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Revised elements and evolutionary status of six eclipsing binary stars

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Abstract. Twelve photoelectric light curves of six eclipsing binary stars namely GG Cas, WX Cep, UZ Cyg, VW Cyg, AQ Peg and ST Per in two different filters (*B* and *V* for the first five binaries and *R* and *I* for ST Per) have been analysed by Kopal's new method of analysis of the light curves of eclipsing system in the frequency domain. The elements of the systems have been evaluated and their evolutionary status discussed.

Key words : eclipsing binaries—photometric elements—absolute stellar dimensions—stellar evolution

1. Introduction

Photoelectric observations of close binary stars are aimed at deriving the orbital inclination and the relative radii of the components. Most of the present estimates of the light elements of binary stars are based on the old conventional methods such as that of Russell & Merrill (1952 ≡ RM). In recent times, various light curve analysis techniques have been developed. The resulting photometric elements are, in general, expected to have greater accuracy than those obtained with old conventional methods (*e.g.* Kopal 1982b). It is therefore essential that the observations of eclipsing binary stars be analysed in the light of modern light curve analysis techniques as far as possible.

In this paper, we present results of our analysis of a total of 12 light curves of six eclipsing binary stars in the frequency domain recently developed by Kopal (1982a) because no photometric elements based on this method have yet been published for these stars. A brief description of the numerical method used and the results of the computations are given in sections 2 and 3. After giving a brief introduction of the individual systems, we discuss our results in section 4. Section 5 contains a discussion of the evolutionary status of the system components based on their absolute dimensions. The concluding section 6 gives a short summary of the results.

Throughout this paper subscript 'h' refers to the hotter star which is eclipsed at the primary minimum while 'c' refers to the cooler star.

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2. Moments of the light curves

This study is based on the photoelectric observations of GG Cas, WX Cep, UZ Cyg, VW Cyg, AQ Peg and ST Per made by various investigators. All these binaries show a flat minimum at phase 0.0 and have several interesting spectral features. The ephemerides of the studied light curves along with the spectral types are listed in table 1 and discussed in section 4.

The criteria used for selecting the light curves of these binaries for the present investigation were as follows: (i) The light curves should be available in two colours; (ii) the light curves should be well covered with observations having been made for sufficiently long integration time (on both the variable and the comparison star) to ensure the accuracy of each differential magnitude; and (iii) all phase intervals should have been observed on at least two different nights.

The moments of the light curves, as represented by figure 1 and defined by the equation (Kopal 1979)

$$A_{2m} = \int_0^{\theta_e} (1 - l)\, d \sin^{2m} \theta, \qquad \ldots(1)$$

where θ_e = outer contact angle;

l = normalized intensity at phase angle θ; and

m = any non-negative integer,

were computed for all the light curves, for $m = 0, 1, 2$ and 3. These 48 moments of the light curves along with their errors are listed in table 2. It is noticed from this table that, for each light curve, $A_{2m} \gg A_{2(m+1)}$.

3. Orbital elements

The four moments of a light curve (A_0, A_2 A_4 and A_6) were calculated to extract the orbital elements and their errors (Kopal 1982a):

$$A_0 = L_1 \alpha_0, \qquad \ldots(2)$$

$$A_2 = L_1\{C_3 + (1 - \alpha_0) \cot^2 i\}, \qquad \ldots(3)$$

$$A_2 = L_1\{C_3^2 + C_2^2 - (1 - \alpha_0) \cot^4 i\}, \qquad \ldots(4)$$

$$A_6 = L_1\{C_3^3 + 3C_2^2 C_3 + C_1 C_2^2 + (1 - \alpha_0) \cot^6 i\}, \qquad \ldots(5)$$

Table 1. Ephemerides of the program stars

No.	Systems	Spectral type pri.	Spectral type sec.	P in days	Filter	N
1	GG Cas	B5	K0	3.758719	B	268
					V	265
2	WX Cep	A2	A5	3.378453	B	291
					V	303
3	UZ Cyg	A3	K1	31.305809	B	176
					V	184
4	VW Cyg	A3	G5	8.4303102	B	205
					V	214
5	AQ Peg	A2	G5	5.5485028	B	297
					V	312
6	ST Per	A3	K1	2.6493273	R	318
					I	309

