Torsion Effects on Neutrino Emission from Dense Collapsing Cores

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Abstract. We consider the effects of torsion on the emission of neutrinos in a hot dense neutron medium and we explore the consequences for the case of SN1987A neutrinos.

Torsionseffekte bei der Neutrinomission aus dichten, kollabierenden Kernen

Inhaltsübersicht. Wir betrachten den Torsionseffekt bei der Neutrinomission in einem heißen und dichten Neutronenmedium und erläutern die Konsequenzen für den Fall der SN1987A Neutrinos.

The detection of the neutrino burst from SN 1987 A in the Large Magellanic Cloud (LMC) reported by the Kamiokande II Collaboration [1], the IMB [2] and other groups has provided a unique opportunity to not only test theories of stellar collapse and supernovae but also supply some information on the basic properties of the neutrino itself. In particular the data provided has already been used to put a limit on the electron-neutrino mass by several authors [3, 4]. Others have tried to look for oscillation patterns and possible effects associated with the MSW mechanism [5]. However it has to be borne in mind that the properties of the source from which the supernova neutrinos are emitted, i.e. for the SN 1987 A case, in particular the density and temperature of the medium as well as the propagation properties of the emitted neutrinos are vastly different from the usual terrestrial and solar experiments.

The neutrinos have to propagate in the very dense medium of the collapsing star and the presence of many heavier nuclei makes coherent neutrino scattering important (the cross-section $\sigma_{\text{coh}}$ for which increases $\propto E_{\nu}^2 A^3$ with neutrino energy $E_{\nu}$ and atomic mass $A$). This enhanced cross-section increases the opacity, reducing the mean free path $\lambda_{\text{coh}}$ so that the neutrinos undergo several scatterings, literally diffusing out of the collapsed star. The diffusion time $t_D \approx \lambda_{\text{coh}} N_{\text{scatt}} c$, $(1/\lambda_{\text{coh}} \approx (a/A m_p) \sigma_{\text{coh}}$; $N_{\text{scatt}} =$ number of scatterings, $m_p$ is proton mass), so that $t_D \approx 0.1 (a/10^{12})$ sec and for $a \approx 10^{14}$ g/cc the neutrinos take about ten seconds to traverse out of the star. The fact that the neutrino burst detected lasted about ten seconds, rather than the hydrodynamical collapse time scale $(1/(G\rho)^{1/2})$ milliseconds) of a few milliseconds is strong evidence that such diffusion did take place. Moreover the several scatterings undergone by the neutrinos before emerging out (the medium being optically thick to neutrinos), reduces their mean energy to about a few tens of MeV (as detected) rather than the electron Fermi energy of...
\[ 28 \text{ MeV}, \text{ corresponding to nuclear density at core bounce. Again it must be remembered that the magnetic field during the collapse may become rather large (}> 10^8 \text{ gauss}) if flux conservation is assumed. This would imply that even neutrinos with very small magnetic moments can flip helicities. For instance the transition rate for relativistic neutrinos with a magnetic moment \( \mu_r \), to flip helicity (i.e. become right-handed) while moving in a region with magnetic field \( B \) is given by [6]

\[ \Gamma \approx \frac{2}{\hbar} |\mu_r| B \hbar. \quad (1) \]

When applied to SN 1987 A neutrinos moving through the intergalactic and galactic magnetic [6] fields, this would constrain \( \mu_r < 10^{-20} \mu_B \), \( \mu_B \) being the Bohr magneton \((= e\hbar/2m_c)\) if \( 1/\Gamma \) is not to exceed the neutrino travel time of \( \approx 6 \cdot 10^{12} \text{ sec} \) from LMC, 55 kpc away. Neutrinos with larger \( \mu_r \) would get depolarized on the way and therefore would not register on the detectors. This constraint on \( \mu_r \) while being of the same order as that predicted by diagrams of electroweak order in the Weinberg-Salam theory, is however much smaller than that required \((\mu_r \approx 10^{-10} \mu_B)\) by the Voloshin-Vysotsky-Okun (VVO) [7] mechanism to resolve the anticorrelation between solar neutrino flux and solar activity. But during the collapse if the magnetic fields become as large as \( 10^8 \) gauss, then \( 1/\Gamma \) would imply that \( 1/\Gamma \) would be shorter than the neutrino diffusion time \( t_D \), even for \( \mu_r < 10^{-20} \mu_B \). For much larger \( \mu_r \) one would end up with equal amount of left and right helicities. More details on the effects of the magnetic field on neutrino propagation in ref. [6].

