dissipation due to anomalous magnetic viscosity. This feature when taken into account in the study of disc dynamics has revealed the existence of two physically distinct states in the middle part of the disc which are thermally stable. A more rigorous treatment of the magnetic field generation due to differential motion of conductive media in discs was made by Galeev *et al.*[36] who found that even the fastest reconnection mechanism is not rapid enough to develop effectively in the inner portions of the disc and that the building up of the magnetic fields within the disc is instead limited by nonlinear effects related to convection. An important result of this analysis is that the disc could develop a magnetically confined structured corona consisting of many smallscale extremely hot coronal loops which could emit both soft and hard x-rays depending upon the disc luminosity. Though some of these studies are quite rigorous, the fact that the analysis is purely Newtonian could become a constraint in certain situations.

Ghosh & Lamb[37] considered in quite detail the accretion by rotating magnetic neutron stars and put some constraints on the possible models. Using the solutions to the two dimensional hydromagnetic equations they calculate the torque and find that the magnetic coupling between the star and the plasma in the outer transition zone is appreciable as a result of which the spin up torque on fast rotators is substantially less than on slow rotators and for sufficiently high stellar angular velocities or sufficiently low accretion rates, the stellar rotation can be braked even as the accretion continues.

4. General relativistic accretion discs with self consistent electromagnetic fields

As the study of dynamics of discs arose in the context of binary stars, it is quite natural to expect situations when the primary compact object is a neutron star with sufficiently high magnetic field Further as the disc matter is in the state of ionized gas (plasma), the rotating plasma discs would generate currents and consequent electromagnetic fields which have to be taken self-consistently with the external magnetic field of the neutron star or the seed field of the background.

Some studies[38] considering accretion discs in the presence of magnetic fields around compact objects have been made and in almost every case the magnetic field was entirely due to the disc plasma alone as the compact object considered was a black hole Prasanna and coworkers[39] have studied this problem systematically starting with the analysis of the charged particle trajectories in electromagnetic fields on curved space time and later on[41-43] discussed the equilibrium configuration of disc structures of plasma with a self consistent electromagnetic field pervading both inside and outside the disk.

As calculated earlier[39] if the plasma density in the disc is very low $N < 5.5\ T^2$, one might be able to understand the dynamics from the orbit theory alone and in this context it has been shown that the presence of even a weak magnetic field around the central compact object would result in the charged particle having stable orbits quite close to the event horizon. Though generally the particle gyrates around the field lines as viewed on the equatorial plane, in the Kerr geometry, due to the dragging of inertial frames, the particle if inside the ergosurface cannot gyrate. Further considering the motion of the charged particle off the equatorial plane it has been shown that due to the presence of the central compact object even a uniform magnetic field outside would develop field gradient which in turn bunches the magnetic field lines in such a way that the particles can get trapped in 'banana' orbits thus possibly giving rise to equilibrium disc-like configurations wherein magnetic field plays an important role.

We then considered the case of a charged fluid disc rotating around a Schwarzschild object with no external field and considered its stability under radial perturbations. Though it was found to be stable we realised that the situation considered was too academic as no dissipative terms were introduced. More recently we have taken up the study of accretion discs around compact objects with electromagnetic fields—test disc and test fields—implying that the background geometry is fully specified by the central compact object and that neither the disc nor the electromagnetic field perturbs the geometry. The matter in the disc is taken to be plasma with viscosity and finite conductivity. However the equations of motion for the disc are obtained through the conservation laws

$$T^{ij}_{;j} = 0, \qquad \ldots(7)$$

$$T^{ij} = (\rho + \bar{p})\, u^i u^j - \bar{p} g^{ij} + 2\eta_s \sigma^{ij} - E^{ij}, \qquad \ldots(8)$$

$$\bar{p} = p - \left(\eta_b - \frac{2}{3}\eta_s\right)\Theta\ , \quad \Theta = u^i_{;i}. \qquad \ldots(9)$$

Here ρ is the density; p the pressure; η_b and η_s the cofficients of bulk and shear viscosity; σ^{ij} the shear tensor; Θ the expansion parameter; E_{ij} the electromagnetic stress energy tensor

$$E_{ij} = F_{ik} F^k_j - \frac{1}{4} g_{ij} F_{kl} F^{kl} \quad \ldots(10)$$

alongwith the Maxwell's equations

$$F^{ik}_{;k} = - J^i, \quad F_{(ik,j)} = 0; \quad \ldots(11)$$

J^i the current being defined through the generalised Ohm's law

$$J^i = \epsilon\, u^i + \sigma F^{ik} u_k; \quad \ldots(12)$$

ϵ and σ are the net charge density and the conductivity respectively. U^i, the velocity four vector, is timelike and satisfies the orthonormality relation

$$g_{ij}\; u^i\, u^j = \pm 1. \quad \ldots(13)$$

Though in principle the system of equations appears sufficient to determine all the unknowns, the use of symmetries (which are essential) reduces some of the equations to identities making the system indeterminate. In such situations, one needs to use an equation of state to close the system.

