Strong gravity, black holes, and hadrons

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Analogies between the properties of black holes (in the framework of strong gravity) and those of elementary particles are discussed especially in connection with recent works on black holes with gauge charges and black-hole thermodynamics.

Recent renewal of interest in general relativity playing some role in elementary-particle physics has drawn attention to certain striking resemblances between the properties of black holes and those of elementary particles. For instance, black holes are characterized by only a few observable parameters such as mass, angular momentum, and charge. These are the measurable parameters for an elementary particle where these quantities occur in discrete or quantized units. One could picture particles as quantum black holes of the strong gravity field (i.e., mediated by massive spin-2) particles as was done in Refs. 2, 3, 6, 7, and 8. Moreover, it is known that a charged rotating black hole, a Kerr-Newman black hole, has a gyromagnetic ratio of 2, the same value as that for an elementary particle. Although they have a magnetic moment, charged black holes with angular momentum do not have an electric dipole moment, which is also true for elementary particles. In a recent paper, Tennakone has pictured the proton to be a black-hole singularity of the Reissner-Nordström metric in the strong gravitational field, assuming that the usual results of general relativity are applicable in the case of strong gravity.

If the structure of space-time in the immediate vicinity of hadrons is presumed to be determined by strong gravity, it is natural to replace the Newtonian constant \( G_N \) by the strong gravitational coupling constant \( G_f \), the dimensionless constant then being of the same magnitude as the strong-interaction dimensionless constant. Now the Einstein field equations \( G_{\mu\nu} = \kappa T_{\mu\nu} \) relate a geometrical invariant quantity (i.e., the Einstein tensor \( G_{\mu\nu} \)) on the left-hand side to an invariant physical quantity (i.e., the conserved energy-momentum tensor \( T_{\mu\nu} \)) on the right-hand side through a proportionality (coupling) constant \( \kappa \). It must be emphasized that the derivation of the equation places no restriction whatsoever on the numerical value of the constant \( \kappa \). For instance, in the standard derivation of the field equation from an action principle with the Lagrangian density \( \mathcal{L} = \kappa^4 R^{\mu\nu} g_{\mu\nu} + \mathcal{L}_m \) (where \( R \) is the curvature scalar and \( g = \text{det}(g_{\mu\nu}) \), \( \mathcal{L}_m = \text{matter Lagrangian density} \), \( \kappa \) is a factor of dimensions \( g^2 \text{ cm}^{-2} \text{ sec}^2 \) whose numerical value is entirely undetermined at this stage. It is only when one uses the field equations as the basis for a relativistic theory of gravitation that one relates to the Newtonian constant \( G_N \). Since Einstein used his field equations to describe a theory of gravitation, he chose \( \kappa = 8\pi G_N / c^4 \), so as to be consistent with Newtonian gravitation theory. Following this it has become customary to always relate \( \kappa \) to the Newtonian constant as a matter of habit since all applications of general relativity have hitherto been to macrophysics.

Apart from this there is no other compelling reason why the coupling parameter \( \kappa \) should be related only to the Newtonian constant. The existence of massive spin-2 meson states (like the \( f \) meson) in nature does suggest the possibility of a short-range strong gravitational interaction which would determine the metrical properties of space-time in the region near an elementary particle. This field would then also be described by an Einstein-type field equation (since by now it is well known that starting from a linear spin-2 relativistic theory and successively adding all self-interactions in a consistent way one does recover field equation of the Einstein type).

With a new metric \( f_{\mu\nu} \) and a strong coupling constant \( \kappa_f = 8\pi G_f / c^4 \) to make it consistent with strong-interaction physics, one would now have a Lagrangian density

\[
\mathcal{L} = \frac{1}{\kappa_f} \sqrt{-f} R(f) + \frac{1}{\kappa_g} \sqrt{-g} R(g) + \mathcal{L}(fg),
\]

where \( R(f) \) is the curvature scalar constructed from \( f_{\mu\nu} \) and its derivatives and \( \mathcal{L}(fg) \) describes interaction between \( f \)-mesons and gravitons. If one drops this term, and considers that \( \kappa_f / \kappa_g \ll 1 \), the only dominant term is \( \kappa_f \sqrt{-f} R(f) \) which leads to an Einstein-type equation for the \( f_{\mu\nu} \) field with a constant \( \kappa_f \). This merely suggests that strong interactions curve the space much more strongly than in the usual Newtonian case, an idea also expressed and justified (independently of strong gravity) in Refs. 9 and 10. Again, as we are interested in the strong gravity field in the immedi-
ate neighborhood of hadronic matter, any exponent-
ial factors of the form $e^{-m^2}$ occurring in the met-
ric components due to the finite range of the strong
gravity can be neglected. This is also done in Ref.
8. Thus we see that the solutions of the new field
equations after these considerations will be the
same as those of the usual Einstein equations, or
in any case are very likely to be a good approxi-
mation to them. The reasonable results obtained
by the author of Ref. 8 as well as the results of this
paper seem to support this.

