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THE HYDROGEN CONTENT OF PROMINENCES

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Abstract—The enormous value obtained by Pannekoek and Doorn for the density of hydrogen in the prominences they observed during the total solar eclipse of 1927 is due to their hypothesis of a condition resembling thermodynamic equilibrium in the prominences

It is shown that such an assumption must automatically lead to high values for the density, independent of the intensities observed and that it is inconsistent with conditions of line-absorption and radiation

It is also sought to explain how the intensity of a particular Balmer line, such as Hγ, can provide us with no clue to the number of hydrogen atoms in the ground level, if the state is not one of thermodynamic equilibrium This number can be found only from a knowledge of the intensities of the Lyman lines.

An attempt is made to estimate the density of hydrogen in prominences, using Pannekoek and Doorn's data of intensities of the Balmer lines to derive the number of atoms in the second quantum state, the probable number of atoms in the first quantum state absorbing the Lyman lines is, in the absence of adequate data, guessed at A very rough upper limit to the density of hydrogen is arrived at of the order of 1,000 atoms per c c

The density of Ca+ atoms in the prominences of Pannekoek and Doorn is recalculated. After applying certain corrections (indicated by Pettit) to the densities of Ca + and hydrogen, it is shown how their partial pressures are comparable with Milne's estimates for the pressure of Ca+ in the chromosphere

The question of the hydrogen content of prominences is of considerable importance, especially the question of the proportion of hydrogen to ionised calcium Pannekoek and Doorn have found[1] that in their prominence "a" of the eclipse of 1927 the number of hydrogen atoms was 1.6×10^{11} per c c. and of ionised calcium atoms only 0.13 per c.c.; in their prominence "b" the number of hydrogen atoms was 33×10^{11} per c.c. and of calcium atoms only 1.6 per c c. In other words, they find the calcium content of prominences to be insignificant, the ratio of the number of hydrogen to calcium atoms being of the order of 2×10^{12}. Pettit has improved[2] upon their estimates of the densities by assuming a more reasonable shape, and hence a better value of the volume for either prominence, and by allowing for the comparative "weakness" of the prominences observed ; but this leaves the proportion of hydrogen to calcium unchanged.

In Milne's theory of selective radiation pressure as the force supporting prominences, the radiation pressure can be effective only on Ca+ atoms, that on other atoms being comparatively insignificant A difficulty of this theory is to explain the presence of hydrogen and helium at all in prominences ; Pannekoek and Doorn's estimate of the enormous excess of hydrogen in prominences increases the difficulty considerably and, indeed, if it were true, would be fatal to the theory of radiation pressure as the supporting force. For, if the prominence is supported by pressure on the calcium content alone, how are we to explain the presence of 2×10^{12} as many atoms of hydrogen ? Even if we could find an explanation of how the lifting force acting on calcium atoms could be communicated to atoms of other elements (for instance, by collisions, "turbulence," or other means), we are unlikely to succeed in explaining in this way an enormous excess of hydrogen of the order of 2×10^{12} times.

[1] Verhand d Koninklijke Akademie v.w t Amsterdam, etc, Deel 14, No. 2
[2] Ap. J. 76, 1 P. 17 seq (1932).

Now Pannekoek and Doorn's result depends on the factor which they have used for ascertaining the number of unexcited hydrogen atoms from the evaluated number of atoms in the fifth quantum state   This factor they have taken as $\frac{1}{12 \times 10^{-13}}$ by assuming that it would be the same as for a gas in thermodynamic equilibrium   It is easily possible to show, without considering the observed intensities at all, that Pannekoek and Doorn's assumption of thermodynamic equilibrium must necessarily lead to a high density, much higher than that obtained when monochromatic radiative equilibrium holds , and further that, in conditions of line absorption and emission their high density leads to results which cannot possibly be true   There appears to be little doubt that their high value for the hydrogen content of prominences is due to the unwarranted (though tentative) assumption of thermodynamic equilibrium to deduce the number of normal atoms

