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Centre-limb Variation of Solar Excitation Temperature as derived from Ti I lines

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ABSTRACT

The centre-to-limb variation of the equivalent widths of 55 lines of the neutral titanium atom in the wavelength range 5250A-4300A have been studied by the method of curve of growth. The majority of the lines studied (47 out of 55) show a systematic *increase* in equivalent width from the centre of the solar disk to the limb; even in the few remaining cases no reliable evidence is found of the existence of a maximum in the variation of equivalent width in the neighbourhood of Cos $\theta = 0 \cdot 4$. The behaviour of the Ti I lines thus appears to be just the opposite of the Fe lines which, according to previous workers, show a steady *decrease* from the centre of the disk to the limb The excitation temperature for the centre of the disk as derived from the Ti lines measured is about 4540° which is in good agreement with the mean of the temperatures determined by Prouse, Menzel, King, and Wright, but is considerably lower than the value obtained by Bruggencate and Houtgast. According to the measurements reported in the present paper the excitation temperature decreases steadily from the disk-centre towards the limb up to the point sin $\theta = 0.96$ where it is about 4080° K.

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INTRODUCTION

In the central frequencies of strong Fraunhofer lines of the solar spectrum the absorption coefficient is so large compared with the continuous absorption coefficient outside the lines that it is reasonable to think of the centres of these lines as being formed above the bulk of the photosphere. However, the levels at which the centres of the various Fraunhofer lines are formed are different for different lines; and there is also a gradual lowering of the level at which the wings of a given Fraunhofer line are formed, as we proceed from the centre of the line, where the level is relatively high, to the extreme wings, where it is almost co-extensive with the photosphere. Furthermore, at the centre of the disc the observer perceives radiation from deeper and hotter layers, while near the limb from more superficial and cooler layers. Thus the constitution of the solar atmosphere is much too complicated to be represented by any simple model derived from observational data so far as available at present. Nevertheless, a study of the Fraunhofer spectrum, from the disc-centre to the limb can furnish much valuable information on the structure and the physical state of the solar atmosphere, not obtainable from a study of the continuous spectrum.

Studies of the centre-to-limb variation of Solar Fraunhofer lines have been made by many observers among whom mention may be made of Royds and Narayan (1936), M.G Adam (1938), Shane (1941), Allen (1949), Charlotte Pecker and Jean Pecker (1949), Barocas and Righini (1951), Suemoto (1951), Houtgast (1942), (1952), Priester (1953) and Scheffler (1955). However, except for the pioneering work of Righini (1933) on the centre and limb spectra and the investigation of two Fe-multiplets near the limb by Bruggencate and Houtgast (1941), very few observational studies appear to have been made of the centre-limb variation of the equivalent widths of the solar Fraunhofer lines by the method of curve of growth with the help of the available experimental gf values* The present paper aims at contributing in some measure towards the filling of this lacuna.

Observational Material.

The experimental arrangement for this investigation was as follows · A metal disc mounted upon an adjustable stand was placed just in front of the slit of a large plane-grating spectrograph** in Littrow mounting. Sun-light reflected by a 45-cm Foucault siderostat was focussed by a 30-cm photovisual objective to a solar image of about 60 mm diameter on this disc. The disc, which was capable of being turned round its centre, had a circle of the same diameter as the solar image engraved upon it. The image was carefully guided over this circle. The disc had a graduated slot through which a brass strip containing a tiny, central hole could be moved. With the help of this device, light from any desired point of the sun's image could be admitted into the spectrograph. The exposures were made for the various points of the solar disc successively on the same photographic plate. In addition to the solar spectra, each plate contained a series of varying density marks for determining its characteristic curve.

The standardisation marks were obtained with an auxiliary plane-grating spectrograph using a previously calibrated step-slit uniformly illuminated by a Philips tungston ribbon lamp fed by a 6-volt accumulator with a current of 15 to 16 amperes. The procedure for impressing the density marks on the plates was the same as described in the work of Das and Ramanathan (1953).

