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Global sausage and Kink modes in coronal Loops

Anasooya Ashok*, A. Satya Narayanan, R. Ramesh, C. Kathiravan
** Dept. of Physics, University of Calicut, Calicut - 673 635*
Indian Institute of Astrophysics, Koramangala, Bangalore - 560 034

MHD Oscillations and their presence in the solar corona is known both from theoretical predictions and observations such as TRACE (Transition Region And Coronal Explorer) and SOHO (SOlar and Heliospheric Observatory). These modes have interesting consequences in coronal seismology, for example in determining the strength of the magnetic field. Magnetic field structures (with strong gradients) are known to exist in the corona and they are often referred to as coronal loops. We model the loop to be made up of a cylindrical tube of constant cross-section. The magnetic field, pressure and density are assumed to be different both inside and outside the tube. The tube admits modes such as sausage (symmetric), kink (asymmetric), surface and body modes. The dispersion relation of the modes for a cylindrical tube which is compressible, infinitely conducting with uniform flows is presented. Limiting cases are discussed briefly. The phase speed of the kink mode is used as a diagnostic for determining the magnetic field of the corona. For different values of the coronal parameters, the magnetic field ranges from a few Gauss to 25 Gauss. The sausage mode does not exist for all wavenumbers. The condition for this mode to exist is to do with the ratio of the length of the loop to its diameter, which in turn depends on the density ratio. An example of quasi-periodic pulsations from radio observations found in the literature is presented. We hope to interpret some of the observations from the Gauribidanur Radioheliograph in terms of these global modes. The analysis is underway and will be reported later.

1. Introduction

Theoretical aspects of magnetohydrodynamic (MHD) waves in the solar coronal plasma have been investigated for decades, but it is only very recently, with the unambiguous detection of such oscillations, that those theories take on a new vigour. There have been several reports on coronal oscillations, in radio wavelengths [1-4]. The theory of coronal loop oscillations has recently been reviewed by [5-7]. However, it is evident that the subject is developing apace, led by the recent observational discoveries which have prompted a re-examination of the theoretical aspects. Loops may also carry upwardly propagating waves, detected with SOHO's Extreme ultraviolet Imaging Telescope (EIT) and Transition Region And Coronal Explorer (TRACE). The longitudinal intensity oscillations are a commonly occurring phenomena in coronal loops[8]. Loops are not the only objects to oscillate in the corona,. prominence oscillations have long been studied and coronal plumes are seen to support waves.

Coronal seismology became an efficient new tool that uses standing MHD waves and oscillations as a tool to explore the physical parameters in the solar corona [9]. There are three basic branches of solutions of the dispersion relation for propagating and standing MHD waves: the slow-mode branch (with acoustic phase speeds), the fast mode branch and the Alfven branch (with Alfvenic phase speeds). Furthermore, each branch has a symmetric and asymmetric solution, termed the sausage and kink modes [10]. All of these MHD oscillation modes have been detected with imaging observations. However, a recent study [11] pointed out that the dispersion relation and oscillation period has been incorrectly applied to the data, because the highly dispersive nature of the phase speed and the long-wavelength cutoff in the wavenumber has been ignored. A recent review on coronal oscillations can be found in [12]. In this paper, we present the dispersion relation for the different modes for a cylinder with uniform cross-section (straight) with flows. We discuss some of the limiting cases. The application of kink modes for determining the magnetic field is presented in the next section. Sausage modes are interpreted as quasi period structures for some radio observations found from the Nobeyama Observatory, Japan recently. We also hope to interpret some of the observations from Gauribidanur RadioHeliograph in terms of these global modes.

2. The model

The coronal loop is assumed to be a straight cylindrical tube (as a first approximation). The plasma inside and outside the tube are assumed to have different densities, magnetic field (though uniform), which is compressible, infinitely conducting. We assume a uniform flow 'U_0' of the plasma inside the tube of radius 'a' as shown in the Figure 1.