It should also be mentioned that Pannekoek and Doorn's deduced density of the "atoms in the fifth quantum state " relates in fact only to those atoms which fall from state 5 to state 2 thereby emitting Hγ, the two aggregates are not identical[1], and it is not legitimate to infer from the density of the excited Hγ particles the density of atoms in state 1

In this paper an attempt is made to calculate the hydrogen content of prominences using Pannekoek and Doorn's observational data but abandoning the assumption of thermodynamic equilibrium   The results indicate an entirely different order of magnitude for the hydrogen content , but until more complete observations are available, it is not claimed that the results here derived do more than indicate the *order of magnitude*

2   The assumed similarity of the condition existing in a prominence to that of a gas in thermodynamic equilibrium is certainly opposed to Milne's views[2] of the solar atmosphere, according to which the state of local thermodynamic equilibrium in lower layers changes to one of monochromatic radiative equilibrium in the upper layers

If the matter were in local thermodynamic equilibrium, whatever the nature of the radiation incident on it, the radiation emitted will have a definite frequency-distribution, and the number of atoms emitting a particular frequency will bear a definite relation to the total number of atoms of the substance participating in the radiation , so that one may infer, as Pannekoek and Doorn did, the number of atoms in state 1 from the number of atoms in state 5   On the other hand, monochromatic radiative equilibrium involves a particular frequency being absorbed and re emitted without change of wavelength by an atom during transitions between two stationary states , the relative numbers of atoms in the two states bear a definite relation to each other, depending on the intensity of the incident radiation   Whereas in the former case, the ratios depend on the temperature at the point and not at all on the incident radiation (which is accordingly redistributed before emission), in the latter case the ratios depend on the intensities of the several frequencies which are, in general, independent, so that there is no necessary relation between the atoms in the various quantum states of the substance in monochromatic radiative equilibrium, and one cannot infer the number of atoms in state 1 from that of atoms in state 5 emitting the frequency

Further, in monochromatic radiative equilibrium, the set of atoms in a particular quantum state, say state 5, emitting a frequency ν_{52} is not, in general, co terminous with the total number of atoms in that state   It is even possible to regard the atoms passing between states 5 and 2 absorbing and emitting Hγ as a statistical aggregate distinct from similar aggregates partaking in the radiations of other frequencies   Even if a particular atom in state 5 passes to another state such as state 1, the principle of detailed balancing requires that an atom should pass at once from state 1 to state 5, and again another make the reverse of the first transition, viz, $5 \rightarrow 2$ It is not enough if the atom passing from state 5 to state 2 is replaced *somehow*, e g , by an atom passing from state 3, the latter being replaced by an atom from state 1 , this would introduce a cycle of

[1] This is explained in greater detail in the following section
[2] Vide several papers in the Monthly Notices of the R A S   A concise account appears in Handbuch d Astrophysik   Bd III, I half, Chap 2

transitions. and there appears to be good reason to taboo cyclic processes.[1] Thus, for every atom passing from state *2* to state *5* there is another passing from state *5* to state *2* Or, statistically regarded, these form a set of atoms making the reversible transition state $2 \underset{\leftarrow}{\overset{\rightarrow}{}}$ state *5*, absorbing and emitting ν_{25}.

If there are n_2 atoms in state *2,* of which the number n_2 (ν_{25}) are capable of absorbing the frequency ν_{25} and arriving at state *5*, the number of atoms which actually make this transition in time dt

$$= B_{25}. \quad n_2 \, (\nu_{25}) \quad \left(\int I_{25} \frac{d\omega}{4\pi} \right).dt$$

where B_{25} is the Einstein probability coefficient for the transition $2 \longrightarrow 5$ in the presence of isotropic radiation of intensity I_{25}. And the number passing from state *5* to state *2* is, by the principle of detailed balancing equal to this.