The spectrograms were obtained in the second order of the grating on several days in the early hours of the morning during the months of December, January and February 1954-55 when the seeing conditions were good. The photographic plates $8\frac{1}{2}'' \times 6\frac{1}{2}''$ used were Ilford Process for the region 4300A-4600A, Ilford Selochrome for the region 4600A-5000A and Ilford Special Rapid Panchromatic for the region 5000A-5300A. All the plates were developed for 5 minutes in M.Q. developer, taking the usual precautions to eliminate Eberhard effects. It was found that an exposure varying between 8 and 20 seconds gave suitable densities with the step-slit for microphotometry in all the regions, while the exposure times for the spectrum of the centre of the solar disc varied between 3 and 7 seconds The ratio between the exposure time for the solar spectrum and that for standardisation was always less than three. For this ratio, it was assumed that the characteristic curve of the plate was not affected. Since we are concerned in this work only with the equivalent widths of the absorption lines, the exposure times were gradually increased as the points away from the centre of the disc were exposed, so that use could be made of one and the same characteristic curve for the various spectra from the centre to the limb. It should be mentioned here that the exposure time for the limb never exceeded that for the standardisation spectra.

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*While the present paper was going through the press, a paper by M Bretz (Veroff Uni Gott Nr 112) dealing with the same problem was received. It is interesting to note that in the range $\sin\theta=0$ to $\sin\theta=0.96$, there is no essential difference between our values and those of Bretz

**This is the same spectrograph as was described and used by A K. Das and K. D Abhyankar in their paper in *Vistas in Astronomy* Volume I., Pergamon Press, London, 1955.

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Microphotometer Records

The plates were analysed with the Cambridge recording microphotometer of this observatory. Each spectrum plate was placed with the direction of dispersion parallel to the direction of its motion in the microphotometer. To ensure exact parallelism, the scanning spot of light was first brought to one end of the spectrum and was adjusted to be on the centre of the spectrum. The plate was then moved so that the other end of the spectrum came under the scanning spot of light. If the direction of dispersion was not parallel to that of the motion of the plate, the spot of light would not fall on the centre of the spectrum. In that case, the plate was rotated around a vertical axis by means of the screw provided for this purpose and was also moved crosswise until the spot of light was on the centre of the spectrum. The plate was then run backward to the original position and the adjustment was made again. This process was repeated until the spot of light remained on the centre of the spectrum all along the entire length of the plate. As the same plate contained the spectra of various points from the centre to the limb, it was necessary to make this adjustment only once for each plate. For the tracing of the Fraunhofer lines with the microphotometer, the fast rotation of the drum was used; this had a magnification of 50 on the recording paper. Thus a mean dispersion of 1·435 A/mm on the plate was magnified into 0·287A/cm on the record. In order to allow for any change in the sensitivity of the microphotometer, calibration marks were registered on the tracings both before and after each run along the spectrum. The scanning slit of the microphotometer was adjusted to a width of 0·03 mm while the slit of the spectrograph had a width of 0·04 mm for photographing the spectra, so that the resolving power of the spectrograph was maintained in the microphotometer tracings.

For each plate four microphotometer tracings of the standardisation spectra were made at equal wavelength intervals and each record was used for about 75 A to convert densities into intensities.

Determination of Equivalent Widths

Fifty-five lines of the neutral Titanium atom chosen according to their membership in multiplets and according to the availability of experimental gf values (see Table I) were indentified by comparing the microphotometer records with those in the Utrecht Photometric Atlas of the solar spectrum. The wavelengths used were those given in the Revised Rowland Table. The density profile of every identified line was converted into an intensity profile from the standardisation curves in the usual way. The value of r, the ratio of the intensity at a point in the line to the intensity of the continuum, was determined; from this, by the trapezoidal rule, the area corresponding to $\int(1\text{-}r)d\lambda$ was obtained giving the equivalent width (W) of the line in question. The equivalent widths of all the lines used were determined as an average of three independent measures.