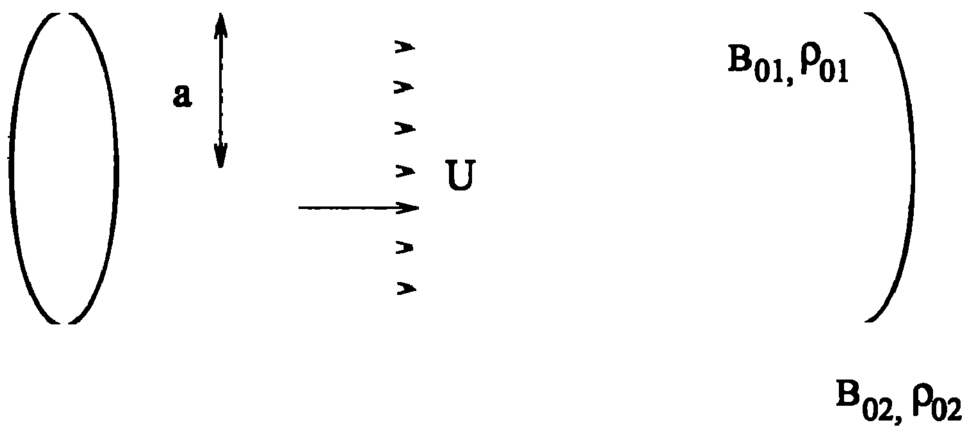

Figure 1: The Model

3. Dispersion relation

The equations of motion governing the electromagnetic and hydrodynamic properties of a compressible, infinitely conducting and moving plasma inside the cylinder of radius 'a' is linearized using the normal mode approach to derive the dispersion relation. The wave equation for the total pressure (gas pressure + magnetic pressure) is derived by algebraic simplifications. For the modes that vary as

$$f(r,\phi,z,t) = \hat{f}(r)exp[\imath(kz + l\phi - \omega t)], \tag{1}$$

where k is the axial wavenumber, l is the azimuthal wavenumber and ω is the angular frequency, it can be shown that the radial dependence of the flow variables satisfies the Bessel differential equation.

By simple algebraic simplifications, the dispersion relation can be shown to be [13,14]

$$\rho_0[\Omega^2 - k^2C_{Ao}^2]m_e + \rho_e[\omega^2 - k^2C_{Ae}^2]m_0F_m(m_0, m_e, ma) = 0, \tag{2}$$

where $\Omega = \omega - kU_0$ is the Doppler shifted frequency and

$$F_m(m_0, m_e, ma) = \frac{K_m(m_ea)I'_m(m_0a)}{K'_m(m_ea)I_m(m_0a)}, \tag{3}$$

$$m_0^2 = \frac{(k^2C_{s0}^2 - \Omega^2)(k^2C_{A0}^2 - \Omega^2)}{(C_{s0}^2 + C_{A0}^2)(k^2C_{T0}^2 - \Omega^2)}, \tag{4}$$

$$m_e^2 = \frac{(k^2C_{se}^2 - \omega^2)(k^2C_{Ae}^2 - \omega^2)}{(C_{se}^2 + C_{Ae}^2)(k^2C_{Te}^2 - \omega^2)}. \tag{5}$$

C_{s0}, C_{se} are the sound speeds inside and outside the tube, respectively. C_{A0}, C_{Ae} are the Alfven velocities inside and outside the tube while C_{T0}, C_{Te} are the tube speeds, respectively.

4. Limiting cases

In the absence of a basic flow $U_0 = 0$, which implies $\Omega = \omega$, the dispersion relation reduces to

$$\rho_0[\omega^2 - k^2C_{A0}^2]m_e + \rho_e[\omega^2 - k^2C_{Ae}^2]m_0F_m(m_0, m_e, a) = 0. \tag{6}$$

The expression for m_0^2 is modified accordingly, with Ω being replaced by ω. In the absence of a magnetic field ($B_{01} = B_{02} = 0$), the dispersion relation with $l = 0$ reduces to

$$\rho_0\Omega^2m_e + \rho_e\omega^2m_0F_m(m_0, m_e, a) = 0. \tag{7}$$

For an incompressible flow, C_{A0} and $C_{Ae} \to \infty$ so that m_0^2 and $m_e^2 \to k^2$. In this case the dispersion relation can be solved analytically with

$$F_m(m_0, m_e, a) = F_m(ka). \tag{8}$$

Here

$$F_m(ka) = \frac{K_m(ka)I'_m(ka)}{K'_m(ka)I_m(ka)}, \tag{9}$$

$$\frac{\omega}{kV_{A0}} = \frac{V \pm [(1 + \eta F_0(ka))(1 + \alpha^2F_0(ka)) - V^2\eta F_0(ka)]^{(1/2)}}{[1 + \eta F_0(ka)]}, \tag{10}$$

where $\alpha = B_{02}/B_{01}, V = U_0/C_{A0}$ and $\eta = \rho_e/\rho_0$.
In the limit $ka \to 0$, the above equation reduces to

$$\frac{\omega}{kC_{A0}} = V \pm 1. \tag{11}$$

It follows that for values $V \leq 1$, there exists only one positive value for ω/kC_{A0}. For $V > 1$, there are two branches. In the limit $ka \to \infty$, $F_0(ka) \to 1$ so that equation (10) becomes,

$$\frac{\omega}{kC_{A0}} = \frac{V \pm [(1 + \eta)(1 + \alpha^2) - V^2\eta]^{(1/2)}}{(1 + \eta)}. \tag{12}$$