Similarly, the number leaving state *5* for any other state, say state *1* will be

$$= B_{15}. \quad n_1 \, (\nu_{15}). \quad \left(\int I_{15} \frac{d\omega}{4\pi} \right) dt$$

The coefficients B_{15}, $B_{25}\cdots\cdots$ are constants for the atom. The intensities I_{15}, I_{25} have no known relations with one another, unless the matter be in local thermodynamic equilibrium. So that the number of atoms leaving state *5* for state *2* so as to emit Hγ is distinct from that of atoms leaving for state *1*, not to mention the total number of atoms in the fifth state Hence it is easily seen that the number of atoms in the fifth state found from the intensity of Hγ-radiation can afford no clue to the total number of atoms in the lowest state, if we regard the conditions in the prominence to be the same as in the chromosphere. All that we can infer is the number of atoms in state *2*—the " normal state " for the Balmer lines—partaking in Hγ-radiation In the same way the intensities of other Balmer lines may give the numbers of hydrogen atoms partaking in the radiation of the corresponding lines, $n_2 (\nu_{23})$, $n_2 (\nu_{24})$, etc. These sets of atoms are not in general coincident with the $n_2 (\nu_{25})$ atoms absorbing ν_{25}. supposing that they do not partially overlap, the maximum number of atoms in state *2* is given by the sum of these separate numbers. Similarly we may find the number of atoms in state *1* if we knew the intensities of the Lyman lines.

3. The objection to the assumption of thermodynamic equilibrium may again be presented from other standpoints.

(*a*) Milne shows[2] that in any steady state, the equation of transfer of radiation can be expressed as

$$\frac{dI_\nu}{d\tau_\nu} = - I_\nu + \frac{\int I_\nu \frac{d\omega}{4\pi} + \eta B_\nu \, (T)}{1 + \eta},$$

where T is a parameter corresponding to an assumed pseudo-Maxwellian distribution of velocities,

τ_ν is the optical depth for colour ν, and

η is the factor depending on the probability coefficients of transition by collision, it is independent of T, and varies as the density ρ .

$$\begin{cases} \eta \longrightarrow \infty & \text{as } \rho \longrightarrow \infty \\ \eta \longrightarrow 0 & \text{as } \rho \longrightarrow 0 \end{cases}$$

At high densities, $\eta \longrightarrow \infty$, and the equation takes the form

$$\frac{dI_\nu}{d\tau_\nu} = - I_\nu + B_\nu(T),$$

which is the equation of transfer for thermodynamic equilibrium. Thus he infers that the more the atoms are battered about by collisions, the more closely will emission correspond to the Kirchoff emission. No wonder then that Pannekoek and Doorn by assuming thermodynamic equilibrium arrived at high densities ; in a sense the reasoning involves a vicious circle.

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(1) Cf. Eddington , Internal constitution of stars, p. 45 seq.　　　　(2) Op cit. p. 163 seq.

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Further, at low densities $\eta \to 0$, and the equation becomes

$$\frac{dI_\nu}{d\tau_\nu} = -1_\nu + \int I_\nu \frac{d\omega}{4\pi},$$

which is the form of the equation of transfer for monochromatic radiative equilibrium We may point out that the converse is easily seen to hold so that, if monochromatic radiative equilibrium were assumed instead of thermodynamic equilibrium, we should get only low values for the density

(b) That Pannekoek and Doorn's high value of density of atoms in the first state is inconsistent with conditions of line radiation can be shown in another way

If the number of atoms in state 1 be denoted by n_1 per c c the number of atoms that absorb the 4th Lyman line (say) in time dt

$$= n_1 \ B_{15}\left(\int I_\nu \frac{d\omega}{4\pi}\right) dt,$$

where I_ν is the intensity of the incident radiation

The amount absorbed per sec per c c $= n_1 \ B_{15}\left(\int I_\nu \frac{d\omega}{4\pi}\right) h\nu_{15}$