N = number of observations used in the present study

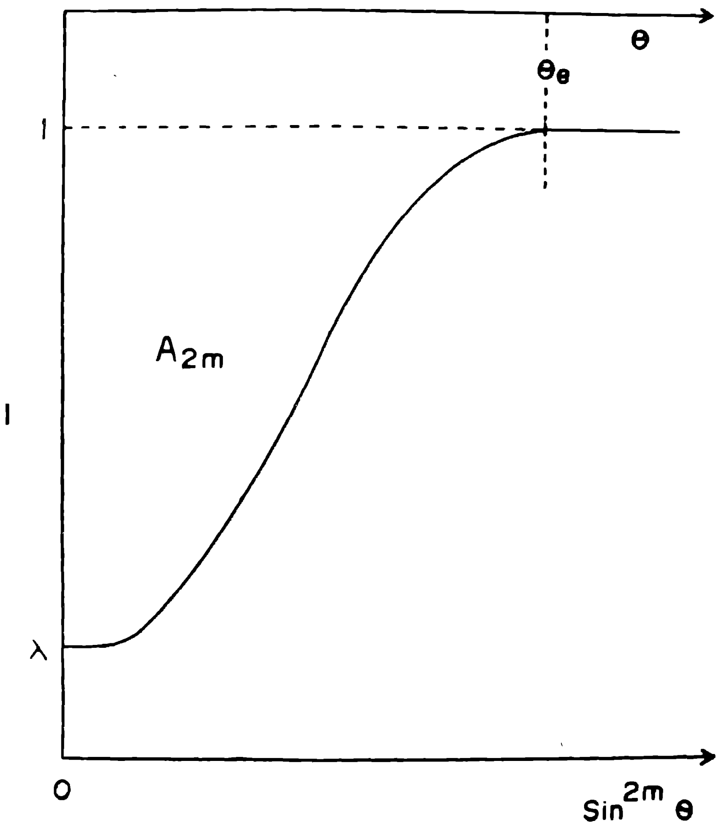

Figure 1. The moment A_{2m} and the light curve of an eclipse.

where L_1 is the luminosity of the eclipsed star expressed in the units of the total light of the system; α_0 is the maximum fractional loss of light of the eclipsed star; i is the angle of inclination of the orbital plane to the plane of the sky;

$$C_3 = \{r_2^2 \operatorname{cosec}^2 i - \cot^2 i\} \sum_{n=0}^{N} \frac{1!C^{(n)}}{(\frac{1}{2}n + 1)}, \qquad \ldots(6)$$

$$C_2^2 = r_1^2 r_2^2 \operatorname{cosec}^4 i \sum_{n=0}^{N} \frac{2!C^{(n)}}{(\frac{1}{2}n + 1)(\frac{1}{2}n + 2)}, \qquad \ldots(7)$$

$$C_1 C_2^2 = r_1^4 r_2^2 \operatorname{cosec}^6 i \sum_{n=0}^{N} \frac{3!C^{(n)}}{(\frac{1}{2}n + 1)(\frac{1}{2}n + 2)(\frac{1}{2}n + 3)}. \qquad \ldots(8)$$

In the above equations, r_1 and r_2 are the fractional radii of the eclipsed and the eclipsing star respectively; and $C^{(n)}$ are functions specifying the distribution of brightness over the apparent disc of the eclipsed star, characterized by the limb darkening coefficients $u_1, u_2, u_3, \ldots, u_n$:

