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Thermal-convective instability in a stellar atmosphere

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The stellar chromospheres, coronae and the interstellar medium may exhibit thermal-convective instability. The Schwarzschild criterion is applicable in the interior of a star, where the photon mean free path is small, the assumption that the motion is adiabatic is justified. The departure from adiabatic motion may be significant in the outer layers of stellar atmosphere, where the effective heat transfer is no longer prevented by opacity. The Schwarzschild criterion for convection has been generalized to include departures from adiabatic motion by Defouw (1970). He has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient, if

$$D = \frac{1}{C_p}(L_T - \rho\alpha L_\rho) + \kappa k^2 < 0 . \qquad \ldots (1)$$

Here L is the energy lost minus the energy gained per gram per second. L_T, L_ρ are partial derivatives of L with respect to temperature and density, both evaluated in the equilibrium state. In general, instability due to inequality (1) may be either oscillatory or monotonic.

Defouw (1970) has also studied, separately, the effects of a uniform rotation and a uniform magnetic field on thermal-convective instability of a stellar atmosphere. For situations of astrophysical interest ($\nu = 0$), inequality (1) has been found to be a sufficient condition for monotonic instability.

Sharma (1974) and Sharma & Prakash (1974) have considered the effects of Hall currents and finite Larmor radius and found that $D < 0$ is a sufficient condition for monotonic instability.

The conditions under which convective motions are important in stellar atmospheres are usually far removed from the considerations of a single-component fluid and rigid boundaries and, therefore, it is desirable to consider one gas component acted on by solute concentration gradient (*e.g.* hydrogen gas acted on by helium as solute) and free boundaries.

Keeping in mind such astrophysical situations, Sharma & Sharma (1984) have considered the thermal-convective instability of a stellar atmosphere in the presence of stable solute gradient, wherein the effects of uniform rotation and uniform magnetic field have also been considered. The criterion for monotonic instability have been derived as

$$D < 0 \text{ and } |(\nu\kappa' k^4 + \Gamma'\beta')D| > \Gamma\left(\beta + \frac{g}{C_p}\right)\kappa' k^2, \qquad \ldots (2)$$

where $\Gamma = \frac{g\,\alpha (k_x^2 + k_y^2)}{k^2}$, $\Gamma' = \frac{g\,\alpha' (k_x^2 + k_y^2)}{k^2}$.

Criteria (2) hold good if Hall current and finite Larmor radius are taken into account (Sharma & Sharma 1980). Recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of Martian atmosphere (Pollack 1975). Thermal-convective instability in the presence of stable solute gradient of a stellar atmosphere has been considered by Sharma & Singh (1987) to include the effect of suspended (dust) particles. The criteria (2) still hold good for monotonic instability.

Generally, the magnetic field has a stabilizing effect on instability. But a few exceptions are there. For example, Kent (1966) has studied the effect of a horizontal magnetic field, which varies in the vertical direction $H(H_0(z), 0, 0)$, on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of a magnetic field the system is known to be stable. In stellar interiors and atmospheres, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. The effect of a variable magnetic field has been considered on thermal-convective instability in the presence of a stable solute gradient.

Consider an infinite horizontal fluid layer of thickness d heated from above and subjected to a stable solute gradient so that temperatures and solute concentrations at bottom surface ($z = 0$) are T_0, C_0; and at the upper surface ($z = d$) are T_1, C_1.