This is a fraction s_ν of the radiation incident on unit volume

$$= s_\nu \int I_\nu \frac{d\omega}{4\pi},$$

the limits of integration being the same as before , for instance, in the case of isotropic radiation, the integration is carried over a complete sphere round an internal point so that the integral reduces to I_ν in either case , while at the boundary, the integration is confined to the lower hemisphere, and the integral becomes $\frac{1}{2} I_\nu$ —

$$n_1 B_{15} \ h\nu_{15} = s_\nu$$

Also $\qquad \dfrac{A_{51}}{B_{15}} = \dfrac{2h\nu_{15}^{3}}{c^{3}} \ \dfrac{q_1}{q_5}$

$$\frac{q_1}{q_5} \ s_\nu = n_1 \ A_{51} \ \frac{c^3}{2\nu^3}$$

$$= n_1 \ A_{51} \ \frac{1}{2} \left(\frac{25}{24R}\right)^{3}$$

Using Pannekoek and Doorn's value for n_1 1 6 × 10^{11}, and Francis Slack's value[1] for $A_{51} = 412 \times 10^7$, in the right hand side we get

$$\frac{q_1}{q_5} \ s_\nu \sim 3 \times 10^7,$$

which is absurd, since the left-hand side is a proper fraction

(c) Pannekoek and Doorn make use of the Schrödinger-Pauli formula for intensities in terms of the series number and temperature—assuming thermodynamic equilibrium— in order to derive the temperature T from their observed values of the intensities for the 5 Balmer lines The curve plotted—log $I_i/\Phi(i)$ against $1/i^2$ must be a straight line whose slope depends upon the temperature T But the attempt to fit a straight line to the plotted values cannot be claimed to be entirely successful—even allowing for the experimental errors mentioned the discrepancy is most glaring in the relative positions of the H_α and $H\beta$. Though the authors suggest the various experimental defects as the cause of the high value of temperature obtained, the error may at least in equal (if not greater) probability, be due to their tentative assumption of thermodynamic equilibrium

4 We may now proceed to estimate the densities of hydrogen atoms in various states, on the assumption that conditions in prominences resemble the condition of the chromosphere, i e , a state of monochromatic radiative equilibrium

[1] Phys Rev 31, 527 (1928)—quoted by Caroll in M.N.R.A.S 90,590

Considering radiation of a particular colour, there will be some relation between the number of atoms in the " excited state" and that in the lower state. For matter in local thermodynamic equilibrium, this relation will depend on the temperature T at the point, and is given by Boltzmann's equation .

$$\frac{n_1}{n_s} = \frac{q_1 \; e^{-\chi_r/kT}}{q_s. \; e^{-\chi_s/kT}}$$

where n_1, q_1, χ_r represent the number of atoms per c c., the "statistical weight," and the internal atomic energy corresponding to state r.

$$\frac{n_1}{n_s} = \frac{q_r}{q_s}. \; e^{(\chi_s - \chi_1)/kT}$$

$$= \frac{q_1}{q_s}. \; e^{h\nu/kT}$$

where ν is the frequency emitted when the atom passes from state s to state r.

As this formula depends on the temperature sheerly in virtue of the velocity-distributions, it may be taken to hold wherever there is a similar velocity-distribution [1] Such may be assumed to be the state in monochromatic radiative equilibrium also. Though we cannot talk of a temperature T (since there is no thermodynamic equilibrium) yet there is a parameter T corresponding to the pseudo-Maxwellian distribution, which will behave just like the temperature T for all intents and purposes, inasmuch as a thermometer exposed to these velocities will receive such a number of collisions of varying magnitudes as will cause it to register a temperature T

But this parameter T will in general vary with each colour, except in the case of local thermodynamic equilibrium ; it is, in fact, measurable only from the observed intensities which, as stated above, have no fixed relations with one another, in a state of monochromatic radiative equilibrium.

For want of definite data, we assume T = 5500° in the following calculations. This is not to mean that a uniform temperature is conceded in the case of the several radiations considered ; on the contrary, 5500° is adopted as the parameter in the hope that it will be roughly of the same order of magnitude. Even so, this is radically different from the assumption of a uniform temperature for the complete continuum of frequencies such as exists in a state of thermodynamic equilibrium.

Thus, for the Balmer lines, ignoring statistical weights,

$$\frac{n_2}{n_r} = e^{hc/\lambda kT}, \quad \text{------ (1)}$$

where T may be taken as $\sim$ 5500°.