Table 2. Moments of the light curves, A_{2m}

Systems	F	A_{2m} $m = 0$	$m = 1$	$m = 2$	$m = 3$
GG Cas	B	0.4455 ± 0.0038	0.02520 ± 0.00032	0.002135 ± 0.000025	0,000210 ± 0.000014
	V	0.2344 ± 0.0035	0.01539 ± 0.00011	0.001725 ± 0.000018	0.000195 ± 0.000010
WX Cep	B	0.4377 ± 0.0025	0.02820 ± 0.00023	0.003090 ± 0.000034	0.000413 ± 0.000021
	V	0.4311 ± 0.0026	0.02931 ± 0.00017	0.033480 ± 0.000031	0.000538 ± 0.000022
UZ Cyg	B	0.9236 ± 0.0014	0.024945 ± 0.00016	0.000848 ± 0.000011	0 000033 ± 0.000002
	V	0.8095 ± 0.0012	0.02323 ± 0.00011	0.000778 ± 0.000009	0.0000284 ± 0.0000013
VW Cyg	B	0.9499 ± 0.0013	0.05085 ± 0.00021	0.003205 ± 0.000016	0.000228 ± 0.000006
	V	0.8781 ± 0 0015	0.04526 ± 0.00018	0.002805 ± 0.000013	0.000198 ± 0.000004
AQ Peg	B	0.9553 ± 0.0016	0.05272 ± 0.00021	0.003895 ± 0.000083	0.000342 ± 0.000008
	V	0.8962 ± 0.0018	0.04956 ± 0.00024	0.003699 ± 0.000090	0.000328 ± 0.000010
ST Per	R	0.8414 ± 0.0015	0.04927 ± 0.00025	0.005032 ± 0.000026	0.000643 ± 0.000012
	I	0.7566 ± 0.0013	0.04521 ± 0.00018	0.004665 ± 0.000027	0.000598 ± 0.000011

$$C^{(n)} = \frac{1 - u_1 - u_2 - u_3 - \ldots - u_n}{1 - \sum\limits_{l=0}^{N} l u_l/(l+2)} \quad \text{for } n = 0 \qquad \ldots(9)$$

and

$$C^{(n)} = \frac{u_n}{1 - \sum\limits_{l=0}^{N} l u_l/(l+2)} \quad \text{for } 0 < n \leqslant N. \qquad \ldots(10)$$

In our calculations, we have assumed the linear law of limb darkening for the distribution of brightness over the apparent disc of the hotter star. The values of limb darkening coefficients have been estimated from Grygar *et al.* (1972). The resulting elements are given in table 3. A comparison of our computed orbital elements with the previous ones, based on the RM model and obtained by earlier investigators from the same data, is made in table 4.

Table 3. Orbital elements of the program stars in the frequency domain

Stars	F	r_h	r_c	i
GG Cas	*B*	0.115 ± 0.002	0.360 ± 0.004	74°.2 ± 0°.3
	V	0.117 ± 0.003	0.353 ± 0.004	74°.7 ± 0°.3
WX Cep	*B*	0.186 ± 0.002	0.257 ± 0.004	84°.5 ± 0°.4
	V	0.184 ± 0.002	0.258 ± 0.003	84°.7 ± 0°.4
UZ Cyg	*B*	0.069 ± 0.001	0.207 ± 0.002	84°.5 ± 0°.3
	V	0.062 ± 0.001	0.195 ± 0.002	84°.8 ± 0°.2
VW Cyg	*B*	0.100 ± 0.001	0.241 ± 0.003	86°.8 ± 0°.4
	V	0.103 ± 0.001	0.235 ± 0.004	87°.0 ± 3°.5
AQ Peg	*B*	0.140 ± 0.001	0.243 ± 0.002	86°.2 ± 0°.5
	V	0.138 ± 0.001	0.242 ± 0.003	86°.5 ± 0°.6
ST Per	*R*	0.216 ± 0.002	0.248 ± 0.003	86°.7 ± 0°.8
	I	0.213 ± 0.001	0.249 ± 0.002	87°.1 ± 0°.5

Table 4. Comparison of our results with the previous ones, based on the RM model

Binary systems	Elements	Present values	Earlier values
GG Cas	i	74°.5	89°.5
	r_h	0.116	0.175
	r_c	0.357	0.250
WX Cep	i	84°.6	88°.8
	r_h	0.185	0.180
	r_c	0.258	0.236
UZ Cyg	i	84°.7	—
	r_h	0.066	—
	r_c	0.201	—
VW Cyg	i	87°.1	88°.5
	r_h	0.102	0.122
	r_c	0.238	0.233
AQ Peg	i	86°.4	85°.7
	r_h	0.139	0.139
	r_c	0.243	0.238
ST Per	i	86°.9	88°.8
	r_h	0.215	0.201
	r_c	0.249	0.247

4. Discussion of the results

It can be seen from table 3 that there is an agreement between the values of r_h, r_c and i in both the filters in the case of all the six binary stars. But for some of the systems analysis in frequency domain gives results appreciably different from the earlier ones based on the RM model.