Sources of Error

The following principal sources of error should be mentioned :

(a) Diffuse light inside the spectrograph. Suitable colour filters were used to allow only a few hundred angstroms to enter the spectrograph. Stops at suitable places inside the spectrograph further reduced stray light. In actual practice, the amount of stray light was found to be negligibly small ; in any event, the presence of a small amount of scattered light would not materially affect the equivalent widths.

(b) Finite resolving power of the spectrograph. The finite resolving power would considerably affect the profile of the spectral line. But it would scarcely affect the equivalent width of the line, particularly with the high dispersion of 1·435 A/mm used here. As we were concerned with the total absorption in the line, no correction was necessary on this account.

(c) Poor guiding of the image: This, especially near the limb, would sometimes produce a non-uniform density across the width of the spectrum. As all the plates used were very nearly uniform, this effect must have been small. Only those plates were used for which it was possible to keep the image steady on the metal disc without any noticeable drift during exposures.

(d) Focussing of the plates : Before making the microphotometer records the plate grains were focussed in the attached microscope. Errors in the focussing were expected to have little effect on the tracings, as had been shown by Thackeray (1933). Nevertheless, the focus was tested, immediately before the tracings were made, over the entire length of the spectrum and it was always found to be uniformly good.

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Construction of the curve of Growth.

Following the method due to R.B. King and K O. Wright (1947) for constructing the empirical curve of growth, the lines were separated into five groups having common excitation potentials of 0 000, 0 034, 0 816, 0 926 and 1.617 volts. A curve of growth was obtained for each group of lines by plotting $\log W/\lambda$ against $\log \eta_f$ where $\log \eta_f = \log \eta_o + \log gf\lambda - \frac{5040}{T} E_l + \log \frac{\bar{K}}{K\lambda} + \triangle \log \eta_o$. In this expression η_f is a quantity proportional to the number of atoms active in absorbing the line,

η_o is the number of atoms in the ground state,

f is the oscillator strength of the line,

g is the statistical weight of the lower level,

T is the excitation temperature,

E_l is the excitation potential of the lower level expressed in volts,

$\bar{K}$ is the mean absorption coefficient,

$K\lambda$ is the continuous absorption coefficient for the wavelength λ, $\log \bar{K}/K\lambda$ and $\triangle \log \eta_o$ are both correction terms and $\log \eta_o$ is an additive constant which can be ignored for the construction of the empirical curve of growth.

The factor $\log \bar{K}/K\lambda$ which corrects for the wavelength variation of the cont nuous absorption coefficient was taken from Münch's observational determination (1945).

The factor $\triangle \log \eta_o$ reduces all lines to a single value of the darkening coefficient B_o/B_1. Wrubel (1950) has given the corrections $\triangle \log \eta_o$ for converting all the measurements to the same value of $B_o/B_1 = 2/3$. With the boundary temperature of 4656°K for the photosphere derived by Munch (1945), B_o/B_1 was calculated from the ormula $B_o/B_1 = 8/3 \quad K/\bar{K}kT_o/hc. \lambda(1 - e^{\frac{-hc}{\lambda kT_o}})$ expressed in the usual notation. It was found that B_o/B_1 varied from 0 23 to 0 33 as λ varied from 4000A to 5500A. As the lowest value of B_o/B_1 in Wrubel's tabulation is 1/3, the correction $\triangle \log \eta_o$ could not be determined in the present case. However, the correction, being always small, would not affect the shape of the curve of growth, nor the determination of the excitation temperature.

