

## On the solar limb darkening in the wings of some atomic hydrogen lines

K. R. Bondal, V. P. Gaur and M. C. Pande

*Uttar Pradesh State Observatory, Manora Peak, Naini Tal 263 129*

Received 1986 November 15; accepted 1987 January 10

**Abstract.** The centre-to-limb behaviour of the first four lines of the Balmer, Paschen, and Brackett series of atomic hydrogen at selected distances from the line centre, i.e. at  $\Delta\lambda = 5, 10, 20$  and  $30 \text{ \AA}$  has been theoretically investigated in three photospheric models. The results have been compared with observations of David (1961) for the Balmer lines, the corresponding observations for Paschen and Brackett lines not being available in published sources. After a detailed comparison of the computed results with the observations, it is concluded that limb-darkening data in the wings of hydrogen series lines can distinguish amongst the various photospheric models.

*Key words* : photospheric models—hydrogen lines—limb darkening

### 1. Introduction

In an earlier paper (Bondal & Gaur 1986) we reported our computed results on the behaviour of the line profiles in the Balmer, Paschen, and Brackett lines (alpha to delta) in the solar disc centre and compared them with the available observations for some of the lines. The photospheric models chosen for these calculations were of Holweger & Muller (1974 = HM 74) and of Vernazza, Avrett & Loeser (1976, 1981 = VAL 76, VAL 81). The comparison showed that at disc centre VAL 76 model yields the best fit with observations, followed by HM 74, and by VAL 81. However, no single model can explain all the hydrogen series lines equally well.

Limb darkening observations of  $H\alpha$ ,  $H\beta$ , and  $H\gamma$  lines near the line centre up to  $\pm 1 \text{ \AA}$  have been made by White (1962) who compared his results with the observations of de Jager (1952) and David (1961). The continuum limb-darkening parameters of Pierce & Waddell (1961) have been used by White (1962) in the conversion of residual intensities relative to the continuum at disc centre. White (1962) found good agreement between his observations and those of David (1961).

The centre-to-limb (CL) behaviour of these Balmer lines in the far wings up to  $\pm 30 \text{ \AA}$  where Stark broadening plays a very dominant role in the line formation have been reported by David (1961) alone. Here, in this paper, we present computed CL behaviour of the first four lines of Balmer, Paschen, and Brackett series of atomic hydrogen at four selected distances from the line centre, i.e. at  $\Delta\lambda = 5, 10, 20,$  and  $30 \text{ \AA}$ . The photospheric models are HM 74, VAL 76, and VAL 81. The problem has been taken up to see whether limb darkening data in the hydrogen line wings can distinguish amongst various photospheric models and can also help in improving the existing models.

## 2. Computational procedure

Following the method outlined in Bondal & Gaur (1986), the residual intensities for the chosen Balmer, Paschen, and Brackett series lines at selected  $\Delta\lambda$ s from the line centre were computed for HM 74, VAL 76, and VAL 81. The selected CL positions were  $\sec \theta = 1, 1.77, 2.0, 2.5, 3.0,$  and  $4.0$ . The residual intensities were then converted into intensities in units of continuum intensity at  $\sec \theta = 1$ , so as to obtain the limb darkening behaviour of a given line at a given  $\Delta\lambda$  value. For converting David's (1961) residual intensities of Balmer series lines into intensities in units of continuum at  $\sec \theta = 1$ , the continuum limb darkening parameters of Pierce & Waddell (1961) were used. Intensities in units of continuum intensity at  $\sec \theta = 1$  for the four Balmer series lines at  $\Delta\lambda = 5, 10, 20$  and  $30 \text{ \AA}$  are given as a function of  $\cos \theta (= \mu)$  in figure 1. For the Paschen and Brackett series lines computed limb darkening data are given in table 1. The results obtained by us