For $U_0 = 0$ and $a \to \infty$, the cylindrical geometry reduces to the case of an infinite fluid with a single interface. In this case the dispersion relation is given by,

$$\psi_1(\omega, k)(m_e^2 + l^2)^{1/2} + \psi_2(\omega, k)(m_0^2 + l^2)^{1/2} = 0, \tag{13}$$

where

$$\psi_{1,2}(\omega, k) = \rho_{1,2}(k^2C_{A1,2}^2 - \omega^2). \tag{14}$$

5 Kink oscillations

The observed properties of these oscillations unambiguously indicate to their interpretation as a kink fast magnetoacoustic mode. The theory of this mode has been developed by [15]. It is known that coronal loops are anchored in the dense plasma of the photosphere, so it is reasonable to assume that any motions in the corona are effectively zero at the base of a loop. A typical picture of the loop oscillation is presented in Figure 2. The first observation of kink oscillations was after the flare on the 14th July 1998 at 12.55 UT [16]. The oscillation was identified as a global mode, with the maximum displacement situated near the loop apex and the nodes near the footpoints.

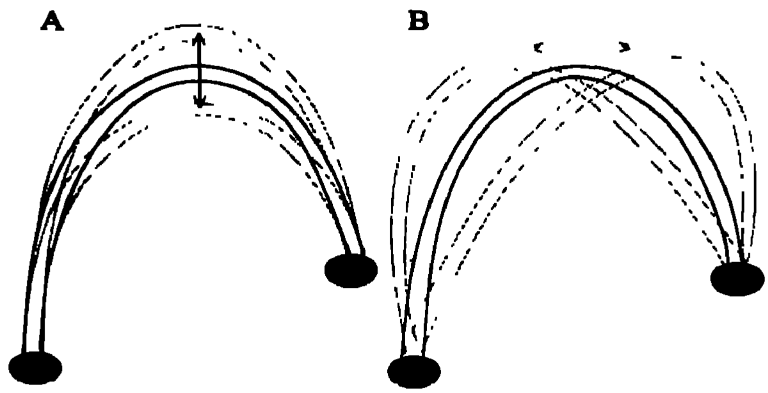

Figure 2: A loop oscillation

Assume the plasma $\beta << 1$. The pressure balance condition is given by,

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}. \quad (15)$$

Define : $\alpha = \rho_e/\rho_0, \epsilon = U_0/C_{A0}, x = \omega/kC_{A0}$.
For low β plasma, it can be shown that

$$m_0 = k[1-(x-\epsilon)^2]^{1/2} = m_0^*, \quad (16)$$

$$m_e = k[1-\alpha x^2]^{1/2} = m_e^*, \quad (17)$$

The dispersion relation for low beta plasma with flow can be written as

$$[(x-\epsilon)^2 - 1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} F(m_0^*, m_e^*, a) = 0, \quad (18)$$

$$F(m_0^*, m_e^*, a) = \frac{K_m(m_e^* a) I'_m(m_0^* a)}{K'_m(m_e^* a) I_m(m_0^* a)}. \quad (19)$$

The above relation is highly transcendental and will have to be solved numerically However, for $ka << 1$, one can show that $F(m_0^*, m_e^*, a) \approx 1$ so that the dispersion relation would reduce to

$$[(x-\epsilon)^2 - 1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} = 0. \quad (20)$$

For long wavelengths, the phase speed of the kink mode is about equal to the so-called kink speed C_k which, in the low-β plasma is

$$C_k \approx \{\frac{2}{1+n_e/n_0}\}^{1/2} C_{A0}, \tag{21}$$

where n_0 and n_e are the plasma concentrations inside and outside the loop, respectively, and C_{A0} is the Alfven speed inside the loop. It was shown by[16] that the formula for the kink speed can be utilized to determine the magnetic field as follows :

$$B_0 = (4\pi\rho_0)^{1/2} C_{A0} = \frac{\sqrt{2}\pi^{3/2} L}{P} \sqrt{\rho_0(1+\rho_e/\rho_0)}. \tag{22}$$

The dependence of the magnetic field on the density (number) is shown in Figure 3 for different coronal parameters. It is evident from equation (22) for determination of the magnetic field B_0 in the corona, that the strength of the magnetic field depends on L (length between the foot points of the loop), P (the period) and the ratio of the plasma densities. This puts a certain constraint on the determination of the field accurately. However, one can deduce the strength if one has good observations of L, P and the ratio of the densities. One can in principle assume a certain density model and work out. It is evident from the figure that the strength of the magnetic field is dependent on the periods of these waves significantly.