Also, the emission by the atoms in the rth state per c.c.

$$= n_1. \; A_{12}. \; h\nu_{r2}. \; \text{ergs per sec. ------ (2)}$$

Denoting by E_ν the intensities given by Pannekoek and Doorn, and the volume of the prominence by V, the emission per c.c. $= \frac{E_\nu}{V}$. From this and (2), we get

$$n_r = \frac{E_\nu}{V} \; \frac{\lambda}{hc} \; \frac{1}{A_{r2}} \quad \text{------ (3)}$$

Using Pannekoek and Doorn's value of the volume of prominence " a " as $5\cdot8 \times 10^{29}$ c c. and their intensity-values for the different images (see column 5 of the following table), and Francis Slack's values[2] (column 4) for the probability co-efficients A_{r2}, n_r can be calculated (column 6) And from this the values of n_2 can be known with the aid of equation (1) (column 7). The number of atoms $(n_2 + n_1)$ taking part in the radiation of each line is given in the last column of the table. Assuming that there is no overlapping,

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(1) In this argument, I follow Milne Op. cit. p. 160.   (2) Loc cit.

the total number of atoms partaking in the radiation of the Balmer lines is obtained by adding up these numbers $\Sigma(n_2 + n_r)$

r	Line	Wavelength A U	Transition probability $A r_2$	Intensities erg/sec E_ν	Number of atoms in excited state, n_r	Number of atoms in second state, n_2	Number of atoms $n_2 + n_r$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
3	Hα	6563	$4\,42 \times 10^7$	1890×10^{22}	0 246	13 028	13 27
4	Hβ	4861	845	512	0 258	54·813	55·07
5	Hγ	4341	254	126	0 189	76 130	76 42
6	Hδ	4102	102	115	0 406	231 910	232 32
7	Hξ	3890	046	188	0 139	112 680	112 82
						Total	489 90

The number of atoms in state *2* is found to be increasing as we pass from Hα to Hδ, because A_{r2} decreases much more rapidly than the observed intensities   In this connexion we have to bear in mind the uncertainties in the measures of the intensities which, in the words of the authors, [1] are " caused by the great density of the prominence images, the extrapolation from the density curves, and the large influence of the Schwarzschild exponent", these factors obviously make the error greater, the denser the image , so that the values of n_2 are probably more and more reduced as we go from Hδ to Hα It is satisfactory to note, however, that the number begins to decrease as we come to Hξ, and perhaps one may conjecture that it will continue to decrease as we go to other members of the series   The total number of atoms partaking in the radiation of the Balmer lines may therefore be taken as of the order of *500*   Taking account of the facts that the different sets of (n_2) atoms found above may overlap to some extent, and that the statistical weights will tend to reduce these numbers, we may safely put *500* as the *maximum* number of atoms

The number of atoms in state *1* can be found as argued above, only from a knowledge of the intensities of the Lyman lines   The ratio used by Pannekoek and Doorn ($1\,20 \times 10^{-12}$) is really the ratio $n_4(\nu_{16})$: $n_1(\nu_{16})$, $n_4(\nu_{16})$ can be found only if the intensity I (ν_{16}) were known   The intensities of the Lyman lines in prominence spectra are not known, but if the intensity I (ν_{16}) were $\sim 10^{-9}$ times that of Hγ, we get the same order of magnitude for the density of atoms in state *1* absorbing and emitting the first Lyman line, for that of the Hγ particles

We arrive at the same result from calculations similar to that made in a previous section (3 c)

Since the fraction $\dfrac{q_1}{q_4}$ $s_\nu < \dfrac{1}{25}$, we get $n_1 < 2\,2 \times 10^2$

Thus we may estimate the number of hydrogen atoms per c c in the prominence to be at most of the order of *1,000*   This produces a pressure of about $7\,5 \times 10^{-16}$ atmospheres, taking a temperature of $5500°$ Or if, with Pettit [2], we regard Pannekoek and Doorn's estimate of the volume of the prominence as 20 times too large, the pressure becomes $1\,5 \times 10^{-14}$ atoms   again, following Pettit in considering that, since the prominence " a " is comparatively " weak," the intensities of lines will be about six times as great in a " representative prominence " such as the prominence " c " of Pannekoek and Doorn, the partial pressure of hydrogen becomes 9×10^{-14} or slightly less than 10^{-13} atmospheres

