SHOCK ACCELERATED PARTICLE ENERGIES IN TYPE-II
SUPERNOVAE AND COSMIC RAY CALCIUM ENHANCEMENT

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Abstract

There is an intensity enhancement in the cosmic ray spectrum in the energy range $10^{14}$-$10^{16}$ eV per nucleus and two independent observations imply a calcium overabundance in this range (Ca/Fe $> 2$) above $10^{14}$ eV per nucleus and perhaps 10). In a type-I supernova (SN-1), although the beta decay of $^{56}$Ni, provides reasonable explanation for the light curve, the predicted Ca/Fe ratio is less than a half. However, models of SN-2 involving 8-15 solar masses predict overabundance of Ca-Fe which can be as high as 10, for a 15 $M_\odot$ model. These models also predict large S/Fe and Ar/Fe ratios. Thus the anomalous Ca enhancement can be associated with SN-2 and the remnant neutron star. We consider the cumulative effect of acceleration from the neutron star as well as the shock acceleration in the supernova remnant in the pre Sedov and Sedov expansion phases. A relation is obtained for the limiting energy which depends on the charge of the particle, the magnetic field, the mean atomic weight of the medium and the SN-2 blast wave parameters. This energy for nuclei around the mass number of calcium is found to be in the range of $10^{15}$-$10^{16}$ eV per nucleus, strengthening the belief that SN-2 are the sources for these particles in cosmic rays.

1. Introduction and Source Anomalies

Intensity enhancement in the energy range of $10^{14}$ to $10^{16}$ eV per nucleus is seen in the cosmic ray spectrum and observations of heavy cosmic rays in this energy range appear to indicate a Ca/Fe ratio perhaps as large as ten times the solar value. Again the data of Burnett et al./1/, showed no anomalous helium or iron abundances but did show an overabundance of calcium at high energies (Ca/Fe $> 2$ above $10^{14}$ eV per nucleus). They also observed enhancements of medium heavy nuclei ranging from carbon to silicon. Observations from the HEAD 3 satellite experiment /2/ also show a steep rise in both the calcium and argon abundances at energies above 500 GeV AMU$^{-1}$. Thus two independent observations at least, imply a calcium overabundance in high energy cosmic rays. If as usually supposed such cosmic rays originate from supernovae then we have to consider which of the two broad types of supernovae can give rise to such anomalous abundances. The type-I (SN-1) indicate no evidence for hydrogen at maximal light output and are observed in all types of galaxies. This type of explosion arises in a mass-accreting carbon white dwarf where runaway carbon deflagration occurs leading to a complete disruption of the white dwarf and in the process producing a substantial ($\sim 1 M_\odot$) amount of $^{56}$Ni. The
resulting successive \( \beta \)-decays of the \( ^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe} \) provides a reasonable explanation for the changes in the SN-I light curve, the lifetime for the decay totalling about 80 days. However, the predicted Ca/Fe ratio in the synthesized elements of SN-I is less than half of the solar value. So the SN-I can be completely rules out as a possible source of the anomalous calcium in these cosmic rays. Type II supernovae (SN-II) involve massive stars with several solar masses as the progenitors showing hydrogen rich spectrum with various light curves, usually occurring only in spiral galaxies. In some nucleosynthesis calculations for a 15 M\(_{\odot}\) model progenitor for SN-II, the \( ^{40}\text{Ca}/\text{Fe} \) ratio can be expected to be as large as ten times the solar value since the explosion energy is so small that only the shell material is ejected. Also interesting to note that a rather high ratio (\( \sim 7 \)) of S/Fe and Ar/Fe relative to solar abundances are predicted. Again in some models involving 8-11 M\(_{\odot}\) stars, core collapse is triggered by electron capture on \( ^{20}\text{Ne} \) and \( ^{24}\text{Mg} \) and \( ^{48}\text{Ca} \) is a most abundant nucleus when nuclear statistical equilibrium is realized above \( 10^{10} \) g/cc. The shock wave generated when the nuclear matter density core bounces leaves behind a neutron star ejecting the overlying layers rich in calcium isotopes. However, for a more massive progenitor (\( \sim 25 M_{\odot} \)) the \( ^{40}\text{Ca}/\text{Fe} \) ratio is predicted to be near to solar value; the remnant core would be too massive to remain a neutron star and would collapse to a black hole. Thus, it is likely that SN-II with progenitor stars in the 10-15 M\(_{\odot}\) mass range, that leave behind a neutron star are the most probable sources of the high energy cosmic rays at energy range \( 10^{14}-10^{16} \) eV, exhibiting the peculiar calcium abundance anomaly. However, it is to be noted that in arriving at the progenitor masses, different workers assume different values of the primordial helium abundance getting different main-sequence masses for a given size of C-O core. The reason still heavier stars may not explode is that if the mass is less, the shock wave requires less energy to start with in order to emerge out and is more likely to produce an explosion. Again observations of the Crab Nebula imply that it is not a typical Type II remnant with the expansion velocities rather small (corresponding to an explosion energy an order of magnitude less than the typical value \( \sim 10^{51} \) ergs and O and Ca abundances close to solar rather than enhancement by factor of \( \sim 10 \).