6 Sausage oscillations

The fast magnetoacoustic mode (sausage mode), associated with perturbations of the loop cross-section and plasma concentration, has been used to interpret periodicities in the range 0.5-5 s, which are usually observed as modulation of coronal radio emission. Quasi periodic oscillations of shorter period (0.5-10 s) may be associated with sausage modes of higher spatial harmonics. There have been quasi periodic pulsations in the periods 14 - 17 s, which oscillate in phase at a loop apex and its foot points which have been observed at radio wavelengths. These modes have a maximum magnetic field perturbation at loop apex and nodes and at the foot points.

The dispersion relation for magneto acoustic waves in cylindrical magnetic flux tubes has many types of long wavelength solutions in the fast mode branch (n = 0, 1, 2, ...) with the lowest ones called the sausage mode (n=0) and kink mode (n=1). Kink mode solutions extends all the way to the long wavelength limit ($ka \rightarrow 0$) while the sausage mode has a cut off at a phase speed

$$v_{ph} = v_{Ae}, \tag{23}$$

which has no solutions for wavenumbers $ka < k_c a$. The cutoff wavenumber k_c is given by

$$k_c = \{\frac{(c_s^2 + v_{A0}^2)(v_{Ae}^2 - c_T^2)}{(v_{Ae}^2 - v_{A0}^2)(v_{Ae}^2 - c_s^2)}\}^{1/2} (\frac{j_0}{a}). \tag{24}$$

Under coronal conditions the sound speed $c_0 \approx 150 - 260 km/s$ and Alfven speed is $v_A \approx 1000$ km/s. Therefore

$$c_s << v_A. \tag{25}$$

Here tube speed is similar to sound speed

$$c_T \approx c_s$$

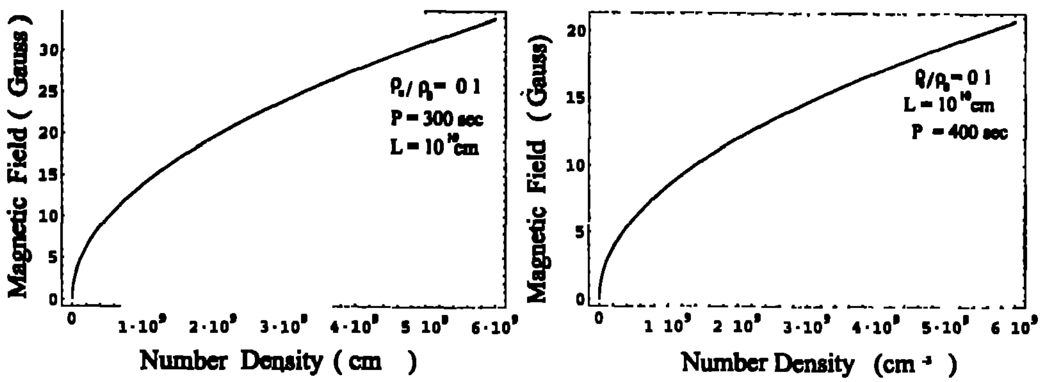

Figure 3: The variation of the magnetic field for different coronal parameters

The expression for the cutoff wavenumber reduces to

$$k_c \approx \left(\frac{j_0}{a}\right) \frac{1}{[(v_{Ae}/v_{A0})^2 - 1]^{1/2}}. \tag{26}$$

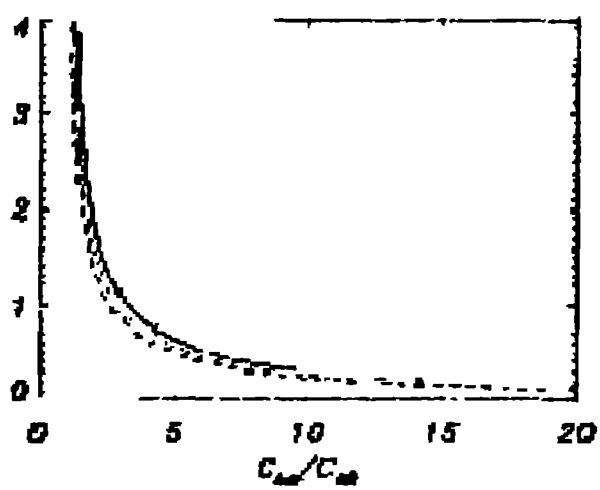

Figure 4: Dependence of the cut-off wavenumber of the sausage mode

For a typical density ratio in the solar corona (0.1 - 0.5), the cut off wavenumber $k_c a$ fall in the range $0.8 \leq k_c a \leq 2.4$. Therefore the long wavelength sausage mode oscillation is completely suppressed for the slender slopes. The occurrence of Global sausage modes therefore requires special conditions : 1) very high density contrast ρ_0/ρ_e, 2) relative thick loops to satisfy $k > k_c$. The high density ratio $\rho_0/\rho_e >> 1$ or $v_{Ae}/v_{A0} >> 1$ yields the following simple expression for the cut off wavenumber k_c.