It is useful to use the spatial 3-velocity $V^\alpha = U^\alpha / U^1$ and rewrite the equations so that one can compare the system with those of ordinary hydrodynamics after appropriately considering the components in local Lorentz frames. The system of governing equations on the background geometry as given by linearized Kerr space time

$$ds^2 = \left(1 - \frac{2m}{v}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2$$

$$- r^2 \sin^2\theta\, d\varphi^2 + \frac{4\,am}{r} \sin^2\theta\, dt\, d\varphi, \quad \ldots(14)$$

where a/m is assumed to be $\ll 1$, is presented in detail by us[41]. I will now briefly discuss various special cases that have been studied.

Case 1

Consider a non-rotating central star (with a dipolar magnetic field at infinity) with an infinitely conducting disc rotating around it:

$$a = 0,\ \eta_s = 0,\ V^r = 0,\ V^\theta = 0,\ \sigma = \infty,$$

The force free condition determines the electric field in terms of B_r, B_θ and V^φ, and consequently the Maxwell's equations provide the constraint equation for V^φ whose general solution is given by

$$V^\varphi = \frac{1}{r \sin\theta}\left(1 - \frac{2m}{r}\right)^{1/2} V^{(\varphi)},$$

$$V^{(\varphi)} = K(1 - 2m/r)^{-1}\; r^{(3-n)/2} \sin^n\theta, \quad \ldots(15)$$

K and n being arbitrary constants. Assuming a thin disc ($\theta = \pi/2$) configuration one can then numerically solve the equation for pressure for the case of incompressible fluid (ρ = constant) and obtain the pressure profiles, using the boundary condition that at the inner edge (r_a) the hydrostatic pressure p_g is equal to the magnetic pressure

$$p_M = B_0^2 R^6 / r_a^6.$$

By choosing the constant appropriately it is possible to have equilibrium configurations with reasonable (plausible) pressure profiles as shown in figure 3 (ref 41).

Case 2

The other factors being same, in the case of finite conductivity one does not have the force free condition and accordingly no direct connection exists between E and B fields As the momentum equations do not give any clue to the nature of V^{φ}, one has to look for possible equilibrium structures for assumed velocity distributions. For relativistic Keplerian angular velocity

$$V^{(\varphi)} = \sqrt{(MG/r)(1-2m/r)}, \qquad \ldots(16)$$

one can obtain plausible equilibrium pressure distributions for the case of incompressible fluids and having again the current components $J^{(\varphi)}$ and $J^{(t)}$ non-zero. However, one notices the difference in the r-dependence of $V^{(\varphi)}$ in the two cases σ infinite, and σ finite, which may be relevant for further discussions.

Further in the case of an infinite-conductivity disc one can introduce a constant $\beta = K^2 m^{(3-n)}/c^2$ in terms of which the angular velocity $V^{(\varphi)}$ becomes

$$(V^{(\varphi)}/c)^2 = \beta R^{(3-n)} (1-2/R)^{-1}, \quad R = r/m, \qquad \ldots(17)$$

such that for $n = 4$ and $\beta = 1$ the velocity is relativistic Keplarian. From the equations it will now become apparent that in order to keep $V^{(\varphi)} < c$ depending upon β and whether $n > 3$ or < 3 the disc will have a constraint on its inner radius or outer radius. For example, $n = 4$, the inner radius $r_a > 2 + \beta$ and thus for $V^{(\varphi)}$ to be relativistic Keplerian with $\beta = 1$, $r_a > 3m$ whereas $\beta < 1$, r_a may be less than $3m$. If $n = 2$ then $\beta = 1$ does not seem to admit any reasonable velocity distribution. In fact for $n < 3$ as $V^{(\varphi)}$ increases with R, β has to be extremely small ($\ll 1$) for any plausible equilibrium configuration. Bhaskaran and Prasanna[42] have shown the pressure profiles for different n and β values clearly showing the type of combinations for which the equilibrium configurations are plausible.