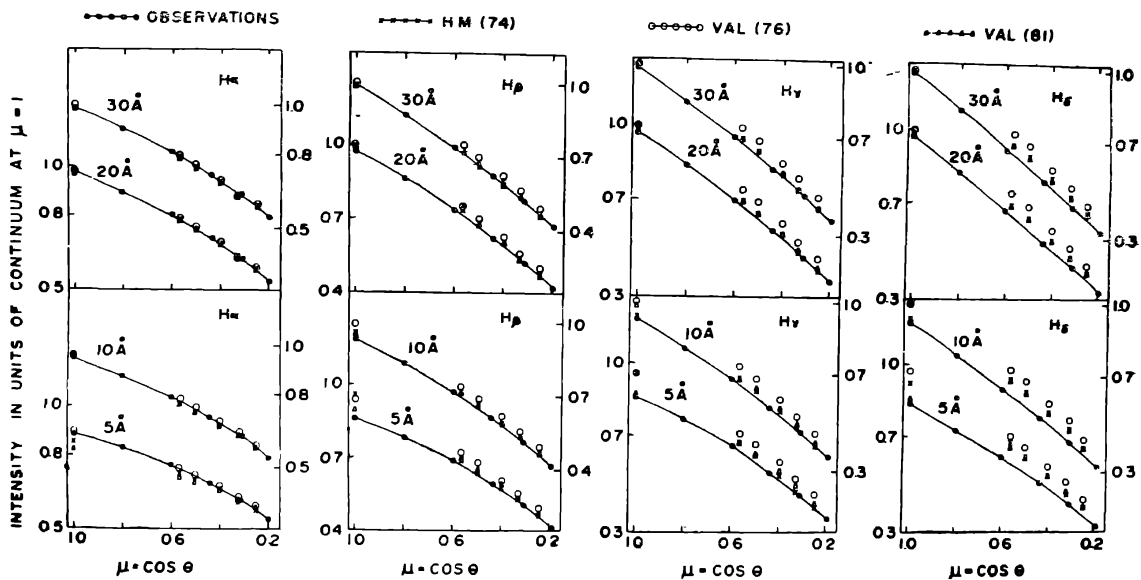


Figure 1. Observed and computed limb darkening values at four wing points  $\Delta\lambda = 5, 10, 20$  and  $30 \text{ \AA}$  for  $H\alpha, H\beta, H\gamma,$  and  $H\delta$  lines in three photospheric models. Each box contains two curves of a given line at two  $\Delta\lambda$  values where the right-hand side ordinate corresponds to the upper curve.

Table 1. Computed line intensities for Paschen and Brackett series in units of intensity at  $\sec \theta = 1$  at  $\Delta\lambda$ s 5, 10, 20 and 30 Å as a function of  $\sec \theta$