The earlier photometric elements of VW Cyg and AQ Peg (the systems with circumstellar material) presented in table 4 are based on the RM model modified to take into account the photometric complications produced by the circumstellar material (Amman *et al.* 1979). But no photometric elements for UZ Cyg were obtained with the modified RM method. In fact one may ask how Kopal's new method could produce solutions apparently so satisfactory if it ignored the photometric complications. The answer is simple. The photometric complications at the primary minimum are small, only of the order of 0.001 light units, and therefore probably could not seriously affect the photometric element which were derived (table 3).

Details of the individual systems are given below.

4.1. GG Cassiopeiae

In GG Cas a brighter K0 giant or supergiant eclipses a smaller fainter B5 star at the time of minimum light. Popper (1956), studying this system spectroscopically, noticed that the spectrum is composite, and the lines of the cooler star dominate in the spectrum longward of λ4100 Å while the hotter star appears strong at λ4100Å. Because of the large scatter in radial velocities, he could not derive the spectroscopic elements of the system.

Srivastava & Kandpal (1970) published the photoelectric observations of the system in *B* and *V* filters and derived its light elements with the RM method. Using their elements, Srivastava (1976) completed the unnoticed depth of secondary minimum for both the light curves which comes out to be $0^m.39$ in *B* and $0^m.58$ in *V*. In this study we have analysed their *B* and *V* observations for photometric elements. Our solution of GG Cas, given in table 3, does not confirm their elements. In spite of the several reasons given by Kopal (1977), probably the RM method, in our opinion, could not produce solution satisfactory for a system having small k value ($k \leqslant 0.3$) like GG Cas. Perhaps because of this reason Amman *et al.* (1979) could not derive the photometric elements of UZ Cyg ($k \lesssim 0.3$) with the RM method.

The large radius of the cooler component and the small surface brightness ratio (cooler over hotter) obtained by using Kopal's method suggests the semidetached model for the system in which the cooler star has filled up its Roche lobe. The value of fractional radius of the cooler component indicates that the masses of both the components are nearly equal. More accurate spectroscopic and photometric orbits are necessary to confirm this conclusion.

4.2. WX Cephei

Spectroscopic observations, made by Sahade & Cesco (1945), suggest that WX Cep is an extremely interesting binary star because its spectrum shows double lines which change in intensity through different phases. They classified the hotter star as A2 and the cooler star as A5.

The only three-colour photometric observations of the system are by Kandpal & Srivastava (1970) who have published the photometric elements in *B* and *V* filters based on the RM method. Our analysis shows that the primary minimum is produced by the occultation of the smaller, fainter, A2 star by the larger, brighter, A5 star. This finding is in aggreement with Kandpal & Srivastava (1970), but our photometric elements of the system differ from their values. The values of fractional radii and surface brightness of the components (table 3) along with their equal masses (Sahade & Cesco 1945) suggest that the eclipsing system WX Cep is a ditached one.

4.3. *UZ Cygni*

Hall & Woolley (1973) have collected and discussed all the times of primary minima for studying period variability and mass transfer in the system. Wyse (1934) observed the star spectroscopically and found no emission lines in the spectra. He classified the hotter star as A3 and the cooler star as K1. No radial velocity study has been done so far.

Amman *et al.* (1979) have published the *UBV* observations and the light curves of the system. We have analysed their *B* and *V* observations only for light elements and found that the ratio of the radii (cooler to hotter) is about 3. The gaseous disc around the hotter component (Amman *et al.* 1979) and unequal surface brightness of the components suggest that the system is semidetached with the cooler star filling up its Roche lobe.

4.4. *VW Cygni*

Photometric elements of the system were obtained by Shapley (1915) from visual observations. Struve (1946a) observed emission lines just before and after the primary eclipse and derived spectroscopic elements of the system. He classified the hotter star as A3 and the cooler star as G5.

The complete *UBV* light curves, photoelectric observations and the orbital elements of this system have been published by Amman *et al.* (1979). We have analysed their *B* and *V* observations only for the orbital elements. The unequal surface brightness of the components along with variability of the orbital period (Hall & Neff 1976) suggests a semidetached model for the system.