However, the metric used by Tennakone does
not enable us to incorporate the angular momen-
tum or spin of the particle. For this we have to
invoke the Kerr metric in the context of strong
gravity; that is, space-time in the vicinity of had-
ronic matter is assumed to be dominated by a
strong gravitational field with a coupling con-
stant $G_p = 6.7 \times 10^{30}$ cgs units. In earlier
papers, we had indicated that, by using quanti-
tized values for the angular momentum $J$ occur-
rning in the Kerr metric, a mass formula for the
hadronic resonance states is obtained as

$$ (G_p/k) m_J^2 = Jc. \tag{1} $$

It is worth noting that Eq. (1) implies a quadratic
dependence of angular momentum on the resonance
mass $m_J$; i.e., a plot of $J$ versus $m_J^2$ would appear as
straight lines with a universal slope given by

$$ S = (G_p/kc) = (1 \text{ GeV})^2. \tag{2} $$

Now, it is a generally accepted experimental feature of
hadron spectroscopy that all hadron reso-
nances lie on rising Regge trajectories, i.e., appear in a
Chew-Frautschi plot as straight lines, the angular momentum being linear in the mass
squared, with a universal slope $\alpha = (1 \text{ GeV})^2$.

Equations (1) and (2) automatically imply a linear
relationship between $J$ and $m_J^2$ (the usual Regge
theory makes no clear-cut prediction on this de-
pendence) with a slope of the right order of mag-
nitude. We can also consider mass formulas re-
sulting from the charged Kerr metric, i.e., the
Kerr-Newman metric which is an exact solution of the
Einstein-Maxwell equations. It seems plausible
as pointed out by Salam$^{11}$ that if internal symme-
tries are incorporated into the structure of Ein-
stein’s field equations then in the corresponding charged Kerr-type solution, the charge $Q^2$ would
be replaced by $I(J + 1)$, where $I$ is the isotropic
spin, and $J^2$ would be replaced by $J(J + 1)$. In ef-
tect this would give a mass formula of the type

$$ m_J^2 b = \left( \frac{I(J + 1)}{m_J^2 b} + I(J + 1) \right), \tag{3} $$

where $a$ is a constant involving $G_p$, $k$, and $c$. This is a typical SU(4)-type formula for a Regge ha-
dronic trajectory. A Kerr-Newman-type metric is
thus seen to provide hadronic mass formulas in-
volving SU(6)-type combinations such as $I(J + 1)$ and
$J(J + 1)$. That an equation like (3) gave reasonable
numerical agreement with actual particle masses
when using the strong-gravity coupling constant $G_p$
was shown in Refs. 3, 6. It is of interest to note
that Eq. (3) also arises by imposing the condition
that a Kerr-Newman black hole be extremely
charged and rotating.

Further, it is interesting to note that the Christo-
doulou-Ruffini mass formula$^{12}$ for charged rotating
black holes (the formula holding for any general
black hole, not merely extreme ones), i.e.,

$$ m^2 = \left( \frac{m_{1/4}}{4m_{1/4}^2} + \frac{Q^2}{4 m_{1/4}^2} \right)^2 + \frac{J^2}{4 m_{1/4}^2} \tag{4} $$