Case 3 (ref. 44)

As the disc configurations are to be linked to final accretion on to the compact star it is necessary to consider the dynamics with the radial velocity of the fluid $V^r \neq 0$. Further as the bulk viscosity coefficient appear only along with the gradient of V^r, one can also look for the effects on equilibrium configurations of η_b. Considering the relevant equations from the general setup the solution for the velocity and electromagnetic fields are given by

$$V^{\varphi} = \frac{L}{r^2 \sin^2\theta}(1-(2m/r)), \qquad \ldots(18)$$

$$V^r u^t = -\frac{c^2}{4\pi\sigma}\frac{1}{r^2}\{km + (2-k)(r-2m)\ [1 + \cot^2\theta\,(1-(3m/r))^{-1}]\}, \qquad \ldots(19)$$

$$u^t = (1 - (2m/r))^{-1/2} (1 - (r^2/c^2))^{1/2}, \quad V^2 = (V^{(r)})^2 + (V^{(\varphi)})^2, \qquad \ldots(20)$$

$$B_r = - A r^{k-1} (1 - (2m/r))^{-k/2} \sin^{k-1}\theta \cos\theta, \qquad \ldots(21)$$

$$B_\theta = A r^k (1 - (3m/r)) (1 - (2m/r))^{-(k/2)-1} \sin^k\theta, \qquad \ldots(22)$$

$$E_r = B_\theta V^\varphi, \quad E_\theta = - B_r V^\varphi, \quad J^r = 0, \quad J^\theta_t = 0, \qquad \ldots(23)$$

$$J^\varphi = - \frac{\sigma}{r^2 \sin^2\theta} B_\theta V^r u^t, \quad J^t = - \sigma \left(1 - \frac{2m}{r}\right)^{-1} V^\varphi V^r B_\theta u^t. \qquad \ldots(24)$$

The continuity equation alongwith the radial momentum equation gives the generalized equation for $\dot{M}$ the accretion rate:

$$(\rho + (\bar{p}/c^2)) (1 - (2m/r)) (1 - (r^2/c^2))^{-1} r^2 V^{(r)} = - \dot{M} \qquad \ldots(25)$$

leaving finally two equations for the two unknowns ρ and p, thus having a completely determinate system. Restricting the discussion to the case of thin disc $\theta = \pi/2$, one can then obtain the equilibrium pressure and density profiles for a given accretion rate $\dot{M}$.

As regards the structure of the magnetic field the solutions (21) and (22) give the field inside the disc. For outside the disc we assume the modified dipole field as obtained by Ginzburg & Ozernoi[38a]. The condition for the continuity of the field lines gives the unknown constant A in terms of the surface magnetic field strength of the compact object B_s. For a given angular momentum L and the field strength B_s one can obtain a relation between the density at the outer edge ρ_0 and the accretion rate $\dot{M}$ through the boundary condition that the pressure at the outer edge is equal to the sum of the magnetic pressure at that radius and one-third the energy density at that point

$$(p)_{rb} = (p_M)_{rb} + \rho_0 c^2/3$$

alongwith equation (25) In fact this relation also relates the parameters B_s and ρ_0 and as one integrates from outer radius to the inner edge one gets the condition on the location of the inner edge (see tables in ref 44).

What is important is to have the disc inner edge move inside $r < 6$m region and this one has already seen is possible in the presence of magnetic field When $V^r = 0$, it was already found that for different velocity distributiuons it is possible to have the disc inner edge close to $r = 3$m. With the inclusion of the radial velocity V^r what we find is that bringing the disc inner edge close to $r = 3$m puts in a natural restriction on the parameters ρ_0, M and B_s. This is in fact very significant for one cannot have the disc very close to the primary with a magnetic field unless the outer density is proportionate. As a specific example it may be seen that for $B_0 = 10^7$G, if r_a has to be close to 3m then $\rho_0 > 5 \times 10^{-10}$ gm cm^{-3} or 5×10^{21} particles m^{-1}, with the corresponding accretion rate $\dot{M} > 7 \times 10^{11}$ gm s^{-1} or 10^{-14} $M_\odot$ yr^{-1}. This matches well with the case of x-ray binaries with a neutron star primary and a giant secondary. In fact from a look at the tables it appears that for a variety of plausible equilibirium disc configurations with the outer density corresponding to about the coronal densities of giant stars giving a mass accretion rate of $\sim 10^{-13} \sim 10^{-15}$ $M_\odot$ yr^{-1}, the disc inner edge would reach almost upto 3m only if the surface magnetic field of the neutron star is $< 10^{10}$ G. The pressure and density profiles for various parameter configurations are given in figures in ref. 44.