4.5. *AQ Pegasi*

Tsesevich (1928) published the photometric elements of the system based on visual observations. In his spectroscopic observations, Struve (1946a) found double peaked emission lines and very strong absorption of H and K Ca II lines. He gave the spectral type of the hotter star as A2 and the cooler star as G5.

Photoelectric observations and the complete *UBV* light curves of the system have been published by Amman *et al.* (1979). The value of fractional radii, unequal surface brightness of the systemic components obtained by us and spectroscopic elements of the system suggest that AQ Peg is semidetached.

4.6. *ST Persei*

Struve (1946b) observed the star spectroscopically and classified the primary as A3 and the secondary as late G or early K.

The photoelectric light curves and photometric orbital elements of the system based on the RM method were published by Srivastava (1970) in *B* and *V* and by Weis & Chen (1976) in *R* and *I*. Srivastava (1970) obtained the depths of the primary minimum as $2^m.4$ and $1^m.8$ in *B* and *V*, respectively. Hall & Weedman (1971) have published the *UBV* magnitude of ST Per and its companion. If one removes the light produced by companion from Srivastava's observations, the depths come out to be $3^m.6$ and $2^m.8$ in *B* and *V*, respectively. These depths are consistent with the $3^m.4$ depth observed by Wood (1946) in visual. Thus we feel that the observations of Srivastava (1970) are unreliable for the present analysis because they have been contaminated with light from a visual companion, 11.6 arcsec away from the variable. Therefore, *R* and *I* observations only have been used in the present analysis.

The orbital solution obtained by us gives two rather large components and their unqual surface brightness suggests that ST Per is semidetached. The variable period and the occurrence of mass transfer from the cooler to the hotter component in the system (Hall & Neff 1976) give additional evidence for semidetached model for the system in which the cooler star has filled up its Roche lobe.

5. Absolute dimensions and evolutionary status

The determination of the absolute dimensions of GG Cas, UZ Cyg, VW Cyg, AQ Peg and ST Per poses a problem. The radial velocity curves of the hotter components of these binaries are distorted and the spectroscopic elements of the systems obtained by using different spectral lines differ. Therefore, in order to determine the absolute dimensions without using their spectroscopic data, we have used the following equations (Hall 1974) :

$$R_h = 215\, r_h \{M_h(1 + q)\, P^2\}^{1/3} \qquad \ldots(9)$$

and

$$\log R_h = 7.48 - 2 \log T_h + 1.94 \log M_h. \qquad \ldots(10)$$

Here T_h is the effective temperature; R_h, the radius in solar units; M_h, the mass in solar units; P, the orbital period in years; and q, the mass ratio (M_c/M_h), which is a function of the fractional radius of the cooler star, and is given by, with error less than 2% (Paczynski 1971),

$$r_c = 0.38 + 0.2 \log q \quad \text{for } 0.3 < q < 20, \qquad \ldots(11)$$

and

$$r_c = 0.4622\, [q/(q + 1)]^{1/3} \quad \text{for } 0 < q < 0.8. \qquad \ldots(12)$$

We have taken the effective temperatures of the hotter stars from Kreiner & Ziolkowski (1978). The resulting absolute elements for GG Cas, UZ Cyg, VW Cyg, AQ Peg, and ST Per are given in table 5. Also listed are the absolute dimensions of the double-line spectroscopic binary WX Cep, computed from the spectroscopic data given by Sahade & Cesco (1945).

Before discussing the evolutionary status of the system components, it is important to estimate how reliable these absolute dimensions are. To do this, let GG Cas serve as an example and consider the various uncertainties involved in the

Table 5. Estimated absolute elements of the program stars

	GG Cas	WX Cep	UZ Cyg	VW Cyg	AQ Peg	ST Per
$M_h/M_\odot$	4.35	1.10	3.42	2.65	2.81	2.62
$M_c/M_\odot$	3.44	1.10	0.31	0.53	0.57	0.57
$R_h/R_\odot$	2.34	2.51	0.25	2.61	2.74	2.55
$R_c/R_\odot$	7.26	3.73	13.02	6.10	4.79	2.96
$A/R_\odot$	20.17	15.05	64.46	25.65	19.73	11.87
M_h^{bol}	$-1^{m}.20$	$+0^{m}.85$	$-0^{m}.17$	$+0^{m}.91$	$+0^{m}.67$	$+0^{m}.93$
M_c^{bol}	$+1^{m}.37$	$+0^{m}.70$	$+0^{m}.25$	$+1^{m}.49$	$+2^{m}.01$	$+3^{m}.45$