2. Blast Wave Dynamics and Acceleration

If the supernova explosion occurs in variable density medium, the density variation assumed of the form: \( \rho = \rho_0 r^{-n} \), \( r \) is the radial co-ordinate, \( n \) is a number; then for an explosion which releases a total energy \( E \), the distance-time evolution of the blast wave is given by:

\[
R_s = \left( \frac{Et^2}{\rho_0} \right)^{1/(5-n)} = \left( \frac{E}{\rho_0} \right)^{1/(5-n)} t^2/(5-n).
\]

The shock velocity then evolves with time as:
\[ V_s(t) = V_{so} \left( \frac{t}{t_0} \right)^{(n-3)/(5-n)}. \]

These relations hold during the so called Sedov phase of the remnant. During the pre-Sedov phase, the free-expansion phase prevails and for a given magnetic field \( B_0 \) (present in the medium and assumed constant), the energy gained by a particle with charge \( Q = Ze \) (\( Z \) is atomic no.) is during this phase given by:

\[ W_0 \approx 0.75 Q B V_{so}^2 t_0, \]

if the shock speed remains constant at \( V_{so} \) between \( t=0 \) to \( t=t_{so} \), when the Sedov phase of the expansion commences. The time available for acceleration can be written as:

\[ t_{acon} \approx \frac{3}{4} \left( \frac{D_1}{V_1} + \frac{D_2}{V_2} \right) \]

where \( V_1, V_2 \) are velocities as measured in the shock rest frame. \( D_1, D_2 \) are diffusion coefficients. \( 1 \) and \( 2 \) refer to upstream/downstream respectively. Smallest \( D \) gives lower limit to timescale. This happens when the particle gyroradius is comparable to the scattering mfp and is given by:

\[ D = \frac{e}{m c} \frac{1}{\lambda}; \quad P \text{ and } V \text{ are the momentum and velocity of the particle. Again } D_2 \ll D_1 \text{ giving } t_{acon} \text{ defined by } (d(\ln p)/dt)^{-1} \text{ as: } t_{acon} \approx \frac{P}{(V_1, V_2, V QB)}. \]

The B.C.s behind the shock are:

\[ P_2 + \frac{4}{3} P_0 R_s^{-n}; \quad V_2 \approx \frac{2}{3} P_0 R_s^{-n} R_s^2, \quad \text{etc.} \]

At time \( t \), a particle which was initially (at \( t_0 \)) at energy \( E_0 \) and momentum \( P_0 \) would have reached the highest momentum \( P_2 \) given as:

\[ \int_{P_0}^{P_2} V \, dP \leq 0.75 Q \int_{t_0}^{t} B_0 V_2 \, dt. \]

Substituting for \( V_2 \) from above we integrate from \( t=t_0 \), when the supernova enters the Sedov phase to some time \( t \). We get for the maximal partial energy at time \( t \):

\[ W < W_0 + 3.75 Q B_0 V_{so}^2 \left[ 1 - \left( \frac{t}{t_0} \right)^{(n-1)/(5-n)} \right] \]

\( W_0 \) is the energy gained in the pre-Sedov free expansion phase.

As we can calculate \( V_1(t) \) and express \( t_0 \) in terms of \( E \) and \( n \) we can express the maximum energy in terms of the explosion energy \( E \) and the density of the ambient medium.

We notice from the formula for \( W \), that at \( t \gg t_0 \), it tends to a maximal limit (provided \( n \geq 2 \)), giving the maximum energy as (substituting for \( W_0 \)):

\[ W_{\text{max}} \leq 4.5 Q B_0 V_{so}^2 t_0, \]

or in terms of \( E \) and the parameters of the ambient medium (assumed to consist of a number density \( N_0 \) of atoms with mean atomic weight \( \mu \) :

\[ W_{\text{max}} \approx \text{Const.} B_0 E^{-1/(3-n)} V_{so}^{1/(3-n)} N_0^{(-1/3+n)} \mu^{-1/3}. \]

For the case of uniform density for the external medium \( N = N_0 \), \( n = 0 \):

\[ W_{\text{max}} \approx \text{Const.} B_0 E^{1/3} V_{so}^{1/3} N_0^{-1/3} \mu^{-1/3}. \]
For a typical SN-II explosion energy $E \approx 10^{51}$ ergs,

$$B_0 \approx 10^{-6} \text{ G, } N_0 \approx 1 \text{ cm}^{-3}, \mu \approx 2, V_{so} \approx 10^{10} \text{ cm s}^{-1},$$

$W_{max} \approx 10^{15}-10^{16}$ eV. Thus, the Ca nuclei which are overabundant by a factor of 10 in the SN-II ejecta can be accelerated to a maximal energy of $\sim 10^{16}$ eV, thus accounting both for the energy range and the calcium anomaly, seen in the observed high energy cosmic ray spectrum. For acceleration from the surface of the neutron star, we have the maximum pot. diff. traversed by a particle emitted at surface to $\infty$ as:

$$V \sim B_s \propto R_s^2 c^2. B_s \approx 10^{12} \text{ G at surface } R_s \approx 10^6 \text{ cm. } \alpha \text{ is angle.}$$

Thermionic current from polar cap can be then estimated from a Richardson-like equation giving for the final value of the energy of particle with $(A Z)$ at the outer boundary

$$W = \int E_{\nu} \cdot dl \cdot Z_\nu \approx 10^{17} \text{ eV. const x } (A^{4/3} Z^{2/3}).$$

This would give too high a value for the Ca nuclei. However, the surface composition of the neutron star would be expected to be predominantly iron, the calcium being ejected out from the envelope (rich in Ca, Ar, Si, etc.) in the SN-II explosion. Thus the calcium nuclei would be accelerated in the shock associated with the Sedov and pre-Sedov phases of the blast wave, for which the above calculation gave $10^{15}-10^{16}$ eV. The cosmic ray Fe energy of $\sim 10^{18}$ eV/nucleus may be explainable as arising from surface acceleration from neutron star.

References