Now it was pointed out [8, 9] that the interaction energy between spin \( S \) and torsion \( Q \) is

\[ E = -\tilde{S} \cdot \tilde{Q}, \quad (2) \]

the torsion tensor \( Q_{ij}^k \) being the asymmetrical part of the affine connection \( \Gamma_{ij}^k \) being related to the spin density tensor \( J^i_j \) by the Einstein-Cartan field equations

\[ Q_{ij}^k = (8\pi G_T/\hbar^2) [J_{ij}^k - (1/2) \delta_i^k J^j_l - (1/2) \delta_j^k J^i_l]. \quad (3) \]

Equation (2) shows a formal analogy to the interaction energy of a magnetic dipole moment in a constant magnetic field \( \vec{H} \), i.e.

\[ E = -\vec{\mu} \cdot \vec{H}. \quad (4) \]

For a number density \( N \) of aligned spins,

\[ \tilde{Q} = (4\pi G_T/\hbar^2) \tilde{S}N, \quad (5) \]

where \( G_T \) is the torsion coupling constant.

Eqs. (2) and (4) would suggest that analogous to eq. (1), the transition rate for flipping of neutrino helicities due to the spin-torsion interaction is given by

\[ \Gamma_r \approx \tilde{S} \cdot \tilde{Q}/\hbar, \quad (6) \]

\( \tilde{Q} \) being given by eq. (5).

As is well known [10–12] the torsionic contact interaction Lagrangian between two spin half particles is formally identical to the weak interaction Lagrangian and may be written in the \((V - A)\) form if at least one of the two fermions is massless. This suggests that the spin torsion coupling constant \( G_T \) be also identified with the weak interaction Fermi constant. This would give \( G_T/G_N \approx 10^31 \), \( (G_N \text{ being the Newtonian constant}) \).

If this value of \( G_T \) is used for the interaction of neutrinos with torsion and if one assumes alignment of neutron spins and magnetic moments as induced by the strong gravitational torsion interaction in a hot protoneutron star formed in the supernova [8, 9, 13], then we can estimate \( \Gamma_r \approx 10^{12} \text{ sec}^{-1} \), (i.e. using \( N \approx 10^{30} \) for the number
density of aligned neutrons in the neutron star with $10^{57}$ neutrons and radius 10 km in eq. (5) and using eqs. (5) and (6). This implies a transition time $t_r \approx 10^{-12}$ sec only, for the helicity flip to occur, which is much shorter than the diffusion time $t_D \approx 10$ sec for the neutrinos to emerge out of the neutron star. The mean free path of the neutrinos at core bounce $\approx 10^2$ cm, which implies that $t_r$ can be smaller than the typical collision times.

If we had used for the torsion coupling constant, the newtonian value $G_N$, then $t_r \approx 10^{20}$ sec, which is very long implying negligible effect of torsion.

In fact for a torsion coupling $G_T \approx 10^{14} G_N$, the neutrino diffusion and the spin-flip transition times become comparable. So if we identify $G_T$ with the Fermi constant $G_F$ (which seems to be appropriate for neutrino interactions as explained earlier), it would imply that the emitted neutrinos emerging out after diffusion through the star, would be a mixture of equal amounts of left and right handed components (as each neutrino would undergo several spin-flips during the diffusion time, $t_r \ll t_D$). This means that each flavour of neutrinos emitted would be effectively reduced by a factor of half as far as their detectability is concerned as only left helicity neutrinos can be detected.

Again it was shown [13, 14] that inside dense matter, torsion can induce flavour oscillations even for zero mass neutrinos. Torsion interaction can induce an energy splitting $\Delta E_T = E_{1T} - E_{2T} = (K_1 - K_2) \tilde{p} \cdot Q$, $Q = 4 \pi G_T \Sigma_i$, $\Sigma_i$ = number of aligned spins per unit volume, as before, and $\tilde{p} = \vec{p}/|\vec{p}|$. This gives an oscillation length

$$l_T = \frac{2\pi}{\Delta E_T} = \frac{c^2}{2G_T \delta \sigma} \delta^2,$$

and with $G_T$ as the Fermi constant, this gives $l_T \approx 10^{-4}$ cm, again implying an oscillation time scale significantly shorter than the diffusion time scale. This can occur even for zero mass neutrinos. For neutrinos with rest mass, the oscillation amplitude can be greatly enhanced in the presence of torsion by a factor $^{13} (l_M/l_T)^2$, where $l_M \sim 4\pi E_\nu / (m_1^2 - m_2^2)$.