calculations. An uncertainty of 0.1 in q in equations (11) and (12) produces uncertainties of $\pm 0.7\ M_\odot$ in M_h, $\pm 0.5\ M_\odot$ in M_c, $\pm 0.01\ R_\odot$ in R_h and $\pm 0.03\ R_\odot$ in R_c. An uncertainty of 0.01 in r_h produces uncertainties of $\pm 0.12\ M_\odot$ in M_h, $\pm$ 0.10 $M_\odot$ in M_c, $\pm 0.02\ R_\odot$ in R_h and $\pm 0.07\ R_\odot$ in R_c. Further an uncertainty of ± 1500 K (the uncertainty of one subclass in the spectral type for B star) in T_h produces uncertainties $\pm$ 0.5 $M_\odot$ in M_h, $\pm 0.4\ M_\odot$ in M_c, $\pm 0.09\ R_\odot$ in R_h and 0.26 $R_\odot$ in R_c. Similarly it can be shown that uncertainties in estimating the absolute dimensions for all studied systems are less than or about 10% of their value. Thus these data are very useful (Andersen *et al.* 1980).

In order to study the evolutionary status of the system components, we have plotted their masses against their radii and their luminosities, respectively, in figures 2 and 3.

The positions of the components in figure 2 reveal that only hotter component of GG Cas is in its core hydrogen burning phase and the others are in the shell hydrogen burning stage. It means that except for GG Cas the hotter components of all the systems belong to higher luminosity class than the main sequence. The spectroscopic observations taken from Struve (1946a) for VW Cyg and AQ Peg also confirm this conclusion in the case of the hotter components. He reports that in both the stars (VW Cyg and AQ Peg) the lines of Fe II are relatively strong—a feature which is usually associated with higher luminosity than is found in the case of the A type main-sequence stars having weak lines of Fe II.

An inspection of figure 3 reveals that the cooler components of UZ Cyg, VW Cyg, AQ Peg and ST Per are overluminous while the cooler component of GG Cas displays significant underluminosity as compared to a main-sequence star of similar mass. This underluminosity may be due to the decrease in its surface temperature during its nuclear evolution. This along with the high luminosity excess ($\Delta M_c^{bol} = -4^{m}.6$) and large radius and mass ($R_c = 7.26\ R_\odot$ and $M_c = 3.44\ M_\odot$) of the cooler component of the system as compared to a K0 type main-sequence star suggests that the cooler component of GG Cas is in an outer-shell hydrogen burning phase, well before the central helium ignition.

The large radius for the low-mass cooler components ($R_c = 13.05\ R_\odot$ for $M_c = 0.31\ M_\odot$) and small mass ratio ($M_c/M_h = 0.10$) coupled with an absence of period change (Hall & Woolley 1973) in case of UZ Cygni suggest that the system is at the end of its first phase of rapid mass transfer. The large orbital period ($P = 31^{d}.35$) may be due to a large amount of mass having been already