Disc luminosity

Page & Thorne[12] while discussing the disc accretion onto black holes have obtained an explicit algebraic expression for the radial dependence of the time averged energy flux emitted from the disc's surface. Assuming the disc to reside on the equatorial plane of a Kerr black hole and be thin with its material moving in nearly circular geodesics with negligible heat transport radially they obtain the time averaged flux of radiant energy f emitted from the disc surface to be

$$f=\frac{3}{2m}\frac{1}{x^2(x^3-3x+2a^*)}\Big\{[x-x_0-\frac{3}{2}a^*\ \ln\ (x/x_0)]$$
$$-\frac{3(x_1-a^*)^2}{x_1(x_1-x_2)(x_1-x_3)}\ln\Big(\frac{x-x_1}{x_0-x_1}\Big)$$
$$-\frac{3(x_2-a^*)^2}{x_2(x_2-x_1)(x_2-x_3)}\ln\Big(\frac{x-x_2}{x_0-x_2}\Big)$$
$$-\frac{3(x_3-a^*)^2}{x_3(x_3-x_1)(x_3-x_2)}\ln\Big(\frac{x-x_3}{x_0-x_3}\Big)\Big\}$$

wherein $a^*=a/m$, $x=(r/m)^{1/2}$, $x_0=(r_{ms}/m)^{1/2}$ with x_1, x_2, x_3 being the three roots of the cubic $x^3-3x+2a^*=0$. As used by Luminet[41] and Hanawa[14] the expression is quite simple for the Schwarzschild spacetime ($a=0$);

$$F=\frac{3GM\dot{M}}{8\pi r^3}\Big(1-\frac{3m}{r}\Big)^{-1}\Big[1-\sqrt{6m/r}+\sqrt{3m/r}\ \ln\Big\{\frac{(1+\sqrt{3m/r})}{(1-\sqrt{3m/r})}\ \frac{(1-(1/\sqrt{2}))}{(1+(1/\sqrt{2}))}\Big\}\Big]$$

If one considers the energy observed by the distant observer E_{DO} measured in terms of the local (disc) observer E_{LO} one finds for the general velocity profile[17] the ratio

$$E_{DO}/E_{LO}=(1-2R^{-1}-\beta\ R^{3-n})^{1/2}$$

for the inclination angle $i=0$ (i.e. when the observer's line of sight is perpendicular to the disc plane). For $i=\ /2$ it is

$$E_{DO}/E_{LO}=(1-2R^{-1}-\beta R^{1-n})^{-1/2}\ (1-(2R^{-1})^{1/2}[(1-(2R^{-1})^{1/2}\pm\beta^{1/2}\ R^{(3-n)/2}]$$

The spectral fit to observed temperature in terms of effective temperature appears as in figures 11-16 of ref. 42 and in figures 1 and 3 of ref. 14 clearly indicating that whereas pure Newtonian analysis would require a fit through several black body temperatures, with the general relativistic analysis the inner region emission can be fitted on by a single black body temperature.

In conclusion one can say that in order to understand the complete picture of the binary x-ray sources and the QPO phenomena, it is very necessary to make a systematic analysis of the dynamics of plasma discs with self consistent electromagnetic fields in strong gravitational fields as described through space-time structure. As most of the interesting physical phenomena would occur in regions very close to highly compact objects it is indeed necessary to use the general relativistic formalism for discussing the equilibrium and stability of disks around compact objects, particularly when the compact object is a member of a binary system.

It is indeed a pleasure to dedicate this article to Professor K. D. Abhyankar on the occasion of his sixtieth birthday.

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Discussion

Bhat : By considering the role of the magnetic field along with the gravitational field, your work shows that it is possible to have 'stable' orbits down to 2-3 m radii. What triggers the instability manifesting itself as QPO?

Prasanna : The instability of QPO would be due to instability in the fluid flow in the regions between 3m and 4m which is in fact a collective effect. Stability of single particle orbits need not guarantee the stability of fluid flow.

Bhat : Can you comment on the nature of the neutron star' which is required in this scenario?

Prasanna . The nature of the neutron star is that it should be highly compact with radius in the range of 2 to 3.5 m.