However it would be difficult to observe any oscillation patterns from the supernova neutrinos. If $r$ is the neutrino source size ($r \approx 10$ km) and $L$ the distance to LMC, the necessary condition for observation of oscillation patterns leads to

$$4\pi E/L < \Delta m^2 < 4\pi E/r.$$  

However in a very hot dense medium, the emitted $\nu$ really has a wavepacket of width $d$ which is determined by collisions between neighbouring charged particles

$$d \sim (KT)^2/4\pi \epsilon \sim 10^{22} [T(\text{MeV})]^2/\epsilon \text{cm}.$$  

For above number density

$$d \approx 10^{-14} \text{ cm and } \Delta m^2 < 2E^2/d/L$$

(the $\nu_1, \nu_2$ components separate by $\Delta L = L\Delta \beta$ after traversing a distance $L$, where $\Delta \beta \approx \Delta m^2/2E_\nu^2$ and if $\Delta L > d$, the $\nu_1, \nu_2$ wave packet can no longer overlap and can't interfere to produce oscillations in $\nu_1, \nu_2$ i.e. coherence is maintained if $L < L_{\text{coh}} = d/\Delta \beta$ giving eq. (10)). This implies

$$\Delta m^2 < 5 \cdot 10^{-23} \text{ eV}^2,$$

whereas $\Delta m^2 \approx 10^{-4}$ eV$^2$ for the MSW effect.

It must be stressed that the torsion effects are operative only as long as the neutrinos move in the hot dense spin-aligned medium. Once the neutrinos emerge out of the star, the torsion effects are no longer present and the interaction falls off as $1/R^4$, with Newtonian $G_N$. The motion of the neutrinos is practically unaffected while moving through intergalactic space. The deviation from the light velocity is only one part in $10^9$, so that the neutrinos can be considered to move along geodesics. However there would
be a general relativistic time delay similar to that observed for photons in the solar system. The time delay in this case is given by the usual formula [15]

\[
\delta t_v \approx G_N M_G/c^2 \ln \left\{ \left( D_L + (D_L^2 + b^2)^{1/2} \right) \left( D_G + (D_G^2 + b^2)^{1/2} \right) \right\},
\]

(12)

\[
M_G = \text{galaxy mass} = 5 \cdot 10^{11} M_\odot \text{ inside 60 kpc} \left[ G_N M_G/c^2 \approx 5 \mu s \right],
\]

\[
b = \text{impact parameter} \approx D_G \approx \text{distance of sun from galaxy center} \sim 10 \text{ kpc},
\]

\[
D_L = \text{distance of LMC from galactic center} \approx 50 \text{ kpc}.
\]

Therefore the expected general relativistic time delay for the neutrinos is \( \delta t_v \approx 5 \cdot 10^6 \text{ sec} \), over a distance of 50 kpc (i.e. \( 6 \cdot 10^{12} \text{ sec} \)). The corrections due to torsion are smaller by a factor \( (G M_G/D_G c^2)^2 \approx 10^{-12} \) (outside the star) and can thus be neglected. In general there can be a post-Newtonian factor \((1 + \gamma)\) in eq. (12). If for instance one believe that the Mont Blanc neutrino signal was coincident with that from a gravitational wave (showing a signal with an amplitude of 135 K against a background noise of 29 K) [16] about 1.4 sec before the first neutrino and if neutrinos and gravitons have different \( \gamma \)'s (i.e. respond differently to the galaxy field) then one can put

\[
(\delta t_v - \delta t_g)/\delta t_v = (1/2)(\gamma_v - \gamma_g) \leq 1.4/(5 \cdot 10^6) \leq 2 \cdot 10^{-7},
\]

showing that the equivalence principle is same for neutrinos and gravitons to within a few parts in 10^7.

So the major time delay for neutrinos is inside the star, when it takes about ten seconds to diffuse out and this major effects is due to torsion inside the star.

References