5   The above estimate of the pressure of hydrogen is comparable with Milne's estimate of the pressure of Ca$^+$ at the top of the chromosphere [3], viz , $\sim 10^{-13}$ atmospheres   Pannekoek and Doorn obtain as low a pressure for Ca$^+$ as $9\,6 \times 10^{-20}$ atmospheres ; but it appears to me that this low estimate is due to an error similar to that in the case of hydrogen

From Żwann's evaluation of the probability co-efficient $A_{42} \rightarrow_{41}$ for the transitions $2S$—$2P$, and $2S$—$2P$ (H and K lines) combined as $1\,55 \times 10^8$, they infer that each atom of Ca$^+$ emits $1\,55 \times 10^8 \times h\mu = 7\,69 \times 10^{-7}$ erg/sec   But certainly, this is the amount emitted by each atom of Ca$^+$ in state *2*, and not the average amount

[1] Pannekoek and Doorn   Op cit p 22
[3] Cf Monthly Notices of the R A S 88, 193   (1928)
[2] Loc cit

emitted by each atom of Ca^+. By regarding 7.69×10^{-4} erg/sec. as emitted by each Ca^+ atom, they obtain from their value of the total emission of H and K radiation by prominence " a " as 6.03×10^{25}, the total number of Ca^+ atoms as 7.8×10^{28} or 1.3 per c c But it is obvious that this is only the number of atoms in the excited state (n_2).

The number of atoms in the lower state (n_1) is given as before by the equation

$$\frac{n_1}{n_2} = \frac{q_1}{q_2} \, e^{\,hc/\lambda kT}, \quad (\lambda = 3950 \mathring{A} \text{ U.})$$

$$= 7.30 \times 10^2 \text{ (omitting statistical weights)}$$

$$\therefore n_1 = .13 \times 730 = 94.9$$

$$n_1 + n_2 = 95.03.$$

The pressure due to $n_1 = 730$ times the pressure due to n_2.

$\therefore$ The pressure of Ca^+ atoms $= 731 \times 9.6 \times 10^{-20}$

$$\sim 7.02 \times 10^{-17} \text{ atmospheres}$$

Correcting, as in the case of hydrogen, for the excess of the assumed volume and the weakness of the lines, the partial pressure of Ca^+ atoms in a " representative prominence "

$$= 6 \times 20 \times 7.02 \times 10^{-17}$$

$$\sim 8.4 \times 10^{-15} \text{ atmospheres}$$

Thus the partial pressure of Ca^+ atoms is $\frac{1}{10}$ of that of hydrogen. The hydrogen content, as measured by its mass, will be only $\frac{1}{4}$ of that of Ca^+.

6 *Conclusion.*—We may therefore conclude that, if we do not assume a state of thermodynamic equilibrium in the prominences, the density of the hydrogen is no longer of immense proportions , on the other hand, it is comparable with the density of Ca^+ in the prominences and, what is more, both these values agree closely with Milne's estimates of the density of Ca^+ at the top of the chromosphere We can be more certain of the estimates of hydrogen-content of prominences, only if we know the intensities of other series of hydrogen lines, especially the first few Lyman lines. If these intensities should happen to be large, the density of hydrogen atoms in the first state will be preponderatingly large, and the condition in the prominence will approximate to one of thermodynamic equilibrium ; if, on the contrary, these intensities should be very low—as we imagine them to be—then the densities will be low as stated above and the condition approximate to one of monochromatic radiative equilibrium. What exactly is the condition existing in the prominence cannot at present be known for certain. Nevertheless one may hazard the conjecture, in the light of Milne's theory and the experience of hydrogen images being less dense than ionised calcium images, that the conditions in prominences correspond more to those in the upper layers of the sun's atmosphere than in the lower layers, that is, to monochromatic radiative equilibrium rather than thermodynamic equilibrium, to lower densities of the gases rather than high, and to lower densities of hydrogen than of calcium.

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