Line	$\Delta\lambda$ (Å)	HM 74				VAL 76				VAL 81											
		$\sec \theta$				$\sec \theta$				$\sec \theta$											
		1.0	1.77	2.0	2.5	3.08	4.0	1.0	1.77	2.0	2.5	3.08	4.0	1.0	1.77	2.0	2.5	3.08	4.0		
Pa $\alpha$	5	.639	.719	.731	.738	.726	.689	.714	.702	.703	.699	.686	.657	.736	.857	.851	.812	.779	.717	.717	
	10	.726	.679	.671	.659	.654	.662	.800	.759	.753	.741	.728	.714	.585	.561	.582	.634	.687	.740	.740	
	20	.818	.789	.775	.747	.723	.699	.896	.829	.816	.795	.778	.758	.722	.704	.697	.678	.660	.657	.657	.657
	30	.846	.796	.789	.775	.758	.732	.928	.867	.854	.828	.804	.779	.893	.737	.722	.709	.703	.693	.693	.693
Pa $\beta$	5	.835	.746	.732	.711	.696	.681	.864	.794	.778	.755	.735	.712	.756	.695	.690	.694	.707	.707	.707	.707
	10	.875	.801	.787	.759	.733	.704	.929	.842	.823	.790	.762	.731	.889	.775	.765	.751	.739	.707	.707	.707
	20	1.000	.840	.813	.774	.744	.713	.976	.867	.844	.806	.774	.739	.955	.846	.819	.776	.742	.707	.707	.707
	30	1.000	.863	.832	.784	.749	.715	.993	.874	.850	.811	.777	.741	.947	.846	.819	.776	.742	.707	.707	.707
Pa $\gamma$	5	.884	.802	.783	.752	.726	.700	.927	.825	.806	.773	.746	.715	.848	.798	.789	.774	.744	.708	.708	.708
	10	.965	.822	.802	.768	.739	.708	.971	.848	.826	.788	.756	.722	.981	.844	.819	.777	.745	.708	.708	.708
	20	1.000	.845	.817	.775	.743	.710	.991	.859	.834	.793	.759	.724	.977	.845	.819	.777	.745	.705	.705	.705
	30	.996	.849	.821	.777	.744	.711	.998	.861	.836	.794	.760	.724	.996	.845	.819	.777	.745	.708	.708	.708
Pa $\delta$	5	.899	.807	.788	.754	.725	.696	.940	.829	.808	.771	.740	.707	.899	.814	.803	.772	.737	.699	.699	.699
	10	.989	.825	.801	.764	.733	.701	.978	.846	.821	.780	.747	.711	.986	.841	.814	.772	.737	.699	.699	.699
	20	.999	.840	.812	.769	.736	.702	.995	.852	.826	.784	.749	.711	.991	.840	.814	.772	.738	.699	.699	.699
	30	.996	.842	.814	.770	.736	.702	.999	.854	.827	.784	.749	.712	1.000	.841	.814	.772	.737	.699	.699	.699
Br $\alpha$	5	.769	.918	.938	.925	.915	.905	1.000	.877	.840	.792	.779	.799	.907	.772	.747	.718	.712	.723	.723	.723
	10	.823	.806	.807	.812	.826	.861	.786	.915	.941	.932	.925	.918	.775	.840	.856	.875	.870	.841	.841	.841
	20	.969	.924	.906	.876	.860	.857	.897	.875	.868	.865	.875	.896	1.000	.953	.915	.861	.835	.830	.830	.830
	30	.897	.936	.934	.921	.903	.884	.911	.904	.904	.901	.897	.896	1.000	.956	.955	.944	.909	.863	.863	.863
Br $\beta$	5	.848	.782	.758	.716	.689	.674	.784	.722	.706	.689	.694	.728	1.000	.829	.779	.707	.672	.672	.672	.672
	10	.727	.743	.756	.779	.791	.784	.823	.790	.786	.784	.784	.780	.712	.886	.884	.855	.836	.821	.821	.821
	20	.842	.781	.775	.765	.760	.757	.886	.845	.837	.825	.815	.804	.786	.732	.738	.763	.795	.821	.821	.821
	30	.878	.827	.814	.793	.779	.767	.929	.869	.859	.843	.830	.816	.826	.804	.797	.788	.789	.821	.821	.821
Br $\gamma$	5	.650	.715	.732	.754	.756	.735	.754	.727	.728	.731	.727	.711	.716	.875	.864	.828	.797	.766	.766	.766
	10	.763	.713	.706	.695	.689	.690	.832	.789	.782	.771	.761	.747	.662	.611	.624	.666	.713	.762	.762	.762
	20	.834	.805	.793	.768	.746	.724	.914	.769	.835	.815	.798	.780	.774	.749	.744	.729	.715	.709	.709	.709
	30	.862	.809	.801	.786	.768	.745	.936	.875	.863	.838	.816	.793	.931	.780	.766	.752	.744	.735	.735	.735
Br $\delta$	5	.671	.639	.644	.664	.688	.712	.765	.731	.723	.711	.706	.705	.518	.678	.714	.769	.782	.745	.745	.745
	10	.815	.736	.722	.703	.691	.679	.846	.799	.790	.773	.759	.744	.716	.635	.618	.602	.609	.643	.643	.643
	20	.834	.798	.792	.776	.758	.732	.916	.865	.851	.825	.802	.777	.862	.726	.717	.708	.701	.687	.687	.687
	30	.965	.815	.801	.781	.765	.744	.967	.882	.868	.843	.821	.793	.977	.824	.791	.746	.705	.675	.675	.675

cannot be compared with observations as observational data in the far wings of these series lines are not available.