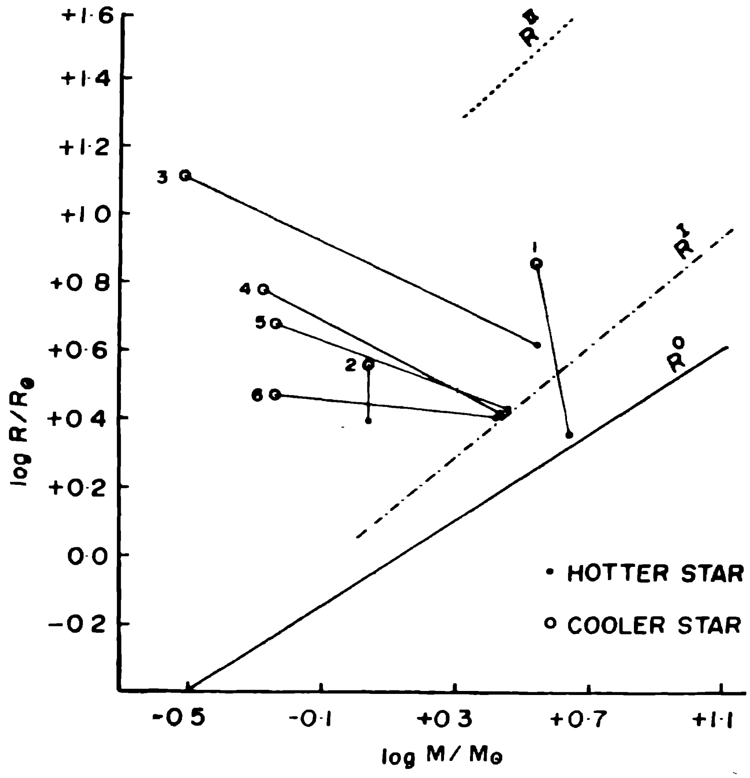

Figure 2. The distributions of the binary components in mass-radius plane. Filled and open circles denote, respectively, the hotter and cooler components. The number near the cooler components corresponds to the identification number of the binary stars in table 1. The R°, R^{I} and R^{II} lines indicate the mass-radius relations for the single star, respectively, at the start of hydrogen burning in the core, at the Chandrasekhar-Schoenberg limit and at the start of helium burning in the core.

transferred from the cooler component to the hotter component. The over luminosity and large size of the hotter component support the conclusion that UZ Cyg represents the end of the first phase of mass transfer in which the cooler component is either in a shell-hydrogen-burning phase, just before central helium ignition, or in central-helium-burning phase.

6. Conclusions

Light elements of six eclipsing binary stars have been evaluated using Kopal's new method of the analysis in the frequency domain. It is found that in some of the systems, analysis in the frequency domain gives results appreciably different from the previous ones which are based on the RM model. It is concluded that GG Cas, UZ Cyg, VW Cyg, AQ Peg and ST Per are semidetached systems whereas WX Cep is detached.

The distribution of the system components in the mass-radius diagram suggests that except for GG Cas, the hotter components of the remaining binary stary lie above the main sequence. It is further concluded that all these semidetached binary stars are highly evolved systems, the cooler components being overluminous for their spectral type and larger than the ZAMS stars of equal masses.

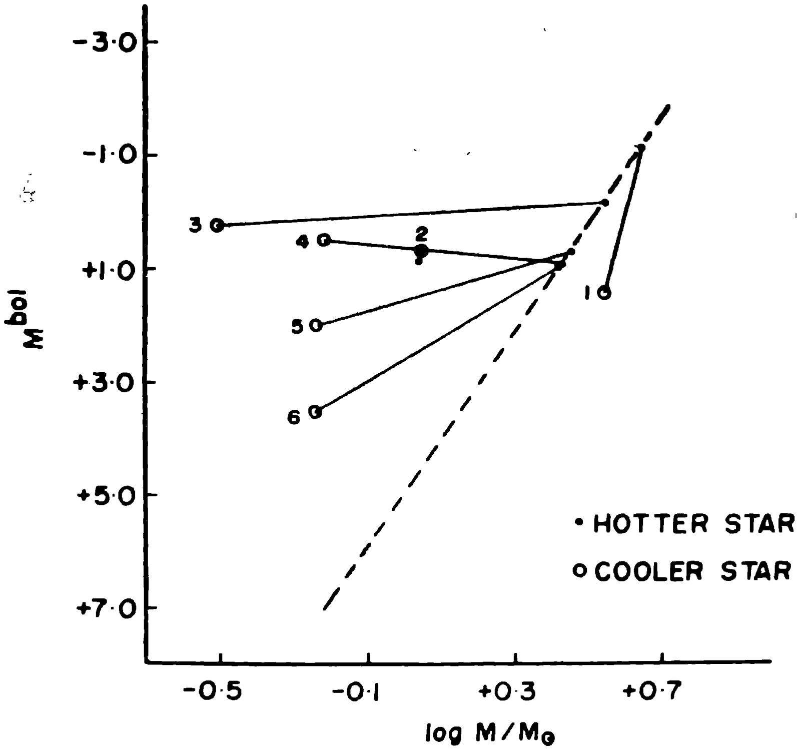

Figure 3. The distributions of the binary components in the mass-luminosity plane. Notation is the same as in figure 2. Dashed line represents the mass-luminosity relation for the main-sequence stars.

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