(where $m_{1/4}$ is the irreducible mass which does not
decrease in black-hole interactions), strongly re-
sembles hadronic mass formulas involving SU(6)
combinations like Eq. (3), if $Q^2$ and $J^2$ are replaced
by $I(J + 1)$ and $J(J + 1)$. However, in order to have a
complete SU(6) formula, we need an additional
parameter to identify with the hypercharge $Y$,
which does not occur in Eqs. (3) and (4). This
could possibly be done through a recent work of
Bekenstein,$^{13}$ who has shown through a new solu-
tion that an additional scalar charge could also be an
observable parameter of a black hole in addi-
tion to mass, angular momentum, and electric
charge. This could then be identified with the hy-
percharge. We would then have a complete SU(6)-
type formula. In this connection, it is of interest
to note that recently$^{14}$ solutions were constructed
for the coupled Einstein–Yang–Mills field equations
which describe the exterior of a rotating black hole
having gauge charges such as isospin and hyper-
charge. These solutions for the Einstein–Yang–
Mills equations are the analog of the Kerr-Newman
solution for the Einstein–Maxwell equations and
now correspond to spinning black holes with gauge
charges instead of or in addition to electric charge,
the equation of the event horizon being written as

$$ \Delta = r^2 - 2Mr + a^2 + \gamma_{sp} Q^2 Q^2, \tag{5} $$

where the constants $M$, $a$, $Q^p$ are interpreted as the
mass, angular momentum per unit mass, and
gauge charges, respectively. The mass formula
for these black holes will now resemble Eq. (4)
with appropriate additional terms for the gauge
charges $Q^p$. The singularity structure of the new
solutions will be the same as that for the Kerr-
Newman solution with the same inequalities be-
 tween $M$, $a$, and $(Q^p)^2 = \gamma_{sp} Q^2 Q^2$ to prevent or per-
mit naked singularities. The gauge fields consid-
ered can belong to any Lie group which has an in-
variant group metric $\gamma_{sp}$, i.e., gauge groups usu-
ally considered by particle physicists. These
gauge groups include the vector mesons mediating
strong and other interactions, the gauge charges
then being the conserved quantities like isospin
and hypercharge derived by applying Noether’s
theorem to the appropriate gauge symmetries.
Moreover, the generalization of the Higgs mecha-
nism to curved space; i.e., solutions of the Ein-
stein-Higgs equations can generate black-hole so-
lutions characterized by massive gauge fields.
Such solutions are not yet known but one would rea-
sonably expect them to exist. The Israel-Carter
uniqueness conjecture\textsuperscript{12} can be extended to these
new black-hole solutions and it leads to the identi-
ﬁcation of various gauge charges as distinct ob-
servable parameters of the black holes. For the
Kerr-Newman solution Wald\textsuperscript{9} has shown that the
inequality $m^2 \geq a^2 + Q^2$ is always maintained, if it
is satisﬁed initially. This would also hold for the
new black-hole solutions. We thus see that with
quantized values for $a$, $Q^2$, etc., and in the con-
text of strong gravity, the mass formulas for these
black holes [Eqs. (3) and (4)] will really look like
the SU(6) mass formulas for hadrons and will give
reasonable numerical values with the strong grav-
itational constant. Of course, we note that we end
up with squared-mass formulas for all hadrons,
i.e., for both mesons and baryons, whereas the
usual Gell-Mann-Okubo (GMO) mass formula is
quadratic for mesons and linear in the mass for
baryons. Whether we should use the linear mass
or (mass)$^2$ has been an unsettled question since
then. Although the linear mass was used for the
original GMO mass formula for baryons the latest
indications\textsuperscript{14} are that the (mass)$^2$ formula is
numerically more accurate even for the baryonic
case. It must be remarked that the unitary sym-
metries [SU(3) or SU(6)] used for the classiﬁcation
of strongly interacting particles are approximate,
their experimental success being attributed to the
rather phenomenological introduction of symmet-
try-breaking terms (which give additional degrees
of freedom such as $I$, $Y$, etc.) as a perturbation on
the mass operator; i.e., we have a theory with a
ﬂat-space invariant $P_\mu P^\mu$ (Poincaré invariant or
mass operator) together with an interaction which
gives a nondegenerate mass spectrum, i.e., mass
splitting.\textsuperscript{9} An “elementary particle” without
intrinsic degrees of freedom such as isospin, hyper-
charge, etc., can be described by a single value
for its mass (i.e., a degenerate state) given by the
invariant mass operator $P_\mu P^\mu = m^2$, i.e., charac-
terized by an irreducible representation of the
Poincaré group as its symmetry group. Symmetry-
breaking interaction terms would now perturb this
ﬂat-space mass operator giving rise to a mass