### 3. Results and discussions

From figure 1, we find that the computed intensities for the  $H\alpha$  lines at the selected  $\Delta\lambda$ s agree best with VAL 76 followed by HM 74 and then by VAL 81. For the  $H\beta$  and  $H\gamma$  lines HM 74 model agrees best with the observations, followed by VAL 81, and by VAL 76. For  $H\delta$  line, the best fit is attained by VAL 81, followed by HM 74. Here, VAL 76 shows maximum deviation.

The deviations that occur at different centre-to-limb values at a given wing point could be due to model effect, such as atmospheric inhomogeneities. These inhomogeneities differently affect the core of the lines and the wings, even in the same model (Basri *et al.* 1979). An account of the three photospheric models has been outlined earlier (Bondal & Gaur 1986), where we had calculated the depths of formation of the  $H\alpha$  and the  $H\delta$  lines in HM 74 and in VAL 76. These depths were found to increase from  $H\alpha$  to  $H\delta$  as the opacity decreases. We had found that VAL 76 explains the  $H\alpha$  line wing better than does HM 74, which is confirmed by the present investigation.

The computed intensities of  $H\beta$ ,  $H\gamma$  and  $H\delta$  lines in HM 74 and VAL 81 are closer to observational values, while those of VAL 76 show considerable departure. The deviation is the largest in  $H\delta$ , which is in agreement with our earlier result (Bondal & Gaur 1986). As we move from centre to limb the deviation of the computed intensities from the observed values decreases, showing that the higher layers in the models represent the observations better than the deeper ones. For the Paschen and Brackett series lines, observations in the wings of these lines are needed for comparison. Mitropol'skaya (1965) has reported her observations for the Brackett  $\gamma$  line ( $\lambda$  2.1655  $\mu\text{m}$ ) at different CL points, but as she has not tabulated her results and the profiles are plotted on an unsuitable scale, we have not used her observed profiles for comparison with our calculated ones. We therefore tabulate our results for these series lines (table 1).

From our results, we conclude that limb darkening data in the wings of the hydrogen series lines can distinguish amongst the various photospheric models and so these lines can be used as a diagnostic tool to improve the existing photospheric models.

### Acknowledgements

We thank the computer centre, department of mathematics and statistics of the G. B. Pant University of Agriculture and Technology, Pantnagar, for the use of their TDC 316 computer facility. Special thanks are due to the programmer, Mr S. C. Tewary.

### References

- Basri, G. S., Linsky, J. L., Bartoe, J. D. F., Brueckner, G., & Van Hoosier, M. E. (1979) *Ap. J.* 230, 924.

- Bondal, K. R. & Gaur, V. P. (1986) *Bull. Astr. Soc. India* **14**, 73.  
David, K. H. (1961) *Zs. f. Ap.* **53**, 37.  
Holweger, H. & Muller, E. A (1974) *Solar Phys.* **39**, 19.  
Mitropol'skaya, O. N. (1965) *Sov. Astr.* **8**, 744.  
Pierce, A. K. & Waddell, J. H. (1961) *Mem. R. A. S.* **68**, 89.  
Vernazza, J. E., Avrett, E. H. & Loeser, R. (1976, 1981) *Ap. J. Suppl.* **30**, 1 ; **45**, 635.  
White, O. R. (1962) *Ap. J. Suppl.* **7**, 333.