The linearized perturbation equations are (Chandrasekhar 1981, Sharma and Sharma 1984)

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_0}\nabla \delta p + \nu \nabla^2 \mathbf{v} - \mathbf{g}(\alpha\theta - \alpha'\gamma) + \frac{\mu_e}{4\pi\rho_0}\Big[(\nabla\times\mathbf{h})\times\mathbf{H} + (\nabla\times\mathbf{H})\times\mathbf{h}\Big], \quad \ldots(3)$$

$$\nabla \cdot \mathbf{v} = 0,\ \nabla \cdot \mathbf{h} = 0, \quad \ldots(4)$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{H}) + \eta\nabla^2\mathbf{h}, \quad \ldots(5)$$

$$\frac{\partial \gamma}{\partial t} - \kappa'\nabla^2\gamma = \beta' w. \quad \ldots(6)$$

First law of thermodynamics is

$$C_v \frac{dT}{dt} = -L + \frac{K}{\rho}\nabla^2 T + \frac{p}{\rho^2}\frac{d\rho}{dt}. \quad \ldots(7)$$

The linearized perturbation form of equation (7), following Defouw (1970), is

$$\frac{\partial \theta}{\partial t} + \frac{1}{C_p}(L_T - \alpha\rho L_\rho)\theta - \kappa\nabla^2\theta = -\left(\beta + \frac{g}{C_p}\right) w. \quad \ldots(8)$$

Boundary conditions. The case of two free boundaries is the most appropriate for stellar atmospheres (Spiegel 1965). Appropriate boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = \gamma = 0. \quad \ldots(9)$$

We seek solutions whose dependence on x, y, z and t is of the form

$$(\sin k_z z)\exp(ik_x x + ik_y y + nt),$$

where $k_z = s\pi/d$ (s is any integer), n is growth rate and $k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$ is the wavenumber of perturbation.

Eliminating u, v, θ, γ and h_x, h_y, h_z from equations (3)-(6) and (8), we obtain the dispersion relation:

$$n^4 + [k^2(\nu + \eta + \kappa') + D]n^3 + \Bigg[\kappa' k^2 D + \nu\eta k^4 + k^2(\nu + \eta)(\kappa' k^2 + D) + \Gamma\left(\beta + \frac{g}{C_p}\right) + \Gamma'\beta' + k_x^2 V_A^2\left(1 - \frac{k_z^2}{k^2}\right)\Bigg] n^2 + \Bigg[\kappa' k^4(\nu + \eta) D + \Gamma'\beta'(D + \eta k^2) + \Gamma\left(\beta + \frac{g}{C_p}\right)(\kappa' + \eta)k^2 + (D + \kappa' k^2)\left\{\nu\eta k^4 + k_x^2 V_A^2\left(1 - \frac{k_z^2}{k^2}\right)\right\}\Bigg] n + \Bigg[\eta k^2\left\{(\nu\kappa' k^4 + \Gamma'\beta')D + \kappa' k^2 \Gamma\left(\beta + \frac{g}{C_p}\right)\right\} + \kappa' k^2 k_x^2 V_A^2 D\left(1 - \frac{k_z^2}{k^2}\right)\Bigg] = 0, \quad \ldots (10)$$

where $V_A^2 = \dfrac{\mu_e H^2}{4\pi\rho_0}$ is the square of the Alfvén velocity.

When inequalities (2) are satisfied, the constant term in equation (10) is negative. This means that equation (10) has one positive real root, implying monotonic instability. The criteria for monotonic instability (2) thus hold good in the presence of a variable horizontal magnetic field on thermal-convective instability in presence of stable solute gradient in a stellar atmosphere and appear to be general result in the presence of various effects of astrophysical interest.

Nomenclature

ν = kinematic viscosity
η = electrical resistivity
κ = thermal diffusivity
κ' = solute diffusivity
β = temperature gradient
β' = solute concentration gradient
ρ = density
α = coefficient of thermal expansion
α' = coefficient of solute expansion
μ_e = magnetic permeability
θ = perturbation in temperature
γ = perturbation in solute concentration
p = pressure
C_p = specific heat at constant pressure.

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Discussion

Trehan : If you have $H_x = H_x(z)$, you will have a certain current distribution in the system. Has this been taken into account?
Sharma : Yes.
Abhyankar : Have you applied your results to any specific situation or stars?
Sharma : These results are applicable to stellar chromospheres, coronae, and interstellar medium. We plan to apply these to specific situations like stars, sun etc.