$$k_c a \approx j_0(v_{A0}/v_{Ae}) = j_0(\rho_e/\rho_0)^{1/2}$$

This is depicted in the Figure 4. The cutoff wavenumber condition $k > k_c$ implies a constraint between the loop geometry ratio (2a/L) and the density contrast ratio (ρ_e/ρ_0) which turns out to be

$$\frac{L}{2a} \approx 0.65\sqrt{\rho_0/\rho_e}.$$

Also it can be shown that the period of the sausage mode satisfies the condition

$$P_{saus} < \frac{2\pi a}{j_0 v_{A0}} \approx \frac{2.62a}{v_{A0}} \tag{27}$$

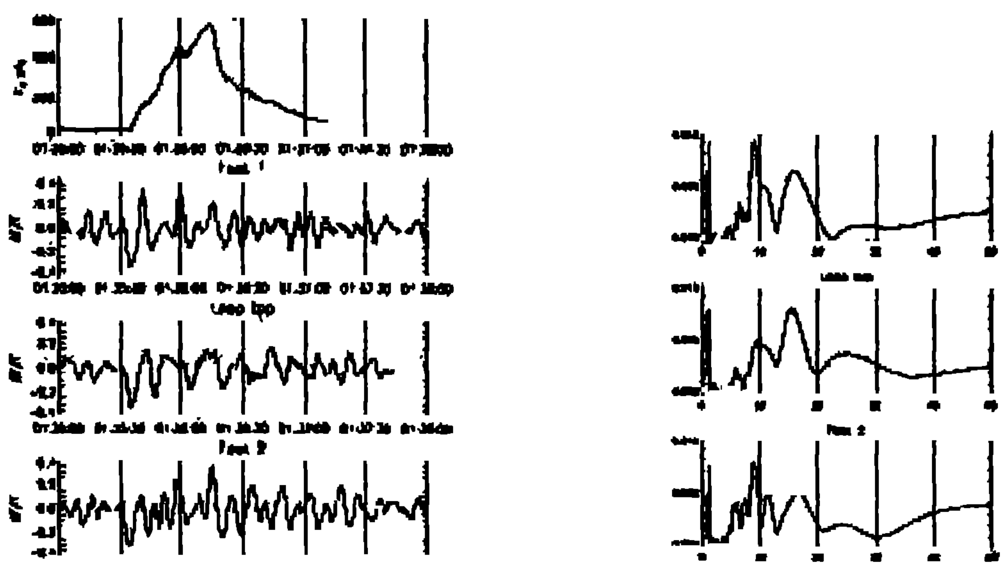

Figure 5: The time profile and the Fourier power spectra of the pulsations

7. An observational example

It is a common belief that microwave bursts are generated by the gyro synchrotron emission which is very sensitive to the magnetic field in the radio source. Causes of microwave flux pulsations with periods $P \approx 1 - 20s$ are believed to be some kind of magnetic field fluctuations that modulate the gyro synchrotron radiation leading to acceleration of particles. The observational proof of the existence of global sausage mode should be based upon the determination of the oscillation period, the longitudinal and transverse size of the magnetic loop and the spatial distribution of the oscillation amplitude along the loop. A good candidate for such a proof is a solar flare which happened on the 12th of January 2000, and observed by Nobeyama Radioheliograph, Japan at two frequencies. Details of the observation can be found in [17]. The following figure (Figure 5) gives the time profile of the 17 GHz flux integrated over the source in the limb on January 12, 2000 and also the radio flux variations for various parts of the flaring loop. It is clear from the figure that these variations are quasi periodic in nature and may be interpreted in terms of sausage oscillations.

8. Conclusions

In this paper we have presented the different modes that are possible in coronal loops. Two modes in particular, the sausage (symmetric) and kink (asymmetric) modes are discussed in detail. The kink mode can be used as a diagnostic for determining the magnetic field of the corona, while the sausage mode can be interpreted as quasi periodic pulsations from radio observations for a flaring loop. We are analyzing the data obtained from the Gauribidanur Radioheliograph and hope to interpret some of the observations in terms of these global modes.

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