The relative gf values for Ti I lines were taken from the work of R.B King and A.S. King (1938). The values of $\log \eta_f$ were obtained for each line by using for E_l the difference between the excitation potential of the lower term of the line and the common excitation potential $\bar{E}_l$ adopted for its group (Boltzmann correction). For this purpose the excitation temperature of 4550°K, determined by Wright (1944), was first adopted Thus within each group, all the lines were reduced to the common excitation potential for the group by subtracting the differences from $\log gf\lambda$. Hence in all cases $\log W/\lambda$ was plotted as ordinate against $\log \eta_f = \log gf\lambda + \log \bar{K}/K - 5040/T(E_l - \bar{E}_l)$ as abscissa The values of $\log \eta_f$ are given in Table I and the equivalent widths W are listed in Table 2. In these tables, the wavelengths are given according to the arrangement in Moore's Multiplet Table Of Astrophysical Interest. If the values of the equivalent widths for the 18 lines of Ti listed by Barocas and Righini (1951) are compared with those obtained in the present paper, one finds that in 14 out of 18 cases the Arcetri values are larger, the average ratio $\frac{\text{W (Arcetri)}}{\text{W (Kodaikanal)}}$ for these 14 cases being 1 17. In the remaining 4 cases however the Arcetri values are slightly smaller, the average ratio $\frac{\text{W (Arcetri)}}{\text{W (Kodaikanal)}}$ being 0.92. Another interesting point which is apparent from Table II is that out of the 55 Ti lines measured 47 lines show a systematic increase of equivalent width from the centre towards the limb up to the point $\sin \theta = 0.96$; in the remaining 8 cases there is some irregularity in the centre-to-limb variation of the equivalent width at some points of the disk, but no reliable evidence of a maximum around $\cos \theta = 0$ 4, which was found by Barocas and Righini It is difficult to be sure as to whether or not a maximum of the kind found by these authors exists at some point farther towards the limb; for this purpose it would be necessary to make a precise determination of the position of the limb. According to the measurements here reported therefore the behaviour of Ti lines, within the limits of accuracy of these measurements, seems to be quite different from that of the Fe lines as observed by previous workers.

Determination of excitation temperatures

Since the curves of growth for different groups of lines were plotted for different common excitation potentials, they were displaced along the log η_f axis according to the excitation potential. There were five groups of lines and hence five shifted segments of the curve of growth The curves were shifted back and forth along the log η_f axis until the best possible fit of all points to a composite curve coinciding with the O-volt curve was obtained. In all cases, the abscissa shift $\triangle \log \eta_f$ given in Table III, was carefully determined This shift was plotted against the excitation potential and the best straight line was fitted to these points by the method of least squares Since $\triangle \log \eta_f = 5040/TE$ the slope *m* of the straight line gave 5040/T from which the excitation temperature was determined This temperature represents the mean taken over the atmospheric layers producing the lines. The third column of Table IV gives the excitation temperatures derived by this method The value of $T = 4540° \pm 90°$ for the centre of the disc according to our measurements of the Ti lines is very similar to the mean of the temperatures determined by Prouse (1942), Menzel (1938), King (1938) and Wright 1944), but markedly lower than the value obtained by Bruggencate and Houtgast (1941) Our measurements also yield a systematic decrease of excitation temperature from the disk-centre to the limb.

In conclusion it is my pleasant duty to thank Dr. A. K. Das, Deputy Director-General of Observatories, for his guidance and encouragement throughout the course of this work.

KODAIKANAL OBSERVATORY,
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S. SANKARANARAYANAN.