spectrum (nondegenerate state) describing elemen-
tary particles with intrinsic degrees of freedom.
The theory in ﬂat space with an interaction can be
reduced to an equivalent description of a group of
motions in curved space; i.e., it can be related to
geometry completely in the spirit of general rela-
tivity where the curvature of space-time is caused
by the presence of energy-momentum tensors gen-
erated by various interactions. Using a curved-
space group (such as the de Sitter group) instead
of the ﬂat-space Poincaré group one automatically
gets a “generalized” mass operator $P_\mu P^\mu$ which
has additional terms\textsuperscript{9,10} giving rise to a mass spec-
trum, i.e., $P_\mu P^\mu$ is no longer a Poincaré invariant,
but terms like $P_\mu P^\mu + J(J+1) + \cdots$ are now invari-
ants of this curved-space group. This seems more
natural than introducing symmetry-breaking terms.
An elementary particle with intrinsic degrees of
freedom would now be described by representations
of the curved-space group (say de Sitter group) as
the symmetry group and would go by contraction\textsuperscript{17}
into an elementary particle with a single degenerate
mass state given by the usual Poincaré invariant.
The use of curved-space groups of motion would be
necessitated\textsuperscript{18} by the presence of the strong
gravitational interaction in the immediate vicinity
of hadronic matter. This is one way of understand-
ing the above results. The irreducible mass in (4)
could be regarded as the rest mass of the degener-
ate ground state (say proton for the baryon spec-
trum).
We would now mention a few more amusing anal-
ogies between the properties of black holes and
those of hadrons. While considering the collision
of two black holes, we have a theorem\textsuperscript{19} due to
Hawking which states that in the interactions in-
volving black holes the total surface area of the
boundaries of the black holes can never decrease.
In stationary processes they remain unchanged at
best. This seems consistent with the observed in-
crease in cross sections in hadron collisions, had-
rons here being assumed to be quantum black holes
of the strong gravity ﬁeld. Further, there are
elegant analogies between the laws of black-hole
dynamics and the laws of ordinary thermodynamics.
One remarkable conclusion arising from this ana-
logy is that a black hole radiates like a black body
whose temperature is inversely proportional to its
mass.\textsuperscript{18,20} We would, therefore, associate with a
black hole of mass $M$ a temperature given by
\begin{equation}
T_b = \frac{M c^3}{8 \pi GM k_B},
\end{equation}
where $k_B$ is the Boltzmann constant. That a black
hole should be assigned a temperature (proportion-
al to its surface gravity) as given by Eq. (6) was
first suggested by Bekenstein\textsuperscript{20} on the basis of anal-
gogies between thermodynamic quantities and the
parameters of black-hole dynamics. Direct calculations by Hawking\(^1\) showed that a black hole does radiate like a body with temperature given by Eq. (6), in agreement with the temperature defined by Bekenstein. The temperature is seen to depend (inversely) only on the black-hole mass and on no other parameter. The smallest black-hole mass will therefore give rise to the highest temperature. In the case of strong gravity, \(G\) in Eq. (6) would be replaced by the strong gravitational constant \(G_f\) and \(M\) would correspond to the mass of a hadron, hadrons being considered as black or "grey" holes of the strong gravity field. Now the hadron with the smallest mass is the (spin-0) pion having \(m_\pi = 140\ MeV\). Equation (6) then gives (with \(M = m_\pi, \ G = G_f\))

\[
T_{\text{br}} = \frac{\hbar c^3}{8\pi G_f m_\pi^2} = 2 \times 10^{12}\ K. \tag{7}
\]

This would be the upper limit for the temperature of hadrons or hadronic matter. It is remarkable that this upper limit for the temperature is the same as the limiting temperature arising in thermodynamic bootstrap models of hadrons (we also recall in this connection Fermi's blackbody radiation model for hadronic matter) such as the Hagedorn model, where usually \(k_B T_{\text{max}} \sim m_\pi c^2 \sim 2 \times 10^{12}\ K\). There is a measure of experimental support for the existence of such a temperature as manifested by a cutoff in the transverse momenta of colliding hadrons in high-energy experiments. The existence of an upper limit for the temperature of hadrons can be conceived of as the fourth law of thermodynamics.\(^{21}\) Equation (7) giving the upper limit for the temperature of black holes would then be a statement of the fourth law of black-hole dynamics, the other three laws being already known.\(^{22}\) Many analogies, therefore, exist between the behavior of black holes and those of elementary particles.

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10. A. Bohm, Phys. Rev. 175, 1767 (1968).