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TABLE I

Multiplet No	λ in A U	E_1 in V	log gf	log $\frac{\bar{K}}{K\lambda}$	log ηf
4	5210 394	048	2 3802	1661	—1 7524
	5192 980	020	2 2553	·1668	—1 8469
	5173 751	000	2 1303	1675	—1 9884
	5152 192	020	1 1584	1684	—2 9457
	5147 484	000	1 1673	1686	—2 9525
5	5064 660	048	2 2553	1718	—1 8838
	5039 966	020	2 1303	1727	—1 9791
	5009 654	020	0 9494	1738	—3 1615
	4997 102	000	1 0792	·1842	—3 0379
6	4681 921	048	2 1206	1865	—2 0380
	4667 596	020	1 9590	1871	—2 1693
	4656 474	000	1 8451	1876	—2 2992
	4693 679	020	0 4150	·1860	—3·7119
7	4562·639	020	0 5563	1912	—3 5777
38	4981 742	·845	3 7160	·1748	—0 3221
	4991·074	832	3 6021	1745	—0·4211
	4999 512	822	3 4983	1741	—0 6352
	5007 223	815	3 3222	1739	—0 8032
	5014 282	·809	3 2041	·1736	—0 9144
	5016 170	·845	2 6628	1736	—1 3736
	5020 033	·832	2 8062	·1734	—1 2156
	5022 876	·822	2 7709	·1733	—1 3614
	5024 852	·815	2 6128	·1732	—1 5118
42	4533·251	·845	3 7076	1924	—0 3539
	4534 789	832	3 5441	·1923	—0 5029
	4535 578	·822	3 2788	·1923	—0 8788
	4535 927	·815	3 0719	·1923	—1 0780
	4536·056	·809	2 9590	·1923	—1 1842
	4555 494	·845	2 7404	·1915	—1 3199
	4552 463	·832	2 8779	1916	—1 1682
	4548 775	·822	2 8663	·1917	—1 2906
	4544·699	·815	2 6990	·1919	—1·4504
	4512 746	·832	2 7404	·1932	—1 3078
	4518 034	·822	2 8751	·1930	—1 2835
	4522 809	·815	2 8692	1928	—1·2815
	45[illegible]7 327	·809	2·7160	1926	—1·4278
43	4326 359	·822	2·1461	·1890	—2 0354
	4314 806	·806	2·8921	·1887	—1 2731
53	4840 886	·896	2·6628	·1805	—1 4386
75	4698 773	1 048	2 2041	·1858	—2 0733
	4722·619	1 048	1 8195	·1850	—2 4565
77	4675·114	1 062	1 8513	·1868	—2 4427
109	5145 470	1 454	2·8261	·1686	—1 1134
	5113·449	1·437	2·6232	1698	—1 2991
110	5035·915	1·454	3·3979	·1729	—0 5466
145	4617·270	1 741	3 5315	·1891	—0 7524
	4623·103	1 732	3 2304	·1888	—1 0433
	4639·673	1·741	3 1139	·1862	—1 1708
	4639·370	1 732	3·1761	·1882	—1 0967
146	4481·275	1 741	3·4472	·1932	—0 8456
	4489·100	1·732	3 2553	·1934	—1 0265
	4465·816	1·732	3 1461	·1927	—1·1387
157	4885·090	1·879	3·5315	1787	—0 8912
	4899·919	1 871	3 4150	·1785	—0·9976
	4913·624	1 865	3 3979	·1755	—1 0099

TABLE II

Equivalent widths in mA.

Sin θ → / λ in A U. ↓	0	0 6	0 8	0 9	0 96	Ratio of equivalent widths. W Arcetri / W Kodaikanal
5210 394	92 3	96 1	106 1	109 8	112 8	1·12
5192 980	77 2	80 3	88 7	91 8	94 2	0 97
5173 751	82 1	85 4	94 4	97 6	100 3	0 94
5152 192	35 2	38 7	42 2	43 8	45·4	0·
5147·484	36 3	39 9	43 7	45 1	46 8	
5064 660	84 2	87 6	96 8	99 8	102 8	0 84
5039 966	79 2	82 4	89 9	93 8	96·5	1·04
5009 654	24 1	27 7	29 6	31 2	33·6	
4997 102	34 2	37 6	41 2	42 4	44 5	
4681 921	82 3	88 9	97 1	99 5	100 5	1·22
4667·596	78 4	81 9	80 7	83 8	87 2	
4656·474	77 3	83 5	91 2	92 8	94 4	1·27
4693·679	12 8	14·1	15·4	17·6	20·3	
4562 639	11·2	12 9	13 8	14·5	15·7	
4981·742	134 6	140 9	144 6	145 8	144 1	
4991 074	106 8	116 7	125 9	130 1	134·9	
4999 512	119 4	124 2	137 3	141 7	134 2	1·09
5007 223	125 7	119 7	130 9	138·2	148 4	
5014 282	115 7	119 6	130 6	136 4	143 2	
5016 170	68 8	73 9	75 6	78 7	83 6	
5020 033	84 7	88 1	94 5	100 2	103 5	1·13
5022 876	76 4	79·5	87 8	90 8	93 4	1·15
5024 852	70 6	73 4	81 2	83 7	86 3	1·22
4533 251	107 5	116 1	126 8	128 7	130 4	0 91
4534 789	104 8	113 2	123 6	126 4	128 2	1 15
4535 578	96 3	97 6	112 1	116 2	121·3	
4535 927	98 7	99 6	109 5	112·1	116 1	
4536 056	78 2	83 9	91 1	97 2	105 2	
4555 494	69 8	75 4	83 4	84 2	85·4	1·06
4552 463	81 2	85 1	89 7	94·8	101 9	
4548 775	80·3	86 7	94 7	96·8	98 1	1 13
4544·699	73·4	68 4	78 6	78 8	81·2	
4512 746	74 2	80 1	87 5	89·6	91·2	1 15
4518 034	78 4	84 7	92 6	94 4	96 2	1·12
4522 809	77 2	79 8	91 8	94 2	98 2	
4527 327	91 2	98 5	107·6	109·8	111 6	1·57
4326 359	30 2	33 2	36 3	37·5	39·4	
4314·806	72 3	74 6	79 6	84 4	95 6	
4840 886	68 1	72 8	74 7	77 8	82·7	
4698 773	39·4	39 8	42·2	44·1	47·2	
4722 619	18·2	19·9	21·8	22·6	23·8	
4675 114	33·7	33 7	34 8	35 2	36 3	
5145 470	34 7	38·2	41 7	42·8	44 8	
5113 449	27 1	29 8	32 6	33 6	34 8	
5035 915	84 7	86 5	87 2	87 8	90·1	

TABLE II—*Contd.*

Sin θ → λ in A.U. ↓	0	0·6	0·8	0·9	0·96	Ratio of equivalent widths. W Arcetri / W Kodaikanal
4617 270	64 7	64 8	65 3	65 2	66 8	
4623 103	55 3	55 9	56 2	58 4	61 7	
4639·673	41 3	42 8	45 4	47 4	50 5	
4639 370	39 8	42 2	44 3	46 2	49 4	
4481·275	68 2	66 9	66 1	67 2	61 4	
4489 100	41·2	45·3	49·5	51·2	53 8	
4465·816	41 3	41 2	46·9	45 7	45 6	
4885 090	66 2	68 5	67 2	66 4	62 4	
4899·919	55·8	56 4	57·2	58 3	60·2	
4913 624	51·2	53·2	54·6	56 2	58 6	

TABLE III

Abscissa shift Δ log η_f

Sin θ → E_s in V ↓	0	0·6	0·8	0 9	0·96
·000	0 00	0 00	0·00	0 00	0 00
·034	0 04	0 04	0 04	0 04	0·05
·816	0·90	0 95	0 97	1 00	1 02
·926	1·02	1 06	1·10	1 10	1·13
1·617	1 80	1 86	1 90	1·95	2 00

TABLE IV

Sin θ	slope (m)	Excitation temperature T
0	1·109	4540± 90° K
0·6	1·152	4380±100° K
0 8	1·180	4270±110° K
0·9	1·205	4180± 50° K
0·96	1·235	4080± 30° K

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